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Class: XII
Subject: Math's
Topic: Differentiability and Differentiation
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1) If $y = x^5 \cot x$, then dy/dx is equal to

(A) $x^4 [-x \operatorname{cosec}^2 x + 5 \cot x]$
(B) $x^4 [-x \operatorname{cosec}^2 x - 5 \cot x]$
(C) $x^4 [x \operatorname{cosec}^2 x + 5 \cot x]$
(D) $x^4 [x \operatorname{cosec}^2 x - 5 \cot x]$

Sol. A

Right Answer Explanation:
Apply product rule and derivative of $\cot x = -\operatorname{cosec}^2 x$.

2) The derivative of $f(x) = 1 + x + x^2 + x^3 + \ldots. + x^{100}$ at $x = 1$ is

(A) 5000
(B) 5050
(C) 5150
(D) 5005

Sol. B

Right Answer Explanation:
$dx^n / dx = nx^{n-1}$
Above expression becomes $0 + 1 + 2x + 3x^2 + 4x^3 \ldots\ldots + 100x^{99}$

3) If $f(x) = (x - 2)^2 (2x - 3)$, then find the value of $f'(1)$.

(A) 4
(B) 1
(C) 2
(D) 3

Sol. A

Right Answer Explanation: find $f'(x)$ and put $x = 1$

4) If $f(X) = x^n + a x^{n-1} + a^2 x^{n-2} + \dots + a^{n-1} x + a^n$ for some fixed real number, then f'(x) is equal to

(A) a [n (n + 1)/2]
(B) a [n^2 (n + 1)/2]
(C) a^{n-1}[n (n + 1)]
(D) a^{n-1}[n (n + 1)/2]

Sol. D

Right Answer Explanation:
Find f'(a) and sum of natural numbers = n(n + 1)/2.

5) If y = (x + 1) (x + 2) (x + 3), then dy/dx is equal to

(A) $3x^3 + 12x^2 + 11x$
(B) $3x^2 + 12x$
(C) $3x^2 + 12x + 11$
(D) $x^2 + 12x + 11$

Sol. D

Right Answer Explanation:
Use: d{ f(x).g(x).h(x)}/d(x) = {d((f(x))/d(x)} g(x). h(x) + f(x){d((g(x))/d(x)} h(x) + f(x)g(x). {d((h(x))/d(x)}

6) If f(x) = x sin x, then $f'\left(\frac{\pi}{2}\right)$ is equal to

(A) 0
(B) 1
(C) -1
(D) $\frac{1}{2}$

Sol. B

Right Answer Explanation:
(2) is the correct answer. As f'(x) = x cos x + sin x

So, $f'\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \cos \frac{\pi}{2} + \sin \frac{\pi}{2} = 1$

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7) What will be the derivative of (x sin x + cos x) (x cos x – sin x)?

(A) $(x^2 \cos 2x + x \sin 2x)$
(B) $(x \cos x - x \sin 2x)$
(C) $(x^2 \cos 2x - x \sin 2x)$
(D) $(-x^2 \cos 2x - x \sin 2x)$

Sol. C

Right Answer Explanation:
use product rule

8) If $y = a e^{mx} + b e^{-mx}$, y_2 is equal to

(A) $-e^{m2\,y}$
(B) $m^2 y$
(C) my_1
(D) none of these

Sol. B

Right Answer Explanation:

$y_1 = am e^{mx} - bm e^{-mx}$

$y_2 = am^2 e^{mx} + bm^2 e^{-mx}$

$= m^2(a e^{mx} + b e^{-mx})$

$= m^2 y$

Hence, 2 is the right answer

9) If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots + \text{to } \infty}}}$, find the value of $\frac{dy}{dx}$.

(A) $\frac{x}{y+1}$
(B) $\sqrt{\frac{x}{y+1}}$
(C) $\frac{1}{2y-1}$
(D) $\frac{1}{2y+1}$

Sol. C

Right Answer Explanation:
$y = \sqrt{(x + y)}$
On squaring both the terms, we gety2 = (x + y)On differentiating/dx = 1/(2y - 1)
Hence, 3 is the right answer.

