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Class: XII
Subject: Mathematics
Topic: Indefinite Integration
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. $\int e^x \frac{1-\cot x}{\sin x}\, dx =$

A. $e^x \cot x$
B. $e^x cosec\ x$
C. $-e^x\ cosec\ x$
D. $-e^x \cot x$

Ans. B

Solution:

$$\frac{1-\cot x}{\sin x} = \frac{1}{\sin x} - \cot x.\frac{1}{\sin x} = \operatorname{cosec} x - \operatorname{cosec} x \cot x$$

$$= f(x) + f'(x) \quad where \quad f(x) = \operatorname{cosec} x$$

$$\therefore \int e^x [f(x) + f'(x)]\, dx \Rightarrow = e^x f(x) = e^x \operatorname{cosec} x + c,$$

2. $\int \frac{dx}{1+e^{nx}} =$

A. $\frac{1}{m}\log(1 + e^{nx})$
B. $x - \frac{1}{m}\log(1 + e^{nx})$
C. $\log(1 + e^{-mx})$
D. $x + \frac{1}{m}\log(1 + e^{nx})$

Ans.B

Solution:

$$\int \frac{dx}{1 + e^{nx}} = \int \frac{1 + e^{mx} - e^{mx}}{1 + e^{mx}} dx = x - \frac{1}{m}\log\left|1 + e^{mx}\right| + c$$

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3. $\int \frac{1}{1-\cos x} dx =$

   A. $\cos x + \cot x$
   B. $-\cot\frac{x}{2}$
   C. $\tan\frac{x}{2}$
   D. $cosec\, x - \cot x$

   Ans. B

   Solution:

   $$\int \frac{1}{1-\cos x} dx = \int \frac{dx}{2\sin^2 x/2}$$
   $$= -\cot\frac{x}{2} + c$$

4. $\int \frac{\sin(2\log x)}{x} dx =$

   A. $-\cos(\log x)$
   B. $\frac{1}{2}\cos(\log x)$
   C. $-2\cos(2\log x)$
   D. None of these

   Ans. D

   Solution:

   2logx = v
   Then (2/x)dx = dv
   $$\int \frac{\sin(2\log x)}{x} dx = \int \frac{\sin v}{2} dv$$
   $$= -\frac{1}{2}\cos(2\log x) + c$$

5. If f(x) = $\int_1^x \frac{dt}{t}$, then f(x) satisfies

   A. $f(xy) = f(x) + f(y)$
   B. $f\left(\frac{x}{y}\right) = f(x) + f(y)$
   C. $f(x+y) = f(x) + f(y)$
   D. $f(x+y) = f(x) + f(y)$
   E. None of these

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Ans. A

Solution:

Put v= log t

Then dt/t=dv

Then f(xy)=f(x)+f(y)

6. $\int \frac{\sin^6 x}{\cos^8 x} dx =$

A. $\frac{\cot^7 x}{7}$
B. $\frac{1}{\cot^7 x}$
C. $\frac{7}{\cot^7 x}$
D. $\frac{\tan^7 x}{7} + c$

Ans. D

Solution:

$$\int \frac{\sin^6 x}{\cos^8 x} dx = \int \tan^6 x \sec^2 x dx = \frac{\tan^7 x}{7} + c$$

7. $\int (e^{a \log x} + e^{x \log a})$ dx, a > 1, is

A. $\frac{1}{a} e^{a \log x} + \frac{1}{\log a} e^x \log a$
B. $\frac{x^a+1}{a+1} + \frac{a^x}{\log a}$
C. $ax^{a-1} + a^x \log a$
D. none of these

Ans. B

Solution:

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$\int(e^{a\log x} + e^{x\log a})\,dx,\ a > 1$

$= \int[x^a + a^x]dx$

$= \dfrac{x^{a+1}}{a+1} + \dfrac{a^x}{\log a} + c$

8. $\int \frac{x^2\,dx}{4+x^6} =$

A. $\frac{1}{3}\sin -1\frac{x^3}{2}$
B. $\sqrt[2]{4+x^6}$
C. $\frac{1}{6}\sinh -1\left(\frac{x^3}{2}\right)$
D. None of these

