

Class: XII
 Subject: Math's
 Topic: Vector Algebra
 No. of Questions: 20
 Duration: 60 Min
 Maximum Marks: 60

1. The values of a , for which the A,B, C with position vectors $2i - j + k$, $i - 3j - 5k$ and $ai - 3j + k$ respectively are vertices of a right with $C = \pi/2$, are
- A. -2 and 1
 B. 2 and 1
 C. 2 and 1
 D. -2 and -1

Ans. C

Sol :

A (2,-1, 1) B (1,-3,-5) C (A,-3, +1)

$$\overrightarrow{AB} = (-1, -2, -6)$$

$$\overrightarrow{BC} = (a-1, 0, 6)$$

$$\overrightarrow{AC} = (2-a, 2, 0)$$

$$\overrightarrow{BC} \cdot \overrightarrow{CA} = 0$$

$$\Rightarrow (a-1)(2-a) = 0$$

$$a = 1 \text{ or } a = 2$$

2. If $\overrightarrow{AB} = 2i + j - 3k$ and the coordinates of A are (1, 2, -1), then the coordinate of B are
- A. (3, 3, 4)
 B. (3, 2, -4)
 C. (3, -3, -4)
 D. (3, 3, -4)

Ans. D

Sol :

$$A(1, 2, -1)$$

$$B(x, y, z)$$

$$\overline{AB} = (x-1, y-z+1) = (2, 1, -3)$$

$$x = 3, y = 3, z = -4$$

3. Let ABC be equilateral triangle whose orthocenter is at origin O

If $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$ then \vec{OC}

- A. $(\vec{a} + \vec{b})$
 B. $-(\vec{a} + \vec{b})$
 C. $2(\vec{a} + \vec{b})$
 D. $-3(\vec{a} + \vec{b})$

Ans. B

Sol:
$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$\vec{c} = -(\vec{a} + \vec{b})$$

4. Which of the following is false?

- A. $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \Rightarrow \vec{a} \perp \vec{b}$
 B. $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| \Rightarrow \vec{a} \perp \vec{b}$
 C. $|\vec{a} \times \vec{b}| = \vec{0} \Rightarrow \vec{a} \parallel \vec{b}$
 D. none of these

Ans. D

Solution:

1,2,3, all are true

None of these is false

5. If $|\hat{a} + \hat{b}| = \sqrt{3}$, then $(\hat{a} + \hat{b}) \cdot (\hat{a} - 2\hat{b})$ is

- A. $-\frac{1}{2}$
 B. $-\frac{3}{2}$
 C. 0

D. None of these

Ans. B

Sol :

$$\begin{aligned} & (\hat{a} + \hat{b})(\hat{a} - 2\hat{b}) \\ &= |\hat{a}|^2 - 2\hat{a}\hat{b} + \hat{b}\hat{a} - 2|\hat{b}|^2 \\ &= 1 - \hat{a}\hat{b} - 2 \\ &= -1 - \hat{a}\hat{b} \\ & \text{and } |\hat{a} + \hat{b}|^2 = (\sqrt{3})^2 \\ & |\hat{a}|^2 + 2\hat{a}\hat{b} + |\hat{b}|^2 = 3 \\ & 2\hat{a}\hat{b} = 3 - 2 \\ & \hat{a}\hat{b} = \frac{1}{2} \\ & \therefore -1 - \hat{a}\hat{b} = -1 - \frac{1}{2} = -\frac{3}{2} \end{aligned}$$

6. If the vectors $(\lambda, 3, 4), (0, -1, 10)$ and $(1, \lambda, 3)$ are coplanar, then $\lambda =$

- A. -2 or 17
- B. 2 or 17
- C. 0 or -2
- D. None of these

Ans. A

Vectors are co-planar then

$$\begin{vmatrix} \lambda & 3 & 4 \\ 0 & -1 & 10 \\ 1 & \lambda & 3 \end{vmatrix} = 0$$

then

$$10\lambda^2 + 3\lambda - 34 = 0$$

$$\lambda = -2, 17$$

7. The position vectors of three consecutive vertices of a parallelogram are $\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{j} + \hat{k}$ and $\hat{j} + 3\hat{k}$. Then its area in square units is
- $\sqrt[2]{30}$
 - $\sqrt{30}$
 - $\frac{1}{2}\sqrt{30}$
 - None of these

