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Class: XII
Subject: Maths
Topic: Differential Equation
No. of Questions: 24

Q1. Consider the differential equation $\frac{dy}{dx} = \frac{y}{2y\log\ y+y-x}$

A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
B. Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for statement – 1.
C. Statement – 1 is true, statement – 2 is false.
D. Statement – 1 is false, statement – 2 is true.

Sol. A

$$\frac{dx}{dy} = \frac{2y\log y + y - x}{y} = 2\log y + 1 - \frac{x}{y}$$

$\Rightarrow$ $\frac{dx}{dy} + \frac{x}{y} = 2\log y + 1$ which is linear in x and y.

$$\text{I.F.} = e^{\int \frac{1}{y} dy} = y. \text{ So}$$

$$\frac{d}{dy}(xy) = 2y\log y + y$$

$$\Rightarrow \quad xy = 2\int y\log y\, dy + \frac{y^2}{2} + C$$

$$= 2\left[\frac{y^2}{2}\log y - \frac{y^2}{4}\right] + \frac{y^2}{2} + C$$

$$= y^2\log y + C.$$

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Q2. Statement – 1: The differential equation of all circles in a plane must be of order 3.
Statement – 2 : There is only one circle passing through three non-collinear points.

A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
B. Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for statement – 1.
C. Statement – 1 is true, statement – 2 is false.
D. Statement – 1 is false, statement – 2 is true.

Sol. (A) The equation of a circle contains three independent constants if it passes through three Non-collinear points.

Q3. Statement -1: Curve satisfying the differential equation y' = y/2x through (2,1) is a parabola with focus (1/4, 0)

Statement – 2: The differential equation y' = y/2x is of variable separable form.

A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
B. Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for statement – 1.
C. Statement – 1 is true, statement – 2 is false.
D. Statement – 1 is false, statement – 2 is true.

Sol. D

$$\frac{dy}{dx} = \frac{y}{2x} \Rightarrow \frac{2dy}{y} = \frac{dx}{x}$$

$\Rightarrow$ log y^2 = log x + const $\Rightarrow$ y^2 = Cx, this passes through (2, 1) if C = 1/2. Thus y^2 = 1/2x which represents a parabola with focus (1/8, 0).

Q4 Let a solution y = y(x) of the differential equation $x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$ satisfy y(2) = 2 $\sqrt{3}$

Statement – 1: y(x) = sec ($\sec^{-1}$ x - $\pi/6$)

Statement – 2: y(x) is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1-\frac{1}{x^2}}$

A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
B. Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for statement – 1.

C. Statement – 1 is true, statement – 2 is false.
D. Statement – 1 is false, statement – 2 is true.

Sol. C

The given equation can be written as

$$\frac{dy}{y\sqrt{y^2-1}} - \frac{dx}{x\sqrt{x^2-1}} = 0$$

Integrating $\sec^{-1}y - \sec^{-1}x = \text{const}$

Putting $x = 2$, const $= \sec^{-1} 2/\sqrt{3} - \sec^{-1}2 = \frac{\pi}{6} - \frac{\pi}{3} = -\frac{\pi}{6}$

so $y = \sec(\sec^{-1}x - \pi/6)$.

Q5. Let $(xy^2 + x)dx + (y - x^2y)dy = 0$ satisfy $y(0) = 0$.

Statement – 1: The curve represented by the solution of the given differential equation is a circle.

Statement – 2: It is circle with radius 1 and centre (0, 0).

A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
B. Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for statement – 1.
C. Statement – 1 is true, statement – 2 is false.
D. Statement – 1 is false, statement – 2 is true.

Sol. C

$$x(1 + y^2)dx + y(1 - x^2)dx = 0$$

$$\Rightarrow \quad \frac{x}{1-x^2}dx + \frac{y}{1+y^2}dy = 0$$

$$\Rightarrow \quad -\log(1 - x^2) + \log(1 + y^2) = \text{Const}$$

$$\Rightarrow \quad 1 + y^2 = C(1 - x^2)$$

Since $y(0) = 0$ so $C = 1$

$$\Rightarrow \quad x^2 + y^2 = 0 \text{ which is a point circle.}$$

Q6. Let $y' + \sin\frac{x+y}{2} = \sin\frac{x-y}{2}$

Statement – 1: A solution satisfying y(0) = π is a periodic function with period 4π.

