Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

askIITians
ENGINEERING | MEDICAL | FOUNDATION

Class: XII
Subject: Math's
Topic: Limits and Derivatives
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1) $\lim_{n\to\infty} \frac{1+2+3+\ldots\ldots+n}{n^2}$, n $\in$ N, is equal to

(A) 0
(B) 1
(C) $\frac{1}{2}$
(D) $\frac{1}{4}$

Sol. C

Right Answer Explanation:

As $\lim_{n\to\infty} \frac{1+2+3+\ldots\ldots+n}{n^2}$

$= \lim_{n\to\infty} \frac{n(n+1)}{2n^2} = \lim_{n\to\infty} \frac{1}{2}\left(1+\frac{1}{n}\right) = \frac{1}{2}$

2) $\lim_{x\to 0} \frac{\sin x}{(1+\cos x)}$ is equal to

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) -1

Sol. A

Right Answer Explanation: B

we have $\lim_{x\to 0} \frac{\sin x}{(1+\cos x)}$ = 0/(1+1) =0

3) $\lim_{x\to 0} \frac{|x|}{x}$ is equal to

(A) 1
(B) -1
(C) 0
(D) Does not exists

Sol. D

Right Answer Explanation:

is the correct answer, since R.H.S. = $\lim_{x\to 0^+} \frac{|x|}{x} = \frac{x}{x} = 1$

and L.H.S. = $\lim_{x\to 0^-} \frac{|x|}{x} = \frac{-x}{x} = -1$

4) $\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$

(A) is equal to 0
(B) is equal to – 1
(C) is equal to 1
(D) does not exist

Sol. A

Right Answer Explanation:
is the correct answer, since

$$\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} = \lim_{y\to 0} \left[\frac{1-\sin\left(\frac{\pi}{2}-y\right)}{\cos\left(\frac{\pi}{2}-y\right)}\right] \left(\text{taking } \frac{\pi}{2}-x=y\right)$$

5) The derivative of f(x) = |x| at x = 0 is

(A) 0
(B) 1
(C) –1
(D) None of these

Sol. D

Right Answer Explanation:
At x = 0, **f(x) = |x| is not differentiable**

6) $\lim_{n\to\infty}\left[\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots\ldots+\frac{1}{n(n+1)}\right]$ is equal to

(A) 0
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) 1

Sol. D

Right Answer Explanation:

$\underset{x\to\infty}{\mathrm{Lt}}\left(\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots\ldots+\frac{1}{n(n+1)}\right)$

$= \underset{n\to\infty}{\mathrm{Lt}}\left(\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right)$

$= \underset{n\to\infty}{\mathrm{Lt}}\left(1-\frac{1}{n+1}\right) = 1-0 = 1$

7) $\underset{x\to 0}{\mathrm{Lt}}\frac{e^x-\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}\right)}{x^4}$ is equal to

(A) $\frac{1}{24}$
(B) 0
(C) $-\frac{1}{24}$
(D) None of these

Sol. A

Right Answer Explanation:

$$\underset{x\to 0}{\text{Lt}} \frac{e^x - \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)}{x^4}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{\frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \ldots\ldots\text{to } \infty}{x^4}$$

$$= \frac{1}{4!} + 0 = \frac{1}{24}$$

8) Find $\underset{x\to\infty}{\text{Lt}}$ $(6x^3 + 19x + 14x + 18)/(5x^3 + 6x^2 + 7x + 12)$.

(A) 6/4
(B) 6/5
(C) 6/11
(D) None of these

Sol. B

Right Answer Explanation:

$\underset{x\to\infty}{\text{Lt}}$ $(x^3 (6+ 19/x^2 + 14/x^2 + 18/x^3)/x^3(5 + 6/x + 7/x^2 + 12/x^3)$

$6+ 19/\infty + 14/\infty + 18/\infty)/(5 + 6/\infty + 7/\infty + 12/\infty) = (6 + 0+ 0 + 0) /(5 + 0 + 0 + 0) = 6/5$

9) $\lim_{x\to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$

(A) is equal to 0
(B) is equal to 1
(C) is equal to – 1
(D) does not exist

Sol. D

Right Answer Explanation:
After solving, we get
L.H.L = -1 and R.H.L =1
Therefore, limit does not exist.

