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Class: XII
Subject: Maths
Topic: Definite Integration
No. of Questions: 25

Q1. $\int_{-1}^{1} \frac{|x+2|}{x+2} dx =$

A. 0
B. 2
C. 1
D. none of these

Right Answer Explanation: B
For -1 ≤ x ≤ 1, 1 ≤ x + 2 ≤ 3

⇒ $|x+2| = x + 2$

Hence, $\int_{-1}^{1} \frac{|x+2|}{x+2} dx = \int_{-1}^{1} 1\, dx = 2$

Q2. $\int_{0}^{\pi/2} \sin^2 x\, dx =$

A. π/4
B. π/3
C. π/2
D. none of these

Right Answer Explanation: A

$$I = \frac{1}{2}\int_0^{\pi/2}(1 - \cos 2x)\,dx$$

$$= \frac{1}{2}\left[x - \frac{\sin 2x}{x}\right]_0^{\pi/2} = \frac{\pi}{4}$$

Q3. $\int_0^{\pi}\sqrt{1-\cos x}\,dx =$

A. 2
B. 1
C. $\sqrt{2}$
D. $2\sqrt{2}$

Right Answer Explanation: D

$$I = \int_0^{\pi}\sqrt{2\sin^2(x/2)}\,dx = \sqrt{2}\int_0^{\pi}\sin(x/2)\,dx = \sqrt{2}\,.\left[\frac{-\cos(x/2)}{(1/2)}\right]_0^{\pi}$$

$$= 2\sqrt{2}\left[-\cos\frac{x}{2} + \cos 0\right] = 2\sqrt{2}.$$

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Q4. $\int_0^{\pi} \sqrt{1+\sin x}\, dx =$

A. 2
B. $2\sqrt{2}$
C. 4
D. none of these

Right Answer Explanation: C

$$I = \int_0^{\pi} \sqrt{\cos^2(x/2)+\sin^2(x/2)+2\sin(x/2)\cos(x/2)}\, dx$$

$$= \int_0^{\pi} [\cos(x/2)+\sin(x/2)]\, dx$$

$$= \left[\frac{\sin(x/2)}{(1/2)} - \frac{\cos(x/2)}{(1/2)}\right]_0^{\pi}$$

$$= 2\,[\sin(x/2)-\cos(x/2)]_0^{\pi} = 4$$

Q5. Evaluate: $\int_0^{2\pi} \sqrt{1+\sin(x/2)}\, dx$

A. 8
B. 2
C. 4
D. None of these

Right Answer Explanation: A

$$1+\sin\frac{x}{2} = \cos^2\frac{x}{4}+\sin^2\frac{x}{4}+2\sin\frac{x}{4}\cos\frac{x}{4}$$

$$I = \int_0^{2\pi}\left(\cos\frac{x}{4}+\sin\frac{x}{4}\right)dx = 4\left[\sin\frac{x}{4}-\cos\frac{x}{4}\right]_0^{2\pi} = 8$$

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Q6. $\int_0^2 \sqrt{\frac{2+x}{2-x}}\,dx =$

A. $\left(\frac{\pi}{2} + 1\right)$
B. $(\pi + 1)$
C. $\left(\pi + \frac{3}{2}\right)$
D. none of these

Right Answer Explanation: D

$I = \int_0^2 \left(\frac{\sqrt{2+x}}{\sqrt{2-x}} \times \frac{\sqrt{2+x}}{\sqrt{2+x}}\right) dx = \int_0^2 \frac{2+x}{\sqrt{4-x^2}}\,dx = 2\int_0^2 \frac{dx}{\sqrt{4-x^2}} + \int_0^2 \frac{x}{\sqrt{4-x^2}}\,dx$

$= 2\left[\sin^{-1}\frac{x}{2}\right]_0^2 - \frac{1}{2}\int_0^2 \frac{-2x}{\sqrt{4-x^2}}$

$= 2 \times \frac{\pi}{2} - \frac{1}{2}\int_4^0 \frac{1}{\sqrt{t}}\,dt$

$= \pi + \frac{1}{2}\int_0^4 \frac{1}{\sqrt{t}}\,dt = \pi + \frac{1}{2} \times \left[2\sqrt{t}\right]_0^4$ [Putting $4 - x^2 = t$]

$= (\pi + 2)$.