Statement – 2: y can be explicitly represented in terms of x.

A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
B. Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for statement – 1.
C. Statement – 1 is true, statement – 2 is false.
D. Statement – 1 is false, statement – 2 is true.

Sol. B

$y' = \sin\frac{x-y}{2} - \sin\frac{x+y}{2} = -2\sin\frac{y}{2}\cos\frac{x}{2}.$

$\Rightarrow$ cosec y/2 dy = – 2 cos (x/2) dx

$\Rightarrow$ 2 log | tan (y/4) | = – 4 sin (x/2) + Const

$\Rightarrow$ log | tan (y/4) | = – 2 sin (x /2) + Const

$\Rightarrow$ | tan (y/4) | = Const $e^{-2\sin(x/2)}$

Since y(0) = π so Const = 1. Thus |tan (y/4)| = $e^{-2\sin x/2}$ so y = 4 $\tan^{-1}\left(\pm e^{-2\sin x/2}\right)$ which is periodic with period 4π.

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Q7. Let f(x) be differentiable on the interval (0, ∞) such that f(1) = 1, and $\lim\limits_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each x > 0. Then f(x) is

A. $\frac{1}{3x} + \frac{2x^2}{3}$

B. $-\frac{1}{3x} + \frac{4x^2}{3}$

C. $-\frac{1}{x} + \frac{2}{x^2}$

D. $\frac{1}{x}$

Sol. A

$$1 = \lim_{t \to x} \frac{t^2 f(x) - t^2 f(t) + t^2 f(t) - x^2 f(t)}{t - x}$$

$$= \lim_{t \to x} \frac{t^2 (f(x) - f(t))}{t - x} + \lim_{t \to x} (t + x) f(t)$$

$$= -x^2 f'(x) + 2x f(x)$$

Thus $f(x)$ is a solution of the differential equation

$$x^2 \frac{dy}{dx} - 2xy = -1 \quad \Rightarrow \quad \frac{dy}{dx} - \frac{2}{x} y = -\frac{1}{x^2} \qquad (1)$$

This is a linear equation with I.F. $= e^{-\int \frac{2}{x} dx} = \frac{1}{x^2}$.

Multiplying (1) with $\left(1/x^2 \right)$ we get

$$\frac{d}{dx}\left(\frac{y}{x^2}\right) = -\frac{1}{x^4} \quad \Rightarrow \quad \frac{y}{x^2} = \frac{1}{3x^3} + C$$

Since $f(1) = 1$ so $C = \frac{2}{3}$ $\therefore$ $y = \frac{2}{3}x^2 + \frac{1}{3x}$.

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Q8. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t),$ where k > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is:

A. e^{-kT}
B. $T^2 - \frac{1}{k}$
C. $I - \frac{KT^2}{2}$
D. $I - K(T)$

Sol. C

$$\frac{dv}{dt} = -K(T-t), K > 0$$

$\Rightarrow \quad dV = -K(T-t)\,dt$

Integrating, we have $V(t) = \frac{k(T-t)^2}{2} + C$

We have $V(0) = I$, therefore

$$I = \frac{KT^2}{2} + C \quad \Rightarrow \quad C = I - \frac{k}{2}T^2$$

Scarp value $V(T) = C = I - \frac{K}{2}T^2$.