10) If $x = a\cos^4\theta$ and $y = a\sin^4\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{3\pi}{4}$ is

(A) -1
(B) 1
(C) $-a^2$
(D) a^2

Sol. A

Right Answer Explanation:

$\frac{dx}{d\theta} = 4x\cos^3\theta\ (-\sin\theta)$

$\frac{dy}{d\theta} = 4a\sin^3\theta\ \cos\theta$

$\therefore \frac{dy}{dx} = -\frac{\sin^2\theta}{\cos^2\theta}$

At $\theta = \frac{3\pi}{4}, \frac{dy}{dx} = -\frac{\sin^2\frac{3\pi}{4}}{\cos^2\frac{3\pi}{4}}$
$= -1$

11) If $x = \frac{2t}{1+t^2}$, $y = \frac{1-t^2}{1+t^2}$, then $\frac{dy}{dx}$ equals

(A) $\frac{2t}{t^2+1}$
(B) $\frac{2t}{t^2-1}$
(C) $\frac{2t}{1-t^2}$
(D) None of these

Sol. B

Right Answer Explanation:

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$\frac{dx}{dt} = \frac{(1+t^2)2 - 2t.2t}{(1+t^2)^2} = \frac{2-2t^2}{(1+t^2)^2}$

$\frac{dy}{dt} = \frac{(1+t^2)(-2t)-(1-t^2)2t}{(1+t^2)^2}$

$= \frac{-4t}{(1+t^2)^2}$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-4t}{2-2t^2} = \frac{2t}{t^2-1}$

12) Let $y = e^{2x}$. Then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is

(A) 1
(B) e^{-2x}
(C) $2\ e^{-2x}$
(D) $-2e^{-2x}$

Sol. D

Right Answer Explanation:

$$\frac{dy}{dx} = 2e^{2x} \text{ and } \frac{d^2y}{dx^2} = 4e^{2x}. \text{ Also } x = \frac{1}{2}\log y,$$

so $\frac{dx}{dy} = \frac{1}{2y}$ and $\frac{d^2x}{dy^2} = -\frac{1}{2y^2} = -\frac{1}{2}e^{-4x}$.

Hence $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right) = -2e^{-2x}$.

13) If x = 2 sin t – sin 2t, y = 2 cos t – cos 2t, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$ is

(A) 2
(B) -1/2
(C) -3/4
(D) -3/2

Sol. B

Right Answer Explanation:

$$\frac{dx}{dt} = 2\cos t - 2\cos 2t = 2\,[2\sin(3t/2)\sin t/2]$$

and $\frac{dy}{dt} = -2\sin t + 2\sin 2t = 2\left[\sin\frac{3t}{2}\cos\frac{t}{2}\right]$.

Hence $\frac{dy}{dx} = \cot(t/2)$ and $\left(\frac{d^2y}{dx^2}\right) = -\frac{1}{2}\ \text{cosec}^2(t/2)$

$\times \frac{dt}{dx}$

Therefore, $\frac{d^2y}{dx^2} = -\frac{1}{2}\text{cosec}^2(t/2)x \times \frac{1}{4\sin(3t/2)\sin(t/2)}$

and $\left.\frac{d^2y}{dx^2}\right|_{t=\pi/2} = \left(-\frac{1}{2}\right) \times (\sqrt{2})^2 \times \frac{1}{4 \times (1/2)} = -\frac{1}{2}$.

