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B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Class: XII
Subject: Mathematics
Topic: Probability
No. of Questions: 30
Duration: 30 Min
Maximum Marks: 90

1. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six is

(a) 3/8
(b) 1/5
(c) 3/4
(d) None of these

Ans. 1 (a)

Solutions

let E denote the event that a six occurs and the event that the man reports that it is a six. We have P(E) = 1/6, P(E') =5/6, P(A|E) =3/4 and P(A|E') =1/4. By the bayes' theorem

$$P(E|A) = \frac{P(E)P(A|E)}{P(E)P(A|E) + P(E')P(A|E')}$$
$$= \frac{(1/6)(3/4)}{(1/6)(3/4) + (5/6)(1/4)} = \frac{3}{8}$$

2. Suppose X follows a binomial distribution with parameters n and p, where 0 < P < 1. If p(X = r)/P(X = n-r) is independent of n for every value of r, then

(a) P = 1/2
(b) P = 1/3
(c) P= 1/4
(d) P= 1/5

Ans. A

Solution

We have $\frac{P(X=r)}{P(x=n-r} = \frac{{}^nC_r P^r (1-P)^{n-r}}{{}^nC_{n-r} P^{n-r} (1-P)^r} = \frac{(1-p)^{n-2r}}{P^{n-2r}} = \left(\frac{1}{P} - 1\right)^{n-2r}$

Note that (1/p) -1 >0. Therefore, the ratio will be independent of n for each r if (1/p) -1 = 1, or p= 1/2.

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3. Nine identical balls are numbers 1, 2, …9 are put in a bag. A draws a ball and gets the number a. the ball is put back in the bag. Next B draws a ball gets the number b. The probability that a and b satisfies the inequality a – 2b +10 > 0 is

(a) $\frac{52}{81}$

(b) $\frac{55}{81}$

(c) $\frac{61}{81}$

(d) $\frac{62}{81}$

Ans. C

Solution:

$a - 2b + 10 > 0$

$\Rightarrow 1 \le b < (a + 10)/2$

Thus, $b \in [1, (a + 10)/2)$

When a =1, 2, $b \in [1, 5]$

When a=3, 4, $b \in [1, 6]$

When a = 5, 6, $b \in [1, 7]$

When a = 7, 8, $b \in [1, 8]$

When a =9 $b \in [1, 9]$

Thus, the inequality a -2b +10 > 0 is satisfied for 2 × 5 + 2 × 6 + 2 × 7 +2 × 8 + 9 = 61 ordered pairs (a, b) total number of ways of choosing (a, b) is 9 × 9 =81

$\therefore \textit{required probability is } \frac{61}{81}$

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4. Let A, B C, be three mutually independent events. Consider the two statements S_1 and S_2.

S_1 : A and $B \cup C$ are independent

S_2 : A and $B \cap C$ are independent

Then

(a) Both S_1 and S_2 are true
(b) Only S_1 is true
(c) Only S_2 is true
(d) Neither S_1 nor S_2 is true.

Ans. A

Solution:

We are given that

$P(A \cap B) = P(A)\, P(B)$

$P(B \cap C) = P(B)\, P(C)$, $P(C \cap A) = P(C)\, P(A)$

And $P(A \cap B \cap C)=P(A)\, P(B)\, P(C)$

$=P(A)\, P(B)\, P(C) = P(A)\, P(B \cap C)$.

$\Longrightarrow$ A and $B \cap C$ are independent. Therefore, S_2 is true.

Also $P[(A \cap (B \cup C)]= P[(A \cap B) \cup (A \cap C)]$

$=P(A \cap B) + P(A \cap C) – P[A \cap B) \cap (A \cap C)]$

$=P(A \cap B) + P(A \cap C) – P(A \cap B \cap C)$

$=P(A)\, P(B) +P(A)\, P(C) – P(A)\, P(B)\, P(C)$

$=P(A)\, [P(B) +P(C)-P(B)\, P(C)]$

$=P(A)\, [P(B) + P(C) –P(B \cap C)] = P(A)\, P(B \cup C)$

$\therefore$ A and $B \cup C$ are independent.