Q9. The solution of $x^3 \frac{dy}{dx} + 4x^2 \tan y = e^x \sec y$ satisfying y(1) = 0 is

A. $\tan y = (x - 2)\, e^x \log x$
B. $\sin y = e^x (x - 1)\, x^{-4}$
C. $\tan y = (x - 1)\, e^x x^{-3}$
D. $\sin y = e^x (x - 1)\, x^{-3}$

Sol. B

Rewriting the given equation in the form

$$x^4 \cos y \frac{dy}{dx} + 4x^3 \sin y = xe^x \quad \Rightarrow \quad \frac{d}{dx}(x^4 \sin y) = xe^x$$

$$\Rightarrow \qquad x^4 \sin y = \int xe^x \, dx$$

$$= (x-1)e^x + C$$

Since $y(1) = 0$, so $C = 0$.

Thus $\sin y = x^{-4}(x-1)e^x$.

Q10. A solution of the equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$ is

A. $y = xe^{cx}$
B. $y^2/x = cx$
C. $y^2 = cx \log x$
D. $\log y = cx$

Sol. A

Putting $y = vx$ in the given equation, we have

$$v + x\frac{dv}{dx} = v(\log v + 1) \quad \Rightarrow \quad x\frac{dv}{dx} = v \log v$$

$$\Rightarrow \quad \frac{dv}{v \log v} = \frac{dx}{x} \quad \Rightarrow \quad \log|\log v| = \log|x| + \text{Const}$$

$$\Rightarrow \quad \log v = \pm Ax = cx \quad \Rightarrow \quad y = xe^{cx}.$$

Q11. The population p(t) at time t of a certain mouse species follows the differential equation $\frac{dp(t)}{dt}$ = 0.5 p(t) – 450. If p(0) = 850, then the time at which the population becomes zero is

A. log 9
B. $\frac{1}{2}\log 18$
C. log 18
D. 2 log 18

Sol. D

The given differential equation in a linear equation with I.F. = $e^{-\int(0.5)dt} = e^{-t/2}$. Multiplying with I.F. we have

$$\frac{d}{dt}\left(e^{-t/2}p\right) = -450e^{-t/2} \Rightarrow e^{-t/2}p(t) = 900e^{-t/2} + C$$

When $t = 0$, $p = 850$,

$$850 = 900 + C \Rightarrow C = -50$$

Thus $p(t) = 900 - 50e^{t/2}$

When $p(t) = 0$, we get $900 - 50e^{t/2} = 0$

$\Rightarrow e^{t/2} = 18 \Rightarrow t/2 = \log 18$ i.e. $t = 2\log 18$

Q12. The degree and order respectively of the differential equation of all parabolas whose axis is x-axis, are:

A. 2, 1
B. 1, 2
C. 2, 2
D. 1, 1

Sol. B

Equation of any parabola whose axis is x – axis is $y^2 = 4a(x + b)$

$$\Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

which is a differential equation of order 2 and degree 1.

Q13. The equation of the curve whose tangent at any point (x, y) makes an angle $\tan^{-1}(2x + 3y)$ with x-axis and which passes through (1, 2) is:

A. $6x + 9y + 2 = 26e^{3(x-1)}$
B. $6x - 9y + 2 = 26e^{3(x-1)}$
C. $6x + 9y - 2 = 26e^{3(x-1)}$
D. $6x - 9y - 2 = 26e^{3(x-1)}$

Sol. A

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$$\frac{dy}{dx} = \tan[\tan^{-1}(2x + 3y)] = 2x + 3y$$

$$\Rightarrow \quad \frac{dy}{dx} - 3y = 2x$$

I.F. $= e^{-3x}$ Multilplying (1) by e^{-3x}, we get

$$e^{-3x}\frac{dy}{dx} - 3e^{-3x}y = 2xe^{-3x} \Rightarrow \frac{d}{dx}[ye^{-3x}] = 2xe^{-3x}$$

$$\Rightarrow \quad ye^{-3x} = \int 2xe^{-3x}\,dx = -\frac{2}{3}xe^{-3x} + \frac{2}{3}\int(1)e^{-3x}\,dx$$

$$= -\frac{2}{3}xe^{-3x} - \frac{2}{9}e^{-3x} + C$$

As this curve passes through (1, 2), we get

$$2 = -\frac{2}{9}(3+1) + Ce^{3} \quad \Rightarrow C = \frac{26}{9}e^{-3}$$

Thus, required curve is

$$y = -\frac{2}{9}(3x+1) + \frac{26}{9}e^{3(x-1)} \quad \Rightarrow 6x + 9y + 2 = 26e^{3(x-1)}$$