14) If (sin x) (cos y) = 1/2 then d^2y/dx^2 at (π/4, π/4) equal to

(A) -4
(B) -2
(C) -6
(D) 0

Sol. A

Right Answer Explanation:

Differentiating the given equation, we have cos *x* cos *y* – sin *x* sin *y* d*y*/d*x* = 0. Putting *x* = *y* = π/4,

we have $\left.\frac{dy}{dx}\right|_{(\pi/4,\,\pi/4)} = 1$. Differentiating again, we get $-\sin x\cos y - \cos x\sin y\,dy/dx - \cos x\sin y\,dy/dx - \sin x\sin y\,(dy/dx)^2 - \sin x\sin y\,d^2y/dx^2 = 0$. Putting $x = y = \pi/4$, we have

$$\left.\frac{d^2y}{dx^2}\right|_{(\pi/4,\pi/4)} = -4.$$

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15) Suppose for a differentiable function f, f (0) = 0, f (1) = 1 and f′(0) = 4 = f′(1). If g (x) = f (e^x) $e^{f(x)}$, then g’ (0) is equal to

(A) 4
(B) 8
(C) 2
(D) None of these

Sol. B

Right Answer Explanation:

$$g'(x) = f'(e^x)\, e^x\, e^{f(x)} + f(e^x)\, e^{f(x)} f'(x),$$

$$\text{so} \quad g'(0) = f'(1)\, e^{f(0)} + f(1)\, e^{f(0)} f'(0) = f'(1) + f'(0) = 8$$

16) $\frac{d}{dx}\left(\log\,(ax)^x\right)$, where *a* is a constant is equal to

(A) 1
(B) log ax
(C) 1/a
(D) log (ax) + 1

Sol. D

Right Answer Explanation:

$$\frac{d}{dx}\left(\log\,(ax)^x\right) = \frac{d}{dx}\left(x \log ax\right) = \log ax + 1.$$

17) If $x^2 + y^2 = t + \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ (xy > 0) then $\frac{d^2y}{dx^2}$ is equals

(A) $1/x^3$
(B) $2/x^3$
(C) $2\sqrt{2}/x^3$
(D) $2\sqrt{2}/x^4$

Sol. B

Right Answer Explanation:

$(x^2+y^2)^2 = \left(t+\frac{1}{t}\right)^2 = t^2 + \frac{1}{t^2} + 2 = x^4 + y^4 + 2$

$\Rightarrow \quad 2x^2y^2 = 2 \Rightarrow x^2y^2 = 1 \Rightarrow xy = 1$

$\Rightarrow \quad x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} = -\frac{1}{x^2}$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{2}{x^3}.$

18) Let f and g be differentiable function satisfying g'(a) = 2, g(a) = b and $g = f^{-1}$ then f' (b) is equal to

(A) 1/2
(B) 2
(C) 2/3
(D) None of these

Sol. A

Right Answer Explanation:

Since $g = f^{-1}$ so $f \circ g(x) = x$.

Therefore, $f'(g(x))\, g'(x) = 1$. Thus, $f'(g(a))\, g'(a) = 1$

$\Rightarrow \quad f'(b)2 = 1 \quad \Rightarrow \quad f'(b) = 1/2.$

19) If $y = \log_e x\,(x-2)^2$ for $x \neq 0, 2$, then y' (3) is equal to

(A) 1/3
(B) 2/3
(C) 4/3
(D) None of these

Sol. B

Right Answer Explanation:

$y = \frac{2}{x}\log(x-2),\ y'(x)$

$= 2\left[\frac{1}{x(x-2)} - \frac{1}{x^2}\log(x-2)\right],\ y'(3) = \frac{2}{3}.$

20) Let y = $\frac{1}{3}\log\frac{x+1}{\sqrt{x^2-x+1}}+\frac{1}{\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}}$

Statement – 1: $\frac{dy}{dx}$ at x = 0 is 1

Statement – 2: $\frac{dy}{dx}=\frac{2}{x^3+2}$

(A) Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
(B) Statement – 1 is true, statement – 2 is true but statement – 2 is not the correct explanation for statement – 1.
(C) Statement – 1 is true, statement – 2 is false.
(D) Statement – 1 is false, statement – 2 is true.

Sol. C

Right Answer Explanation:

$$\frac{dy}{dx}=\frac{1}{3}\left[\frac{1}{x+1}\times\frac{1}{2}\frac{3-3x}{x^2-x+1}\right]$$

$$+\frac{1}{\sqrt{3}}\frac{2/\sqrt{3}}{1+\left(\frac{2x-1}{\sqrt{3}}\right)^2}=\frac{1}{x^3+1}$$