5. A group of 2n boys and 2n girls is randomly divided into two equal groups. The probability that each group contains the same number of boys and girls is

(a) 1/2
(b) 1/n
(c) 1/2n
(d) None of these

Ans. D

Solution:

Total number of ways of choosing a group is ${}^{4n}C_{2n}$. The number of ways in which each group contains equal number of boys and girls is $({}^{2n}C_n)({}^{2n}C_n)$

$$\therefore Required\ \ probability = \frac{({}^{2n}C_n)^2}{{}^{4n}C_{2n}}$$

6. A fair coin is tossed 100 times. The probability of getting tails 1, 3, ….,49 times is

(a) 1/2
(b) 1/4
(c) 1/8
(d) 1/16

Ans. B

Solution:

Let P= probability of getting a tail in a single trial =1/2.

n = number of trials =100

and X = number of tails in 100 trials.

We have $P(X = r)={}^{100}C_rP^rq^{n-r}$

$$= {}^{100}C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{100-r} = {}^{100}C_r\left(\frac{1}{2}\right)^{100}$$

Now, P(X = 1) +P(X = 3)+…+P(X = 49)

$$= {}^{100}C_1\left(\frac{1}{2}\right)^{100}\left(\frac{1}{2}\right)^{100-r} = {}^{100}C_3\left(\frac{1}{2}\right)^{100} + ... + {}^{100}C_{49}\left(\frac{1}{2}\right)^{100}$$

$$=({}^{100}C_1 + {}^{100}C_3 + \ldots + {}^{100}C_{49})\left(\frac{1}{2}\right)^{100}$$

But $^{100}C_1 + {}^{100}C_3 + \ldots + {}^{100}C_{49} = 2^{99}$

Also, $^{100}C_{99} = {}^{100}C_1$,

Thus,

$(2^{100}C_1 + {}^{100}C_3 + \ldots + {}^{100}C_{49} = 2^{99}$

$\Longrightarrow 2^{100}C_1 + {}^{100}C_3 + \ldots + {}^{100}C_{49} = 2^{98}$

$\therefore$ Probability of required event $\frac{2^{98}}{2^{100}} = \frac{1}{4}$.

7. A natural number x is chosen at random from the first one hundred natural numbers. The probability that $\frac{(x-20)(x-40)}{x-30} < 0\ is$

(a) 1/50
(b) 3/50
(c) 3/25
(d) 7/25

Ans. D

Solution:

Let $E = \frac{(x-20)(x-40)}{x-30} = \frac{(x-20)(x-30)(x-40}{(x-30)^2}$

Sign of E is same as that of sign of (x – 20) (x – 30) (x – 40) =F (say).

Note that F < 0 if and only if 0 < x <20 or 30 < x < 40.

$\therefore$ E < 0 in (0, 20) ∪ (30, 40)

Thus E is negative for x =1, 2, .., 19,31, 32, …,39, that is E, <0 for 28 natural numbers

$\therefore$Required probability = $\frac{28}{100} = \frac{7}{25}$.

8. A person writes 4 letters and 4 addresses on 4 envelopes. If the letters are placed in the envelopes at random, the probability that not all letters are placed in correct envelopes is

(a) 1/24
(b) 11/24
(c) 5/8
(d) 23/24

Ans. D

Solution:

Required probability = 1 –P (all the letters are put in correct envelops)

The number of ways of putting the letters in the envelops =$^4P_4$ -4!

The number of ways of putting letters in correct envelops =1

∴Required probability =$1-\frac{1}{24}=\frac{23}{24}$

Paragraph Questions (9-13)

A chess match between two grandmasters X and Y is won by whoever first wins a total of two games. X's chances of winning. Drawing or losing any particular game are a, b, c respectively. The games are independent and a + b +c =1.

9. The probability that X wins the match after (n + 1) games (n ≥ 1) is
(a) $na^2 b^{n-1}$
(b) $a^2(nb^{n-1} + n(n-1)b^{n-2} c)$
(c) $na^2 bc^{n-1}$
(d) None of these

Ans. B

Solution:

X can win after the (n + 1) th game in the following two mutually exclusive ways

(i) X wins exactly one of the first n games draws (n – 1) games and wins the (n + 1)th game
(ii) X loses exactly one of the first n games, wins exactly one of the first n games and draws (n – 2) games and wins the (n +1)th game.

For (i) the probability is $(^nP_1\ ab^{n-1})a$ and for (ii) the probability is $(^nP_2)(ac)b^{n-2}\ a$

∴ The probability X wins after the (n + 1)th game

$P_n = na^2 b^{n-1} + n(n-1)\ a^2 b^{n-2} c$

$=a^2(nb^{n-1} + n(n-1)\ b^{n-2} c)$.