Ans. D

Solution:

Put $x^3 = v$
Then $3x^2dx = dv$

$\int \dfrac{x^2\,dx}{\sqrt{4+x^6}} = \dfrac{1}{3}\log\left|x^3 + \sqrt{4+x^6}\right| + c$

9. $\int \sqrt{\frac{1-\cos x}{1+cosx}}\,dx =$

A. $\log\sec\frac{x}{2}$
B. $2\log\sec\frac{x}{2}$
C. $4\log\sec\frac{x}{2}$
D. None of these

Ans. B

Solution:

$\int \sqrt{\dfrac{1-\cos x}{1+\cos x}}\,dx = \int \tan x/2\,dx = 2\log\sec x/2 + c$

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10. $\int \frac{cosec^2 x}{4+9+\cot^2 x} dx =$

A. $\frac{1}{9}\log(4 + 9\cot^2 x)$
B. $-\frac{1}{3}\tan -1 \left(\frac{3\cot x}{2}\right)$
C. $-\frac{1}{6}\tan -1 \left(\frac{3\cot x}{2}\right)$
D. $-\frac{1}{6}\tan -1 \left(\frac{3\cot x}{2}\right)$

Ans. D

Solution:

$$\int \frac{\text{cosec}^2 \text{ x}}{4 + 9 \cot^2 \text{ x}} \text{dx}$$

$$= \int \frac{\text{cosec}^2 \text{ x}}{4\left(1 + \frac{9}{4}\cot^2 \text{ x}\right)} \text{dx}$$

$$putting... \frac{3}{2}\cot x = v$$

$$-\frac{3}{2}\cos ec^2 x dx = dv$$

$$= -\frac{1}{6}\tan^{-1}\left(\frac{3\cot \text{ x}}{2}\right) + c$$

11. $\int \frac{1+\cot x}{sing\ x} dx =$

A. - Cosec x + log sin x
B. log (cosec x - cot x) - cosec x
C. log (cosec x - cot x) + cosec x
D. none of these

Ans. B

Solution:

$$\int \frac{1 + \cot \text{ x}}{\sin \text{ x}} \text{dx} = \int (\cos ec x + \cot x \cos ec x) dx = \log(\cos ec x - \cot x) - \cos ec x + c$$

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12. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B$ log sin (x - α) + c, then the value of (A,B) is

A. $(-\cos a, \sin a)$
B. $(\cos a, \sin a)$
C. $(-\sin a, \cos a)$
D. $(\sin a, \cos a)$

Ans. B

Solution:

$$\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin (x-\alpha) + c,$$

$$putting...x - \alpha = v$$

$$dx = dv$$

$$\int \frac{\sin(v+\alpha)dv}{\sin v} = x\cos\alpha + \sin\alpha \log(x-\alpha) + c$$

13. If |a| > 1, $\int \frac{dx}{x^2+2ax+1} =$

A. $\frac{1}{2\sqrt{a^2-1}} \log \frac{x+a-\sqrt{a^2-1}}{x+a+\sqrt{a^2-1}}$
B. $-\frac{1}{x+a}$
C. $\frac{1}{2\sqrt{a^2-1}} \log \frac{\sqrt{a^2-1+x+a}}{\sqrt{a^2-1-x-a}}$
D. None of these

Ans. A

Solution:

$$\text{If } |a| > 1, \int \frac{dx}{x^2 + 2ax + 1}$$

$$= \int \frac{dx}{(x+a)^2 - (a^2-1)}$$

$$= \frac{1}{2\sqrt{a^2-1}} \log \frac{x + a - \sqrt{a^2-1}}{x + a + \sqrt{a^2-1}} + c$$