Ans. B

Solution:

Three consecutive vertices of a parallelogram are

$$\hat{i} - \hat{j} + 2\hat{k}, 2\hat{j} + \hat{k} \text{ and } \hat{j} + 3\hat{k}$$

$$\vec{a} = -\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{b} = -\hat{j} + 2\hat{k}$$

$$\text{area} = \left| \vec{a} \times \vec{b} \right| = \sqrt{30}$$

8. The tangent of the angle between $\hat{i} + 3\hat{j}$ and $3\hat{i} - \hat{k}$ is
- ∞
 - $\frac{1}{3}$
 - $\frac{3}{10}$
 - None of these

Ans. D

$$\text{Sol: } \cos \theta = \frac{(\hat{i} + 3\hat{j})(3\hat{i} - \hat{k})}{|\hat{i} + 3\hat{j}| |3\hat{i} - \hat{k}|}$$

$$\cos \theta = \frac{3}{10}$$

$$\tan \theta = \frac{3}{10}$$

9. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ and $\vec{a} + t\vec{b}$ is orthogonal to \vec{c} , then $t =$
- A. 2
B. 4
C. 5
D. 8

Ans. D

Solution:

$$\vec{a} + t\vec{b} = (2-t)\hat{i} + (2+2t)\hat{j} + (3+t)\hat{k}$$

and $\vec{a} + t\vec{b}$ is orthogonal to \vec{c}

Then their dot product = 0

We get $3(2-t) + 2 + 2t = 0$

Then $t = 8$

10. Which of the following is true?

A. $(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}) \times \lambda \vec{c}$ where $\lambda = [\vec{b} \vec{a} \vec{c}]$

B. If $\vec{a}, \vec{b}, \vec{c}$ are mutually \perp unit vectors, then $|\vec{a} + \vec{b} + \vec{c}| = 3$

C. If $\vec{a}, \vec{b}, \vec{c}$ are the p.v.s of the vertices of a Δ then its area is $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

D. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$ then $|\vec{a} + \vec{b} + \vec{c}| = 5$

Ans. D

11. If $\vec{a} = (2, -3, -1)$ and $\vec{b} = (1, 4, 2)$, then $\vec{a} \times \vec{b} =$
- A. $(-2, -5, 10)$
B. $(2, 5, -11)$
C. $(-2, -5, 11)$
D. None of these

Ans. C

$$\hat{a} \times \hat{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ 1 & 4 & 2 \end{vmatrix}$$

$$\begin{aligned} \text{Sol: } &= \hat{i}(-6+4) - \hat{j}(4+1) + \hat{k}(8+3) \\ &= -2\hat{i} - 5\hat{j} + 11\hat{k} \end{aligned}$$

Sol :

12. Let $\vec{a} = 2i + j + k$, $\vec{b} = i + 2j - k$ and a unit vector \vec{c} be coplanar. If \vec{c} is \perp \vec{a} , then $\vec{a} =$

- A. $\frac{1}{\sqrt{2}}(k - j)$
 B. $\frac{1}{\sqrt{3}}(-i - j - k)$
 C. $\frac{1}{\sqrt{5}}(i - 2j)$
 D. $\frac{1}{\sqrt{3}}(i + j - k)$

Ans. A

Sol :

$$\vec{c} \perp \vec{a}$$

$$\text{let } \vec{c} = (l, m, n)$$

since, \vec{c} is unit vector

$$l^2 + m^2 + n^2 = 1 \quad \text{--- 1}$$

$$\vec{c} \perp \vec{a}$$

$$\therefore 2l + m + n = 0 \quad \text{--- 2}$$

ans, $\vec{a} \times \vec{b} \perp \vec{c}$ since a, b, c are coplanar

now solve for $(l, m, n) = ?$

13. If $\vec{a} = i - 2j + 3k$, $\vec{b} = -3i + j - k$, $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$, then a unit vector in the direction of \vec{r} is

- A. $\frac{1}{\sqrt{6}}(-2i + j - k)$
 B. $-\frac{1}{3}(2i + j + 2k)$
 C. $\frac{1}{3}(-2i - j + 2k)$
 D. None of these

Ans. C

14. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then which of the following is true always?