10. The probability that Y wins the match after the 4^{th} game is
(a) $3bc^2(b + 2a)$
(b) $bc^2 (3b +a)$
(c) $2ac^2(b + c)$
(d) abc (2a + 3b)

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Ans. A

Solution;

In question 9, put n = 3, interchange a and c to obtain

$C^2[3b^2 + (3)(2)ba]$

$=3bc^2(b+2a)$

11. The probability that X wins the match is

(a) $\frac{a^3+a^2c}{(a+c)^3}$

(b) $\frac{a^3+3a^2c}{(a+c)^3}$

(c) $\frac{a^3}{(a+c)^3}$

(d) None of these

Ans. B

Solution:

X will win the match with probability

$$\sum_{n=1}^{\infty} p_n = a^2\sum_{n=1}^{\infty} nb^{n-1} + a^2c\sum_{n=1}^{\infty} n(n-1)^{n-2}$$

$$=\frac{a^2}{(1-b)^2}+\frac{2a^2c}{(1-b)^3}$$

$$=\frac{a^2(a+c)+2a^2c}{(a+c)^3}=\frac{a^3+3a^2c}{(a+c)^3}.$$

12. The probability that Y wins the match is

(a) $\frac{b^3+b^2c}{(b+c)^3}$

(b) $\frac{c^3+3c^2a}{(a+c)^3}$

(c) $\frac{c^3}{(b+c)^3}$

(d) None of these

Ans. B

Solution:

Interchange a by c in question 9.

13. The probability that there is no winner is

(a) (1 – a) (1 – c)
(b) (1 – a) b (1 – c)
(c) b
(d) 1

Ans. D

Solution:

Since $= \frac{a^3 + 3a^2c}{(a+c)^3} + \frac{c^3 + 3c^2a}{(a+c)^3} = 1.$

14. A box contains 20 cards of these 10 have letter J printed on them and the remaining 10 have E. printed on them. 3 cards are drawn from the box, the probability that we can write JEE with these cards is

(a) 9/80
(b) 1/8
(c) 4/27
(d) 15/38

Ans. D

Solution:

$\frac{{}^{10}C_1 \times {}^{10}C_2}{{}^{20}C_3}$

15. A fair coin is tossed repeatedly. If head and tail appear alternatively on first 7 tosses: then the probability that head appears on the eighth toss is

(a) 1/2
(b) 1/128
(c) 1/256

(d) 7/256

Ans. A

Solution:

Events are independent.

16. A lottery sell n^2 tickets and declares n prizes. If a man purchases n tickets, the probability of his winning at least one prize is

(a) $(n^2 - n)! / (n^2)!$
(b) 1/2n
(c) $(n-1)!^2/(n^2)!$
(d) None of these

Ans. D

Solution;

$$1 - P(no\ prize) = 1 - \frac{{}^{(n^2-n)}C_n}{{}^{n^2}C_n}$$

17. Let x be a non-zero real number. A determinant is chosen from the set of all determinants of order 2 with entries x or –x only. The probability that the value of the determinant is non-zero is

(a) 3/16
(b) 1/4
(c) 1/2
(d) None of these

Ans. C

Solution:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0 \Leftrightarrow \frac{a}{b} = \frac{b}{d} = 1, -1.$$

Now, a = c = x ⟹ a = c = x and b = d = x or b = d =- x similarly,6 other choices.

18. Two non-negative integers are chosen at random. The probability that the sum of the square is divisible by 11 is

(a) 9/16
(b) 1/121
(c) 9/17
(d) None of these

Ans. B

Solution:

Use x = 11m + r where $0 \le r \le 10$

19. If four positive integers are taken at random and multiplied together, then the probability that the last digit is 1, 3, 7 or 9is
(a) 1/8
(b) 2/27
(c) 1/625
(d) 16/625

Ans. D

Solution:

Unit's digit of the product will be 1, 3, 7 or 9 if and only if each of the four numbers and in 1, 3, 7 or 9. Thus, required probability is $(4/10)^4$.

20. Two contestants play a game as follows: each is asked to select a digit from 1 to 9. If the two digits match they both win a prize. The probability that they will win a prize in a single trial is
(a) 1/81
(b) 7/81
(c) 1/9
(d) 3/11

Ans. C

Solution:

Total number of ways is 81 out of which 9 are favorable.