Q14. If for the differential equation y ' = $\frac{y}{x} + \phi\left(\frac{x}{y}\right)$ the general solution is y = $\frac{x}{\log|Cx|}$ then ϕ (x/y) is given by

A. $-x^2/y^2$
B. y^2/x^2
C. x^2/y^2
D. $-y^2/x^2$

Sol. D

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Putting $v = y/x$ so that $\frac{x\,dv}{dx} + v = \frac{dy}{dx}$, we have

$$\frac{x\,dv}{dx} + v = v + \phi(1/v) \Rightarrow \frac{dv}{\phi(1/v)} = \frac{dx}{x}$$

$$\Rightarrow \log|Cx| = \int \frac{dv}{\phi(1/v)} \text{ (being constant of integration.)}$$

But $y = \frac{x}{\log|(Cx)|}$ is the general solution so

$$\frac{x}{y} = \frac{1}{v} = \int \frac{dv}{\phi(1/v)} \Rightarrow \phi(1/v) = -1/v^2$$

$$\Rightarrow \phi(x/y) = -y^2/x^2.$$

Q15. The solution y(x) of the differential equation $\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$ and $y(0) = 0$ is

A. $-\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$

B. $-\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x$

C. $-\frac{\cos 3x}{3} + e^x + \frac{x^4}{12} + \frac{1}{3}x + 1$

D. none of these

Sol. A

Integrating the given differential equation, we have

$$\frac{dy}{dx} = -\frac{\cos 3x}{3} + e^x + \frac{x^3}{3} + C_1$$

but $y_1(0) = 1$ so $1 = -\frac{1}{3} + 1 + C_1 \Rightarrow C_1 = \frac{1}{3}$

Again integrating, we get

$$y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x + C_2$$

but $y(0) = 0$ so $0 = 1 + C_2 \Rightarrow C_2 = -1$. Thus

$$y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1.$$

Q16. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

A. variable radii and a fixed centre (0, 1)
B. variable radii and a fixed centre (0, – 1)
C. fixed radius 1 and a variable centres along the x-axis
D. fixed radius 1 and variable centres along the y-axis

Sol. C

$$dx = \frac{y}{\sqrt{1-y^2}} dy$$

$$\Rightarrow \quad c + x = \frac{1}{2}\int \frac{2y}{\sqrt{1-y^2}} dy = -\sqrt{1-y^2}$$

$$\Rightarrow \quad (x+c)^2 + y^2 = 1$$

which represents a family of circles of fixed radius 1 and variable centre on the x-axis.

Q17. The orthogonal trajectories of the family of curves $a^{n-1} y = x^n$ are given by

A. $x^n + n^2 y = \text{const}$
B. $ny^2 + x^2 = \text{const}$
C. $n^2 x + y^n = \text{const}$
D. $n^2 x - y^n = \text{const}$

Sol. B

Differentiating, we have (see theory)

$$a^{n-1}\frac{dy}{dx} = nx^{n-1} \quad \Rightarrow \quad a^{n-1} = nx^{n-1}\frac{dx}{dy}$$

Putting this value in the given equation, we have

$$nx^{n-1}\frac{dx}{dy}y = x^n$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we have $ny = -x\frac{dx}{dy}$

$$\Rightarrow \quad ny\,dy + x\,dx = 0 \quad \Rightarrow \quad ny^2 + x^2 = \text{const.}$$

which is the required family of orthogonal trajectories.

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Q18. The solution of the differential equation $\frac{d^2x}{dr^2} + x = 0;$ x(0) = 1, x'(0) = 0

A. approaches infinity as t $\to \infty$
B. is a periodic function
C. is always greater than or equal to unity
D. does not exist

Sol. B

Consider the equation $m^2 + 1 = 0$, $m = \pm i$.
Hence $x = C_1 \cos t + C_2 \sin t$ (see theory)
Now $x(0) = 1 \Rightarrow C_1 = 1$ and $x' = -C_1 \sin t + C_2 \cos t$ so $C_2 = 0$. Hence $x = \cos t$ is the required solution which is a periodic function.