- A. $|\vec{a} + \vec{b}| = |\vec{c}|$
- B. $|\vec{a}| + |\vec{b}| + |\vec{c}|$
- C. $\vec{a}, \vec{b}, \vec{c}$ from sides of the Δ
- D. None of these

Ans. A

Solution:

$$|\vec{a} + \vec{b}| = |\vec{c}| \text{ is true}$$

15. Let \vec{a} and \vec{b} be two non-collinear unit vectors. If $\vec{u} = \vec{a} - (\vec{a} \cdot \vec{b})\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$ then $|\vec{v}| =$

- A. $|\vec{u}|$
- B. $|\vec{u}| + |\vec{u} \cdot \vec{a}|$
- C. $|\vec{u}| + |\vec{u} \cdot \vec{b}|$
- D. $|\vec{u}| + \vec{u}(\vec{a} + \vec{b})$

Ans. C

16. Let $\vec{a} = 2i + j - 2k$ and $\vec{b} = i + j$ if \vec{c} is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = \sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and \vec{c} is 30° , then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is

- A. $\frac{2}{3}$
- B. $\frac{3}{2}$
- C. 2
- D. 3

Ans. B

17. If $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$ And $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} =$

- A. -14
- B. -7
- C. 7
- D. None of these

Ans. A

Solution:

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\text{then } (\vec{a} + \vec{b} + \vec{c})^2 = 0$$

$$2(ab + bc + ca) = -14$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -7$$

18. If $\vec{a} = 2\hat{i} + \hat{j}$, $\vec{b} = 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j}$ Then $\left(\vec{a} - 2\vec{b} \right) \cdot \vec{c} =$
- A. -2
B. 1
C. 0
D. None of these

Ans. D

Solution:

$$\left(\vec{a} - 2\vec{b} \right) \cdot \vec{c} = (2\hat{i} + \hat{j} - 4\hat{j} + 2\hat{k}) \cdot (\hat{i} + \hat{j}) = -1$$

19. If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 4\hat{j} - 3\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k}$, then $\vec{a} \times (\vec{b} \times \vec{c}) =$
- A. $11\vec{b} + 5\vec{c}$
B. $11\vec{c} - 5\vec{b}$
C. $3\vec{a} - 5\vec{b}$
D. None of these

Ans. D

Solution:

$$\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - 4\hat{j} - 3\hat{k} \text{ and } \vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k}, \text{ then } a \times \left(\vec{b} \times \vec{c} \right) =$$

$$= (a \cdot c)b - (a \cdot b)c = i - 2j - 7k$$

20. If $(\vec{a} \times \vec{b})(\vec{c} \times \vec{d}) = \alpha(\vec{b} \cdot \vec{d}) - \beta(\vec{b} \cdot \vec{c})$ then, $(\alpha, \beta) =$

- A. $(\vec{a} \cdot \vec{c}, \vec{a} \cdot \vec{b})$
- B. $(\vec{a} \cdot \vec{b}, \vec{a} \cdot \vec{d})$
- C. $(\vec{a} \cdot \vec{c}, \vec{d} \cdot \vec{d})$
- D. None of these

Ans. C

Solution:

$$\left(\vec{a} \times \vec{b} \right) \left(\vec{c} \times \vec{d} \right) = \left(\vec{a} \cdot \vec{c} \right) \left(\vec{b} \cdot \vec{d} \right) - \left(\vec{a} \cdot \vec{d} \right) \left(\vec{b} \cdot \vec{c} \right)$$