Q7. $\int_3^6 4[x]\,dx =$

A. 24
B. 36
C. 48
D. 12

Right Answer Explanation: C

$I = 4\left[\int_3^4 [x]dx + \int_4^5 [x]dx + \int_5^6 [x]dx\right]$

$= 4 . \left[\int_3^4 3\,dx + \int_4^5 4\,dx + \int_5^6 5\,dx\right]$

$= 4 . \left[[3x]_3^4 + [4x]_4^5 + [5x]_5^6\right]$

= 4 . [3 + 4 + 5] = 48.

Q8. $\int_0^2 [2x]\,dx =$

A. 2
B. 3
C. 4
D. none of these

Right Answer Explanation: B

$I = \int_0^{1/2} [2x]dx + \int_{1/2}^1 [2x]dx + \int_1^{3/2} [2x]dx + \int_{3/2}^2 [2x]dx = \int_0^{1/2} 0\,dx + \int_{1/2}^1 1\,dx + \int_1^{3/2} 2\,dx + \int_{3/2}^2 3\,dx$

$= [x]_{1/2}^1 + [2x]_1^{3/2} + [3x]_{3/2}^2 = \left(\frac{1}{2} + 1 + \frac{3}{2}\right) = 3.$

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Q9. $\int_{-1}^{2} x|x|\,dx =$

A. 3
B. 7/3
C. 2
D. none of these

Right Answer Explanation: B

[1 ≤ x < 0 ⇒ | x | = x] and
[0 ≤ x < 2 ⇒ | x | = x]

$$\int_{-1}^{0} x|x|dx + \int_{0}^{2} x|x|dx = \int_{-1}^{0} -x^2\,dx + \int_{0}^{2} x^2\,dx = \left[\frac{-x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{2} = \left(\frac{-1}{3}+\frac{8}{3}\right) = \frac{7}{3}$$

Q10. $\int_{0}^{\pi} |\cos\theta - \sin\theta|\,d\theta =$

A. 2 √2
B. 2
C. √2
D. none of these

Right Answer Explanation: A

[0 ≤ θ < $\frac{\pi}{4}$ ⇒ cos θ > sin θ] &

$\left[\frac{\pi}{4} \le \theta < \pi\right]$ ⇒ cos θ ≤ sin θ.

$$I = \int_{0}^{\pi/4} |\cos\theta - \sin\theta|\,d\theta + \int_{\pi/4}^{\pi} |\cos\theta - \sin\theta|\,d\theta$$

$$= \int_{0}^{\pi/4} |\cos\theta - \sin\theta|\,d\theta + \int_{\pi/4}^{\pi} (\sin\theta - \cos\theta)\,d\theta$$

$$= [\sin\theta + \cos\theta]_{0}^{\pi/4} + [-\cos\theta - \sin\theta]_{\pi/4}^{\pi} = 2\sqrt{2}\,.$$

Q11. $\int_1^4 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx =$

A. 14/3
B. 20/3
C. 8/3
D. None of these

Right Answer Explanation: B

$I = \int_1^4 x^{1/2}dx + \int_1^4 x^{-1/2}dx$

$= \left[\frac{2}{3}x^{3/2}\right] + \left[2\sqrt{x}\right]_1^4 = \frac{20}{3}$

Q12. Evaluate: $\int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}} dx$

A. π/2
B. 0
C. π/4
D. None of these

Right Answer Explanation: C

Let $I = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}} = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx \quad (1)$