Alternatively

We have $2\frac{dx}{dt}\frac{d^2x}{dt^2} = -2x\frac{dx}{dt} \Rightarrow \left(\frac{dx}{dt}\right)^2 = -x^2 + C$

Since $x'(0) = 0$ and $x(0) = 1$ so $C = 1$. Hence $\frac{dx}{dt} = \pm\sqrt{1-x^2}$

$\Rightarrow \quad \frac{dx}{\sqrt{1-x^2}} = \pm t$

$\Rightarrow \quad \sin^{-1} x = t + C_1$ or $\cos^{-1} x = t + C_2$

$\Rightarrow \quad x = \sin(t + C_1)$ or $x = \cos(t + C_2)$

When $t = 0, x = 1$ so $C_1 = \pi/2$ and $C_2 = 0$

Hence $x = \cos t$.

Q19. The solution of differential equation $\frac{dy}{dx} = \frac{y}{x} + 2\frac{\phi(y/x)}{\phi'(y/x)}$ is

A. $x^2\phi(y/x) = k$
B. $y^2\phi(y/x) = k$
C. $\phi(y/x) = kx^2$
D. $\phi(y/x) = ky^2$

Sol. C

Putting $\frac{y}{x} = u$ we have $\frac{dy}{dx} = u + x\frac{du}{dx}$. The given differential equation can be written as $u + x\frac{du}{dx} = u + 2\frac{\phi(u)}{\phi'(u)}$

$$\Rightarrow \quad x\frac{du}{dx} = 2\frac{\phi(u)}{\phi'(u)} \quad \Rightarrow \quad \frac{\phi'(u)}{\phi(u)}\,du = 2\frac{dx}{x}$$

Integrating, we get $\log \phi(u) = \log x^2 + \log k$, so $\phi(u) = kx^2$
i.e. $\phi(y/x) = kx^2$, k being an arbitrary constant.

Q20. The degree of the differential equation satisfying $\sqrt{1+x^2} + \sqrt{1+y^2} = A\left(x\sqrt{1+y^2} - y\sqrt{1+x^2}\right)$ is

A. 2
B. 3
C. 4
D. None of these

Sol. D

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Put $x = \tan\theta$ and $y = \tan\phi$. Then $\sqrt{1+x^2}$ $= \sec\theta$, $\sqrt{1+y^2} = \sec\phi$, and the equation becomes

$$\sec\theta + \sec\phi = A(\tan\theta\sec\phi - \tan\phi\sec\theta)$$

$$\Rightarrow \quad \frac{\cos\phi + \cos\theta}{\cos\theta\cos\phi} = A\left(\frac{\sin\theta - \sin\phi}{\cos\theta\cos\phi}\right)$$

$$\Rightarrow \quad \cos\phi + \cos\theta = A(\sin\theta - \sin\phi)$$

$$\Rightarrow \quad 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} = 2A\sin\frac{\theta-\phi}{2}\cos\frac{\theta+\phi}{2}$$

$$\Rightarrow \quad \cot\frac{\theta-\phi}{2} = A \quad \Rightarrow \quad \theta - \phi = 2\cot^{-1}A$$

$$\Rightarrow \quad \tan^{-1}x - \tan^{-1}y = 2\cot^{-1}A.$$

Differentiating this, we get $\frac{1}{1+x^2} - \left(\frac{1}{1+y^2}\right)\frac{dy}{dx} = 0,$

which is a differential equation of degree 1.

Q21. Write the degree of the differential equation $\left(\frac{dy}{dx}\right)^4 + 3x\frac{d^2y}{dx^2} = 0.$ [Delhi 2013]

Sol. Given differential equation is

$$\left(\frac{dy}{dx}\right)^4 + 3\times\left(\frac{d^2y}{dx^2}\right) = 0,$$

Degree = 1

Here, we see that, differential coefficient is free from radical sign.

