

Class: XII
Subject: Maths
Topic: Differentiation and Differentiability
No. of Questions: 26

1. If $f(3) = 2$, then $\lim_{h \rightarrow 0} \frac{f(3+h^2) - f(3-h^2)}{2h^2}$ is
- A. 1
B. 2
C. 3
D. 1/2
2. If $f(x) = |\log_{10} x|$ then at $x = 1$
- A. f is not continuous
B. f is continuous but not differentiable
C. f is differentiable
D. the derivative is 1
3. If $f(x) = |x - a| + |x + b|$, $x \in \mathbb{R}$, $b > a > 0$, then
- A. $f'(a+) = 1$
B. $f'(a+) = 0$
C. $f'(-b+) = 0$
D. $f'(-b+) = 1$
4. The function $f(x) = \sin^{-1}(\tan x)$ is not differentiable at
- A. $x = 0$
B. $x = -\pi/6$
C. $x = \pi/6$
D. $x = \pi/4$

5. If $f(x) = \log_x^2(\log x)$ then $f'(x)$ at $x = e$ is
- A. 0
B. 1
C. e^{-1}
D. $(2e)^{-1}$
6. Let $f(x) = \sin x$; $g(x) = x^2$ and $h(x) = \log x$. If $F(x) = h(f(g(x)))$, then $F''(x)$ is
- A. $2 \operatorname{cosec}^3 x$
B. $2 \cot x^2 - 4x^2 \operatorname{cosec}^2 x^2$
C. $2x \cot x^2$
D. $-2 \operatorname{cosec}^2 x$
7. Let $f'(x+y) = f'(x)f'(y)$ for all x and y . If $f'(5) = 2$ and $f'(0) = 3$ then $f'(5)$ is equal to
- A. 5
B. 8
C. 0
D. None of these
8. If $\cos^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = \log a$ then $\frac{dy}{dx}$ is
- A. y/x
B. x/y
C. x^2/y^2
D. y^2/x^2
9. The set onto which the derivative of the function $f(x) = x(\log x - 1)$ maps the ray $[1, \infty)$ is
- A. $[1, \infty)$
B. $(0, \infty)$
C. $[0, \infty)$
D. None of these

10. Let $g = f^{-1}$ and $f'(x) = \frac{1}{1+x^4}$ then $g'(x)$ is equal to
- A. $(1 + (g(x))^4)^{-1}$
 - B. $(1 + (f(x))^4)^{-1}$
 - C. $1 + (f(x))^4$
 - D. $1 + (g(x))^4$
11. The solution set of $f'(x) > g'(x)$ where $f(x) = (1/2)5^{2x+1}$ and $g(x) = 5^x + 4x \log 5$ is
- A. $(1, \infty)$
 - B. $(0, 1)$
 - C. $[0, \infty)$
 - D. $(0, \infty)$
12. For a whole number n , if $f(x) = x^{n-1} \sin(1/x)$, $x \neq 0$ and $f(0) = 0$ then in order that f is differentiable at all x , the value of n can be
- A. 1
 - B. 2
 - C. 3
 - D. 0
13. If the function f is differentiable and strictly increasing in a neighborhood of 0, then $\lim_{x \rightarrow 0} \frac{f(x^5) - f(x)}{f(x) - f(0)}$ is equal to
- A. 1
 - B. 0
 - C. -1
 - D. 5/2

14. $\frac{d^2 x}{dy^2}$ equals

- A. $\left(\frac{d^2 y}{dx^2}\right)^{-1}$
B. $-\left(\frac{d^2 y}{dx^2}\right)^{-1} \left(\frac{dy}{dx}\right)^{-3}$
C. $\left(\frac{d^2 y}{dx^2}\right) \left(\frac{dy}{dx}\right)^{-2}$
D. $-\left(\frac{d^2 y}{dx^2}\right) \left(\frac{dy}{dx}\right)^{-3}$

15. If $2f(\sin x) + f(\cos x) = x$ for all $x \in \mathbb{R}$ then $y = f(x)$ satisfies

- A. $(1 - x^2) y'' - xy' = 0$
B. $x^2 y'' - (1 - x^2)y' = 0$
C. $(1 + x^2) y'' + xy' = 0$
D. $(1 - x^2) y'' + xy' = 0$

16. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be any function. Define $g : \mathbb{R} \rightarrow \mathbb{R}$ by $g(x) = |f(x)|$ for all x . Then g is

- A. g may be bounded even if f is unbounded
B. one-one if f is one
C. continuous if f is continuous
D. differentiable if f is differentiable

17. The function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos|x|$ is not differentiable at

- A. 1
B. 0
C. -1
D. 2

18. If $y = \tan^{-1} \left(\frac{\sqrt{1+x^2} - 1}{x} \right)$, then $y'(0)$ is

- A. 1/2
- B. 0
- C. 1
- D. Doesn't exist

19. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$ then $f^2(24) + g^2(24)$ is

- A. 32
- B. 24
- C. 64
- D. 48

20. If $y = \log \cos \left(\tan^{-1} \frac{e^x - e^{-x}}{2} \right)$

Statement - 1 : $y'(0) = 0$

Statement - 2 : $y'(x) = -\frac{e^x - e^{-x}}{1+x^2}$

- A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
- B. Statement – 1 is true, statement – 2 is true but statement – 2 is not the correct explanation for statement – 1.
- C. Statement – 1 is true, statement – 2 is false.
- D. Statement – 1 is false, statement – 2 is true.

21. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$

Then prove that $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$.

[Delhi 2013C]

OR

If $y = (\sin x)x + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

[Delhi 2013C, 2009; All India 2009C]

22. Differentiate the following function with respect to x.

$$(\log x)^x + x^{\log x}$$

[Delhi 2013]

23. If $y = \log[x + \sqrt{x^2 + a^2}]$, then show that

$$(x^2 + a^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0.$$

[Delhi 2013]

24. If $(\cos x)^y = (\cos y)^x$, then find $\frac{dy}{dx}$.

[Hots; Delhi 2012]

25. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, ($x \neq y$), then prove

$$\text{That } \frac{dy}{dx} = -\frac{1}{(1+x)^2}.$$

[Hots; Foreign 2012; Delhi 2011 C]

26. If $y = Ae^{mx} + Be^{nx}$, then show that

$$\frac{d^2y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0.$$

[All India 2009 C]