Then, $I = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}$

$dx = \int_0^{\pi/2} \frac{\sqrt{\sin x}\, dx}{\sqrt{\sin x}+\sqrt{\cos x}}$ - (2)

Adding (1) and (2), we get

$2I = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

Q13. The value of $\int_0^{\pi/2} \sin x\, e^{\cos x}\, dx$ is

A. $e^{\pi/2} - 1$
B. e
C. -1
D. e - 1

Right Answer Explanation: D
Put cos x = t, we get

$\int_0^{\pi/2} \sin x\, e^{\cos x}\, dx$

$= -\int_1^0 e^t\, dt = e - 1$

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Q14. $\int_0^{\pi/2} \frac{\sqrt{\cot x}\,dx}{\sqrt{\cot x} + \sqrt{\tan x}}$ dx =

A. π/4
B. π/2
C. π
D. 0

Right Answer Explanation: A

Use $\int_0^a f(x)$ dx = $\int_0^a f(a-x)$ dx, then the given integral, say,

I = $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}}$ dx - (1)

or I = $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}$ dx - (2)

Adding (1) & (2), we get
2 I = π/2
So, I = π/4

Q15. $\int_{-8}^{8} \left(\sin^{93} x + x^{295}\right) dx$ =

A. 0
B. a number different from 0
C. $2\,(8^{295} + 1)$
D. $2 + 8^{295}$

Right Answer Explanation: A

As $f(x) = \sin^{93} x + x^{295}$ is odd function and $\int_{-a}^{a}$ f(x) dx = 0 whenever f(x) is an odd function

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Q16. $\int_0^{\pi/2} \sin x \sin 2x \, dx$

A. 2/3
B. 1/3
C. π/3
D. none of these

Right Answer Explanation: A

$\int_0^{\pi/2} \sin x \sin 2x \, dx = 2 \int_0^{\pi/2} \sin^2 x \cos x \, dx$

$= 2 \left[\frac{\sin^3 x}{3}\right]_0^{\pi/2} = 2/3$

Q17. $\int_0^{\frac{\pi}{2}} e^{3\cos x} \sin x \, dx =$

A. $1/3\,(1 - e^3)$
B. $1/3\,(e^{3} - 1)$
C. $(1 - e^{-3})$
D. none of these

Right Answer Explanation: B

Putting cos $x = t$, we get,

$I = \int_1^0 e^{3t} dt = \int_0^1 e^{3t} dt = \left[\frac{1}{3} e^{3t}\right]_0^1 = \frac{1}{3}(e^3 - 1)$

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$$\int_0^1 \frac{dx}{(1+x^2)^{3/2}} =$$

Q18.

A. 3/√2
B. 1/√2
C. √3/2
D. none of these

Right Answer Explanation: B

Putting x = tan θ and dx = $\sec^2\theta$ d θ

$$I = \int_0^{\pi/4} \frac{\sec^2\theta}{(\sec^2\theta)^{3/2}}\, d\theta = \int_0^{\pi/4} \cos\theta$$

$$= [\sin\theta]_0^{\pi/4} = \frac{1}{\sqrt{2}}$$

$$\int_0^{\frac{\pi}{2}} \sin^4 x \cos^3 x \, dx =$$

Q19.

A. π/16
B. π/32
C. 16/π
D. 2/35

Right Answer Explanation: D

Putting sin x = t, we get,

$$I = \int_0^{\pi/2} \sin^4 x\,(1 - \sin^2 x) \cos x \, dx$$

$$= \int_0^1 t^4(1-t^2)\,dt = \int_0^1 (t^4 - t^6)\,dt = \left[\frac{t^5}{5} - \frac{t^7}{7}\right]_0^1$$

$$= \left(\frac{1}{5} - \frac{1}{7}\right) = \frac{2}{35}$$

$$\int_0^{\frac{\pi}{2}} \sin^7 x \; dx =$$

Q20.