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Q22. Write the differential equation representing the family of curves y=mx, where m is and arbitrary constant [All India 2013]

Sol. Given, family of curves y = mx …..(i)

Where, m is and arbitrary constant.

Now, differentiating Eq. (i) w. r. t. x, we get

$$\frac{dy}{dx} = m$$

On putting m = $\frac{dy}{dx}$ in Eq. (i), we get

$$y = \frac{dy}{dx}\,x \qquad (1)$$

Q23. Find the differential equation of family of circles touching y-axis at the origin.

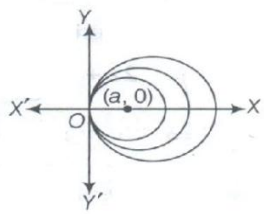
[Hots; Delhi 2010C; All India 2009C]

Sol.

Let radius of family of circle = a

$\therefore$ Centre of circle = $(a, 0)$

Now, equation of family of circle with centre $(a, 0)$ and radius a is



$$(x-a)^2 + y^2 = a^2 \quad \textbf{(1)}$$

[On putting $(h, k) = (a, 0)$ and $r = a$

in $(x-h)^2 + (y-k)^2 = r^2$] **(1)**

$$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = a^2$$

$$\Rightarrow \quad x^2 - 2ax + y^2 = 0 \quad \ldots(i)$$

On differentiating both sides w.r.t. x, we get

$$2x - 2a + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad a = x + y\frac{dy}{dx} \quad \textbf{(1)}$$

On putting above value of a in Eq. (i), we get

$$x^2 + y^2 - 2\left(x + y\frac{dy}{dx}\right)x = 0$$

$$\Rightarrow \quad 2xy\frac{dy}{dx} + x^2 - y^2 = 0$$

$$\Rightarrow \quad 2xyy' + x^2 - y^2 = 0$$

where, $\quad y' = \dfrac{dy}{dx}$

which is the required differential equation. **(1)**

Q24. Form the differential equation representing family of ellipses having foci on X-axis and centre at the origin. [Hots;Delhi 2009C]

Sol.

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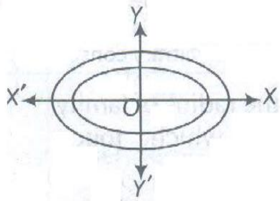
We know that, the equation of family of ellipse having foci on X-axis and centre at origin is given by



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

where, $a > b$ ...(i) **(1)**

On differentiating Eq. (i), w.r.t. x both sides, we get

$$\frac{2x}{a^2}+\frac{2yy'}{b^2}=0 \quad \left[\text{Let } \frac{dy}{dx}=y'\right]$$

$$\Rightarrow \quad \frac{x}{a^2}=\frac{-yy'}{b^2}$$

$$\Rightarrow \quad \frac{yy'}{x}=\frac{-b^2}{a^2} \quad \text{...(ii) (1)}$$

Again, on differentiating Eq. (ii), w.r.t. x both sides, we get

$$\frac{\left[x\cdot\frac{d}{dx}(yy')-yy'\cdot\frac{d}{dx}(x)\right]}{x^2}=0$$

$$\left[\text{Using quotient rule of differentiation in LHS and } \frac{d}{dx}\left(\frac{-b^2}{a^2}\right)=0\right]$$

$$\Rightarrow \quad x\left[y\cdot\frac{d}{dx}(y')+y'\cdot\frac{d}{dx}(y)\right]-yy'\cdot 1=0 \quad \text{(1)}$$

$$\Rightarrow \quad x\,[yy''+y'\cdot y']-yy'=0$$

$$\left[\because \frac{d}{dx}(y')=y'' \text{ and } \frac{d}{dx}(y)=y'\right]$$

$$\therefore \quad xyy''+x(y')^2-yy'=0$$

which is the required differential equation. (1)