

Class: XII
Subject: Maths
Topic: Relations and Functions
No. of Questions: 20
Duration: 60 Min

Q1. If R is a relation from a non-empty set A to a non-empty set B, then

- A. $R = A \times B$
- B. $R = A \cup B$
- C. $R \subseteq A \times B$
- D. $R \subset A \times B$

Right Answer Explanation: C

A relation from a non-empty set A to a non-empty set B is defined as a subset of $A \times B$ or A relation R from the non-empty set A to another non empty set B is a subset of their Cartesian product $A \times B$, i.e. $R \subseteq A \times B$.

Q2. Let R be the relation on N defined as $x R y$, if $x + 2y = 8$. The domain of R is

- A. {2, 4, 8}
- B. {2, 4, 6, 8}
- C. {2, 4, 6}
- D. {1, 2, 3, 4}

Right Answer Explanation: C

Domain of $R = \{x \in N : x R y \text{ for some } y \in N\} = \{2, 4, 6\}$ ($\because x = 8 - 2y, y \in N = \begin{cases} 6 \text{ when } y=1 \\ 4 \text{ when } y=2 \\ 2 \text{ when } y=3 \end{cases}$) Note that in this case $R = \{(2, 3), (4, 2), (6, 1)\}$

Q3. The range of the function $f(x) = -\sqrt{-x^2 - 6x - 5}$ is equal to

- A. [0, 2]
- B. [-2, 0]
- C. [-2, 2]
- D. None of these

Right Answer Explanation: B

Note that $D_f = [-5, -1]$

(f(x) is real if $-x^2 - 6x - 6 \geq 0$ i.e. if $(x + 5)(x + 1) \leq 0$)

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For R, let $y = f(x)$

$$\Rightarrow y = -\sqrt{-x^2 - 6x - 5}$$

$$\begin{aligned} \Rightarrow y^2 &= -x^2 - 6x - 5, y \leq 0 \\ \Rightarrow x^2 &= 6x + (5 + y^2) = 0, y \leq 0 \\ \Rightarrow 6^2 - 4 \cdot 1 (5 + y^2) &\geq 0 \\ (x \text{ is real, disc } &\geq 0) \\ \Rightarrow y^2 &\leq 4 \\ \Rightarrow |y| &\leq 2 \text{ and } y \leq 0 \\ R_f &= [-2, 0] \end{aligned}$$

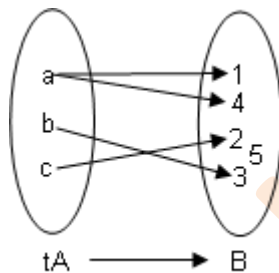
Q4. The number of relations that can be defined on the set $A = \{a, b, c, d\}$ is

- A. 24
- B. 16
- C. 4^4
- D. 2^{16}

Right Answer Explanation: D

As $n(A) = 4$, so $n(A \times A) = 16$. Hence, the number of subsets of $A \times A = 2^{16}$. So, the number of relations on A is 2^{16} .

Q5. The diagram given below shows that

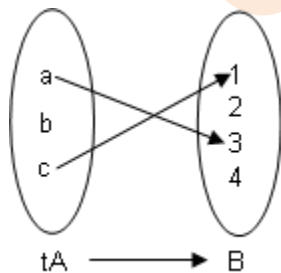


- A. f is function from A to B
- B. f is a one-one function from A to B
- C. f is an onto function from A to B
- D. f is not a function

Right Answer Explanation: D

As $f(a)$ is not unique, f is not a function.

Q6. The diagram given below shows that



- A. f is a function from A to B
- B. f is a one-one function from A to B
- C. f is a bijection from A to B
- D. f is not a function

Right Answer Explanation: D

As $f(b)$ is not defined, so f is not a function.

Q7. If $A = \{0, 1\}$ and $B = \{1, 0\}$, then $A \times B$ is equal to

- A. $\{(0, 1), (1, 0)\}$
- B. $\{(0, 0), (1, 1)\}$
- C. $A \times A$
- D. none of these

Right Answer Explanation: C

Here, the option covered all the ordered pairs.

Q8. If $f = \{(1, 4), (2, 5), (3, 6)\}$ and $g = \{(4, 8), (5, 7), (6, 9)\}$, then $(g \circ f)$ is

- A. $\{\}$
- B. $\{(1, 8), (2, 7), (3, 9)\}$
- C. $\{(1, 7), (2, 8), (3, 9)\}$
- D. none of these

Right Answer Explanation: B

$$(g \circ f)(1) = g(f(1)) = f(4) = 8,$$

$$(g \circ f)(2) = g(f(2)) = g(5) = 7$$

$$\text{and } (g \circ f)(3) = g(f(3)) = g(6) = 9$$

Q9. The domain of the function $f = \{(1, 3), (3, 5), (2, 6)\}$ is

- A. 1, 3 and 2
- B. $\{1, 3, 2\}$
- C. $\{3, 5, 6\}$
- D. 3, 5 and 6

Right Answer Explanation: B

Domain of f is a set of those points on which f is defined. So, $D_f = \{1, 2, 3\}$.

Q10. The range of the function $f(x) = |x - 1|$ is

- A. $(-\infty, \infty)$
- B. $(0, \infty)$
- C. $[0, \infty)$
- D. $(-\infty, 0)$

Right Answer Explanation: C

Since $|x|$ assumes every value in $[0, \infty)$, so $|x - 1|$ also takes every value in $[0, \infty)$. Hence, range of $f(x) = |x - 1|$ is $[0, \infty)$.