A. 3π/16
B. 16π/35
C. 16/35
D. none of these

Right Answer Explanation: C

By walli-s formula, we get,

$$\int_0^{\pi/2} \sin^7 x \; dx = \frac{6\times4\times2}{7\times5\times3} = \frac{16}{35}$$

Q21. Using the properties of definite integrals,

Evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx.$ [All India 2008C]

Right Answer Explanation:

Let $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ ...(i)

$\therefore$ $I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{a-(a-x)}}dx$ **(1)**

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

$\Rightarrow$ $I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$ ...(ii) **(1)**

On adding Eqs. (i) and (ii), we get

$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$ **(1)**

$\Rightarrow$ $2I = \int_0^a dx$

$\Rightarrow$ $2I = [x]_0^a$

$\Rightarrow$ $2I = a - 0 = a$

$\therefore$ $I = \frac{a}{2}$ **(1)**

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Q22. Evaluate $\int_0^{\pi/4} \log(1+\tan x)\,dx$. [Delhi 2013C; All India 2010, 2008C]

Right Answer Explanation:

Let $I = \int_0^{\pi/4} \log(1+\tan x)\,dx$

$\therefore \quad I = \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right] dx$

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

$= \int_0^{\pi/4} \log\left(1+\frac{\tan\frac{\pi}{4}-\tan x}{\tan\frac{\pi}{4}+\tan x}\right) dx \qquad \textbf{(1)}$

$= \int_0^{\pi/4} \log\left(1+\frac{1-\tan x}{1+\tan x}\right) dx$

$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right) dx$

$= \int_0^{\pi/4} (\log 2)\,dx - \int_0^{\pi/4} \log(1+\tan x)\,dx \qquad \textbf{(1)}$

$= \int_0^{\pi/4} (\log 2)\,dx - I$

$\Rightarrow \quad 2I = \int_0^{\pi/4} \log 2\,dx \qquad \textbf{(1)}$

$= \log 2\,[x]_0^{\pi/4} = \log 2\left[\frac{\pi}{4}-0\right]$

$\therefore \quad I = \frac{\pi}{8}\log 2 \qquad \textbf{(1)}$

Q23. Evaluate $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$. [Delhi 2011; All India 2010C]

Right Answer Explanation:

Let $I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ ...(i)

Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, we get

$$I = \int_0^{\pi/2} \frac{\left[\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)\right]}{\sin^4\left(\frac{\pi}{2} - x\right) + \cos^4\left(\frac{\pi}{2} - x\right)} dx \quad \textbf{(1)}$$

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$$\Rightarrow I=\int_0^{\pi/2}\frac{\left(\frac{\pi}{2}-x\right)\cos x\sin x}{\cos^4 x+\sin^4 x}dx$$

$$\left[\because \sin\left(\frac{\pi}{2}-x\right)=\cos x \text{ and } \cos\left(\frac{\pi}{2}-x\right)=\sin x\right]$$

$$\Rightarrow I=\frac{\pi}{2}\int_0^{\pi/2}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx-\int_0^{\pi/2}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx \quad \text{...(ii) (1)}$$

On adding Eqs. (i) and (ii), we get

$$2I=\frac{\pi}{2}\int_0^{\pi/2}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$\Rightarrow \quad 2I=\frac{\pi}{2}\int_0^{\pi/2}\frac{\sin x\cos x}{\sin^4 x+(1-\sin^2 x)^2}dx$$

$$[\because \cos^2 x=1-\sin^2 x] \text{ (1)}$$

Now, put $\sin^2 x=t$

$$\Rightarrow \quad 2\sin x\cos x\,dx=dt$$

$$\Rightarrow \quad \sin x\cos x\,dx=\frac{dt}{2}$$

Also, when $x=0$, then $t=\sin^2 0°=0$

and when $x=\frac{\pi}{2}$, then $t=\sin^2\frac{\pi}{2}=1$

$$\therefore \quad 2I=\frac{\pi}{4}\int_0^1\frac{dt}{t^2+(1-t)^2}$$

$$\Rightarrow \quad I=\frac{\pi}{8}\int_0^1\frac{dt}{t^2+(1-t)^2}$$

$$=\frac{\pi}{8}\int_0^1\frac{dt}{t^2+1+t^2-2t}=\frac{\pi}{8}\int_0^1\frac{dt}{2t^2-2t+1} \text{ (1)}$$