14. $\int e^{\cos 2x}$ sin 2x dx =

A. $e\cos^2 x$
B. $e - \cos^2 x$
C. $-e\cos^2 x$
D. $-e\cos^2 x$

Ans. D

Solution:

Put $\cos^2 x = v$
Then $-\sin 2x dx = dv$

$$\int e^{\cos^2 x} \sin 2x\, dx = \int -e^{v} dv = -e^{\cos^2 x} + c$$

15. $\int \frac{dx}{\sqrt{x}.(x+1)} =$

A. $2\tan -1\sqrt{x} + c$
B. $2\log(1+\sqrt{x})$
C. $\frac{1}{2}\tan -1\sqrt{x+c}$
D. $\tan -1\sqrt{x+c}$

Ans. A

Solution:

Putting $x=v^2$
Then $dx=2vdv$

$$\int \frac{dx}{\sqrt{x}\,.\,(x+1)} = \int \frac{2v\,dv}{v(v^2+1)} = 2\tan^{-1}\sqrt{x} + c$$

16. $\int \sin x \,.\, e^{\cos x}$ dx =

A. $ecosx$
B. $esinx$
C. None of these
D. $-e^{\cos x}$+ e

Ans. D

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Solution:

1
$cosx = v$

$-sinx dx = dv$

$$\int \sin x . e^{\cos x} dx = \int -e^{v} dv = -e^{\cos v} + c$$

17. $\int \frac{\sec x}{\sec x+\tan x} dx =$

A. tan x – sec x + c
B. log (1 + sin x) + c
C. sec x + tan x + c
D. none of these

Ans. A

Solution:

$$\int \frac{\sec x}{\sec x + \tan x} dx$$

$$= \int \frac{\sec^2 x - \sec x \tan x}{\sec^2 x - \tan^2 x} dx = \tan x - \sec x + c$$

18. $\int \frac{x^3+x^2+1}{x+1}$dx =

A. $\frac{x^2}{2} + \log(x+1)$
B. $\frac{x^3}{3} + \log(x+1)$
C. $\frac{x^4}{4} + \frac{x^3}{3} + \log(x+1)$
D. *None of these*

Ans. B

Solution:

$$\int \frac{x^3 + x^2 + 1}{x + 1} dx = \int \left[x^2 + \frac{1}{x+1}\right] dx$$

$$= \frac{x^3}{3} + \log (x + 1) + c$$

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19. $\int \frac{\cos 2x}{\cos x} dx =$

A. $2x$
B. $2sin\, x - sec\, x\, tan\, x$
C. $\log(\sec x + \tan x) - 2\sin x$
D. $2\sin x + \log(\sec x + \tan x)$

Ans. D

Solution:

$$\int \frac{\cos 2x}{\cos x} dx = \int \frac{2\cos^2 x - 1}{\cos x} dx$$
$$= \int [2\cos x - \sec x] dx = 2\sin x + \log(\sec x + \tan x) + c$$

20. $\int \frac{dx}{\sqrt{(x-a)(b-x)}} =$

A. $2\sin^{-1}\sqrt{\left(\frac{x-a}{b-a}\right)} + c$

B. $\sin^{-1}\sqrt{\left(\frac{x-a}{b-a}\right)} + c$

C. $2\sin^{-1}\sqrt{\left(\frac{x+a}{b-a}\right)} + c$

D. None of these

Ans. A

Solution:

Put $x = a\cos^2\theta + b\sin^2\theta$ the given integral becomes.

$$I = \int \frac{2(b-a)\sin\theta\cos\theta d\theta}{\{(a\cos^2\theta + b\sin^2\theta - a)(a\cos^2\theta + b\sin^2\theta - b)\}^{\frac{1}{2}}}$$

$$= \frac{2(b-a)\sin\theta\cos\theta d\theta}{(b-a)\sin\theta\cos\theta} = \left(\frac{b-a}{b-a}\right)\int 2d\theta = 2\theta + c = 2\sin^{-1}\sqrt{\left(\frac{x-a}{b-a}\right)} + c$$