Q11. Number of relation that can be defined on a set $A = \{a, b, c\}$ is

- A. 24
- B. 512
- C. 4^4
- D. 2^{16}

Right Answer Explanation: B

$A = \{a, b, c\}$

If A set contains n elements, then no. of relations on A is 2^{n^2}

Here $n = 3$, so number of relations = 2^9

Q12. Let $A = \{1, 2, 3\}$, then the domain of the relation $R = \{(1, 1), (2, 3), (2, 1)\}$ defined on A is

- A. $\{1, 2\}$
- B. $\{1, 3\}$
- C. $\{1, 2, 3\}$
- D. None of these

Right Answer Explanation: A

Domain of $R = \{x \in A : (x, y) \in R \text{ for some } y \in A\} = \{1, 2\}$

Q13. Let $A = \{a, b, c\}$. Then, the range of the relation $R = \{(a, b), (a, c), (b, c)\}$ defined on A is

- A. $\{a, b\}$
- B. $\{a, b, c\}$
- C. $\{c\}$
- D. $\{b, c\}$

Right Answer Explanation: D

Range of $R = \{y \in A : (x, y) \in R \text{ for some } x \in A\} = \{b, c\}$

Q14. Let $A = \{1, 2, 3\}$. The total no. of distinct relations which can be defined over A is

- A. 6
- B. 8
- C. 2^9
- D. None of these

Right Answer Explanation: C

If a set A has x distinct elements, then the number of all relations is 2^{x^2}

The domain of $f(x) = \sin \left(\log \left(\sqrt{\frac{4-x^2}{1-x}} \right) \right)$ is

Q15.

- A. $(-2,1) \cup (2, \infty)$
- B. $(-2, \infty)$
- C. $(2, \infty)$
- D. None of these

Right Answer Explanation: A

$$\text{Given } f(x) = \sin \left(\log \sqrt{\frac{4-x^2}{1-x}} \right)$$

domain of $\sin x$ is \mathbb{R} . But domain of $\log x$ is $x > 0$. Hence domain of given function is values of x such

$$\text{that } \sqrt{\frac{4-x^2}{1-x}} > 0 \Rightarrow \frac{4-x^2}{1-x} > 0$$

$$\Rightarrow \frac{(x-2)(x+2)}{x-1} > 0$$

$$\Rightarrow x \in (-2,1) \cup (2, \infty)$$

Q16. Let $f(x) = \frac{1}{1-x}$, then $(f \circ f \circ f)(x)$ equals to

- A. x
- B. $\frac{1}{1-3x}$
- C. $\frac{x}{1-3x}$
- D. None of these

Right Answer Explanation: A

Firstly, find $(f \circ f)(x)$ say $g(x)$

Then find $f(g(x)) = x$.

Q17. Which of the following functions is an even function?

- A. $f(x) = x(x^2 - x^4)$
- B. $f(x) = \sin(5x + x^3)$
- C. $f(x) = x^2 - 3x + 4$
- D. $f(x) = x^2 + |x|$

Right Answer Explanation: D

For a function to be an even function, $f(-x) = f(x)$ for all x .

Consider (1): $f(x) = x(x^2 - x^4) = x^3 - x^5$

$f(-x) = (-x)^3 - (-x)^5 = -x^3 + x^5 = -(x^3 - x^5) = -f(x) \neq f(x)$

Thus, $f(x)$ is not an even function. It is an odd function as, $f(-x) = -f(x)$.

Consider (2): $f(x) = \sin(5x + x^3)$

$f(-x) = \sin(-5x + (-x)^3) = \sin(-5x - x^3) = \sin(-(5x + x^3)) = -\sin(5x + x^3)$

$f(-x) \neq f(x)$ This function is not an even function. It is an odd function.

Consider (3): $f(x) = x^2 - 3x + 4$; $f(-x) = x^2 + 3x + 4 \neq f(x)$

Also, $f(-x) \neq -f(x)$ This function is neither an even function nor an odd function.

Consider (4): $f(x) = x^2 + |x|$

$f(-x) = x^2 + |-x| = x^2 + |x| = f(x)$

This function is an even function.

Q18. Domain of the real function $\sqrt{2-x} - \frac{1}{\sqrt{9-x^2}}$ is

- A. $(-3, 1)$
- B. $[-3, 1]$
- C. $(-3, 2]$
- D. $[-3, 1)$

Right Answer Explanation: C

Calculate the domain of $\sqrt{2-x}$ and $\frac{1}{\sqrt{9-x^2}}$ and then find out where it intersects.

Q19. If the binary operation $*$, defined on \mathbb{Q} , is defined as $a*b = 2a + b - ab$, for all $a, b \in \mathbb{Q}$ find the value of $3*4$.

- A. 2
- B. 4
- C. -2
- D. None of these

Solution: C

Given that, $a*b = 2a + b - ab$, $\forall a, b \in \mathbb{Q}$.

On putting $a = 3$ and $b = 4$, we get

$$3*4 = 2 \cdot 3 + 4 - 3 \cdot 4$$

$$= 6 + 4 - 12 = -2$$

Q20. Let $*$: $R \times R \rightarrow R$ is defined as $a*b = 2a + b$. find $(2*3)*4$.

- A. 24
- B. -18
- C. 14
- D. None of these

Solution: D

Given that, $*$: $R \times R \rightarrow R$

Such that $a*b = 2a + b$.

On putting $a = 2$ and $b = 3$, we get

$$(2*3) = 2(2) + 3 = 4 + 3 = 7$$

$$\therefore (2*3)*4 = 7*4 = 2(7) + 4 = 14 + 4 = 18$$

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