$$=\frac{\pi}{8\times 2}\int_0^1\frac{1}{t^2-t+\frac{1}{2}}dt$$

$$=\frac{\pi}{16}\int_0^1\frac{1}{t^2-t+\frac{1}{2}+\frac{1}{4}-\frac{1}{4}}dt$$

$$=\frac{\pi}{16}\int_0^1\frac{1}{\left(t-\frac{1}{2}\right)^2-\frac{1}{4}+\frac{1}{2}}dt$$

$$\left[\because t^2-t+\frac{1}{4}=\left(t-\frac{1}{2}\right)^2\right]$$

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$$= \frac{\pi}{16}\int_0^1 \frac{1}{\left(t-\frac{1}{2}\right)^2+\frac{1}{4}}\,dt$$

$$= \frac{\pi}{16}\int_0^1 \frac{1}{\left(t-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}\,dt \qquad \textbf{(1)}$$

$$= \frac{\pi}{16}\cdot\frac{1}{\frac{1}{2}}\tan^{-1}\left[\frac{t-\frac{1}{2}}{\frac{1}{2}}\right]_0^1$$

$$\left[\because \int\frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}\right]$$

$$= \frac{\pi}{8}[\tan^{-1}1-\tan^{-1}(-1)]$$

$$= \frac{\pi}{8}\left[\tan^{-1}\left(\tan\frac{\pi}{4}\right)-\tan^{-1}\left\{\tan\left(-\frac{\pi}{4}\right)\right\}\right]$$

$$= \frac{\pi}{8}\left[\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right]$$

$$\left[\because 1=\tan\frac{\pi}{4} \text{ and } -1=\tan\left(-\frac{\pi}{4}\right)\right]$$

$$\therefore \qquad I=\frac{\pi^2}{16} \qquad \textbf{(1)}$$

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Q24. Evaluate $\int_1^3 (x^2 + 5x)$ dx as the limit of a sum. [Delhi 2008C]

Right Answer Explanation:

Here, $I = \int_1^3 (x^2 + 5x)\, dx$

On comparing with $\int_a^b f(x)\, dx$, we get

$$a = 1,\ b = 3,\ f(x) = x^2 + 5x \qquad \textbf{(1)}$$

$$\because \quad \int_a^b f(x)dx = \lim_{h \to 0} h[f(a) + f(a+h)$$
$$+ f(a+2h) + \ldots + f\{a + (n-1)h\}] \qquad \ldots(i)$$

where, $nh = b - a$

$$nh = 3 - 1 = 2$$

Now, $f(a) = f(1) = (1)^2 + 5(1) = 6$

$$f(a+h) = f(1+h)$$
$$= (1+h)^2 + 5(1+h)$$
$$= 1 + h^2 + 2h + 5 + 5h$$
$$f(1+h) = h^2 + 7h + 6$$

Now, $f(a+2h) = f(1+2h)$

$$= (1+2h)^2 + 5(1+2h)$$
$$= 1 + 4h^2 + 4h + 5 + 10h$$
$$\therefore \quad f(1+2h) = 4h^2 + 14h + 6 \qquad \textbf{(1)}$$

....