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

10) The set of all points where the function f (x) = $\sqrt{1-e^{-x^2}}$ is differentiable is

(A) (0, ∞)
(B) (– ∞ , ∞)
(C) (– ∞ , ∞) ~ {0}
(D) None of these

Sol. C

Right Answer Explanation:

$$\text{For } x \neq 0, f'(x) = \frac{1}{2}\,\frac{1}{\sqrt{1-e^{-x^2}}}\,[-(-2x)e^{-x^2}]$$

$$= \frac{x\,e^{-x^2}}{\sqrt{1-e^{-x^2}}}$$

$$\text{Also } f'(0+) = \lim_{h\to 0+}\frac{f(h)-f(0)}{h}$$

$$= \lim_{h\to 0+}\left(\frac{e^{-h^2}-1}{-h^2}\right)^{1/2} = 1$$

$$\text{and } f'(0-) = -\lim_{h\to 0-}\left(\frac{e^{-h^2}-1}{-h^2}\right)^{1/2} = -1.$$

Hence the set of all points of differentiability is (– ∞, ∞) ~ {0}.

11) f (x) = $\begin{cases} |x-4| & \text{for } x \geq 1 \\ x^3/2 - x^2 + 3x + 1/2 & \text{for } x < 1 \end{cases}$, then

(A) f (x) is continuous at x = 1 and at x = 4
(B) f (x) is differentiable at x = 4
(C) f (x) is continuous and differentiable at x = 1
(D) f (x) is only continuous at x = 1

Sol. A

Right Answer Explanation:

Since g(x) = |x| is a continuous function and $\lim_{x\to 1+} f(x) = 3 = \lim_{x\to 1-} f(x)$ so f is a continuous function.
In particular f is continuous at x = 1 and x = 4. f is clearly not differentiable at x = 4. Since g(x) = |x| is not

differentiable at x = 0. Now

$$f'(1+) = \lim_{h \to 0+} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0+} \frac{|-3+h| - 3}{h} = -1,$$

$$f'(1-) = \lim_{h \to 0-} \frac{(1/2)(1+h)^3 - (1+h)^2 + 3(1+h) + 1/2 - 3}{h}$$

$$= \lim_{h \to 0-} \frac{(1/2)\left(h^3 + 3h^2 + 3h\right) - \left(h^2 + 2h\right) + 3h}{h}$$

$$= \frac{5}{2}.$$

Hence f is not differentiable at $x = 1$.

12) The set of all points where the function f (x) = 2x |x| is differentiable is

(A) $(-\infty, \infty)$
(B) $(-\infty, \infty) - \{0\}$
(C) $(0, \infty)$
(D) $[0, \infty)$

Sol. A

Right Answer Explanation:

$$f(x) = \begin{cases} 2x^2, & x \geq 0 \\ -2x^2, & x < 0 \end{cases}$$

Since x^2 and $-x^2$ are differentiable functions, $f(x)$ is differentiable except possible at $x = 0$

$$\text{Now } f'(0+) = \lim_{h \to 0+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0+} \frac{f(h)}{h}$$

$$= \lim_{h \to 0+} \frac{2h^2}{h} = 0$$

$$\text{and } f'(0-) = \lim_{h \to 0-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0-} \frac{f(h)}{h}$$

$$= \lim_{h \to 0-} \frac{-2h^2}{h} = 0.$$

Hence f is differentiable everywhere.

13) Let [ ] denote the greatest integer function and f (x) = [tan^2 x]. Then

(A) $\lim_{x\to 0}$ f (x) does not exist
(B) f(x) is continuous at x = 0
(C) f(x) is not differentiable at x = 0
(D) f'(0) = 1

Sol. B

Right Answer Explanation:

For $0 < x < \pi/4$, $0 < \tan^2 x < 1 \Rightarrow [\tan^2 x] = 0$ for $0 < x < \pi/4$. As $\tan^2 x$ is an even function, so $\lim_{x\to 0+} f(x) = \lim_{x\to 0+} f(x) = 0$. So f is continuous at $x = 0$. Now

$$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{f(h)-(0)}{h} = \lim_{h\to 0}\frac{0}{h} = 0.$$

Hence f is differentiable at $x = 0$ and $f'(0) = 0$.

14) Suppose f is differentiable at x = 1 and $\lim_{h\to 0}\frac{1}{h}$ f(1 + h) = 5 then

(A) f'(1) = 4
(B) f'(1) = 3
(C) f'(1) = 6
(D) none of these

Sol. D

Right Answer Explanation:
Since f is differentiable so it is continuous also. Thus

$$f(1) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0} h\frac{f(1+h)}{h} = (0)(5) = 0$$

$$\text{Hence } f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{f(1+h)}{h} = 5.$$

15) If f''(x) is continuous at x = 0 and f''(0) = 4, then the value of $\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is

(A) 4
(B) 8
(C) 12
(D) None of these

Sol. C

Right Answer Explanation:

Required limit

$$= \lim_{x \to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x} \left(\frac{0}{0} \text{form}\right)$$

$$= \lim_{x \to 0} \frac{2f''(x) - 12f''(2x) + 16f''(4x)}{2}$$

$$= 3f''(0) = 12.$$

16) Let f (x) be a function satisfying f (x + y) = f (x) f (y) for all x, y $\in$ R and f (x) = 1 + x g(x) where $\lim_{x \to 0}$ g(x) = 1 then f′(x) is equal to

(A) g′(x)
(B) g(x)
(C) f(x)
(D) none of these

Sol. C

Right Answer Explanation:

$$f'(x)$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{f(x)f(h) - f(x)}{h}$$

$$= f(x) \lim_{h \to 0} \frac{f(h) - 1}{h} = f(x) \lim_{h \to 0} \frac{1 + hg(h) - 1}{h}$$

$$= f(x) \lim_{h \to 0} g(h) = f(x)$$

17) The values of a and b such that the function f defined as $f(x) = \begin{cases} ax^2 - b, & |x| < 1 \\ -1/|x|, & |x| \ge 1 \end{cases}$ is differentiable are

(A) a = 1, b = – 1
(B) a = 1/2, b = 1/2
(C) a = 1/2, b = 3/2
(D) none of these

Sol. C

Right Answer Explanation:

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Since every differentiable function is continuous, so we must have

$$\lim_{x \to 1-} f(x) = f(1) \quad \Rightarrow \quad a - b = -1.$$

For f to be differentiable, $f'(1-) = f'(1+)$

$$\Rightarrow \quad \lim_{h \to 0-} \left[\frac{a(1+h)^2 - b + 1}{h}\right] = \lim_{h \to 0+} \left[\frac{-1/|1+h|+1}{h}\right]$$

$$\Rightarrow \quad \lim_{h \to 0-} \left[\frac{a(2h + h^2)}{h}\right] = \lim_{h \to 0+} \frac{h}{h(1+h)} \text{ (as } a - b = -1)$$

$\Rightarrow \quad 2a = 1$. Hence $a = 1/2$ and $b = 3/2$.

18) Let f(x) = $||x - 1| - 1|$, then all the points where f(x) is not differentiable is (are)

(A) 1
(B) 1, 2
(C) $\pm$ 1
(D) 0, 1, 2

Sol. D

Right Answer Explanation:

$$f(x) = \begin{cases} |x-1|-1 & \text{if } |x-1| \geq 1 \\ 1-|x-1| & \text{if } |x-1| < 1 \end{cases}$$

$$= \begin{cases} -x & x \leq 0 \\ x & 0 < x \leq 1 \\ 2-x & 1 < x < 2 \\ x-2 & x \geq 2 \end{cases}$$

f is not differentiable at 0, 1, 2 as these are corner points on the graph.

19) If f' (c) exists and non-zero then $\lim_{h \to 0} \frac{f(c+h) + f(c-h) - 2f(c)}{h}$ is equal to

(A) f'(c)
(B) 0
(C) 2f'(c)
(D) none of these

Sol. B

Right Answer Explanation:

$$\lim_{h\to 0}\frac{f(c+h)+f(c-h)-2f(c)}{h}$$

$$=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}+\frac{f(c-h)-f(c)}{h}$$

$$=f'(c)-\lim_{h\to 0}\frac{f(c-h)-f(c)}{-h}$$

$$=f'(c)-f'(c)=0$$

20) If f(x) = $\begin{cases} x\sin(1/x) & \text{for } x\neq 0 \\ 0 & \text{for } x=0 \end{cases}$ then

(A) f is a continuous function
(B) f′ (0 +) exists but f′ (0 –) does not exist
(C) f′ (0 +) = f′ (0 –)
(D) f′ (0 –) exists but f′ (0 +) does not exist.

Sol. A

Right Answer Explanation:
LHL = 0=RHL (as it take form 0x sin(1/0) which 0xfinite quantity=0)
So f will be continuous