....

$$f[a + (n-1)h] = f[1 + (n-1)h]$$
$$= [1 + (n-1)h]^2 + 5[1 + (n-1)h]$$
$$= 1 + (n-1)^2h^2 + 2(n-1)h$$
$$+ 5 + 5(n-1)h$$
$$\therefore \ f[1 + (n-1)h] = (n-1)^2h^2 + 7(n-1)h + 6 \qquad \textbf{(1)}$$

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On putting above values in Eq. (i), we get

$$I=\int_1^3 (x^2+5x)\,dx = \lim_{h\to 0} h\,[6+(h^2+7h+6) + (4h^2+14h+6) + \ldots + (n-1)^2h^2 + 7(n-1)h + 6]$$

$$= \lim_{h\to 0} h[(6+6+6+\ldots+6) + h^2\{1+4+\ldots+(n-1)^2\} + 7h\{1+2+\ldots+(n-1)\}]$$

$$= \lim_{h\to 0} h[6n + h^2\{1^2+2^2+\ldots+(n-1)^2\} + 7h\{1+2+\ldots+(n-1)\}]$$

$$= \lim_{h\to 0} h\left[6n + h^2\frac{n(n-1)(2n-1)}{6} + 7h\frac{n(n-1)}{2}\right] \quad \textbf{(1)}$$

$$\left[\because 1^2+2^2+\ldots+(n-1)^2 = \frac{n(n-1)(2n-1)}{6} \text{ and } 1+2+\ldots+(n-1) = \frac{n(n-1)}{2}\right]$$

$$= \lim_{h\to 0}\left[6nh + \frac{nh(nh-h)(2nh-h)}{6} + \frac{7nh(nh-h)}{2}\right]$$

$$= \lim_{h\to 0}\left[6(2) + \frac{2(2-h)(4-h)}{6} + \frac{7\times 2(2-h)}{2}\right] \quad \textbf{(1)}$$

$$[\because nh = 2]$$

$$= 12 + \frac{2\times 2\times 4}{6} + \frac{14\times 2}{2}$$

$$= 12 + \frac{8}{3} + 14 = \frac{36+8+42}{3} = \frac{86}{3} \quad \textbf{(1)}$$

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Q25.Evaluate $\int_0^1 \cot^{-1}(1-x+x^2)\,dx.$ [Hots; Delhi 2008]

Right Answer Explanation:

Let $I = \int_0^1 \cot^{-1}(1-x+x^2)\,dx$

$\therefore \quad I = \int_0^1 \tan^{-1}\left[\frac{1}{1-x+x^2}\right]dx$

$\left[\because \cot^{-1}x = \tan^{-1}\frac{1}{x}\right]$

$\Rightarrow \quad I = \int_0^1 \tan^{-1}\left[\frac{x+(1-x)}{1-x(1-x)}\right]dx$ **(1)**

$\Rightarrow \quad I = \int_0^1 [\tan^{-1}x + \tan^{-1}(1-x)]\,dx$

$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$\Rightarrow I = \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}[1-(1-x)]\,dx$ **(1)**

$\left.\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \text{ use in second integral}\right]$

$\Rightarrow \quad I = \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}x\,dx$

$\Rightarrow \quad I = 2\int_0^1 \tan^{-1}x\,dx = 2\int_0^1 \underset{\text{I}}{\tan^{-1}x}\cdot \underset{\text{II}}{1}\,dx$

On applying integration by parts, we get

$= 2\left[[x\tan^{-1}x]_0^1 - \frac{1}{2}\int_0^1 \frac{2x}{1+x^2}dx\right]$ **(1½)**

$= 2\left[[1\tan^{-1}(1) - 0] - \frac{1}{2}[\log(1+x^2)]_0^1\right]$

$\left[\begin{array}{l}\because \int \frac{2x}{1+x^2}dx \\ \text{put } 1+x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dt = \frac{dt}{2} \\ \therefore \int \frac{2x}{1+x^2}dx = \int \frac{dt}{t} = \log t = \log(1+x^2)\end{array}\right]$

(1½)

$= 2\left[\frac{\pi}{4} - \frac{1}{2}(\log 2 - \log 1)\right] = 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right]$

$\therefore \quad I = \frac{\pi}{2} - \log 2$ **(1)**