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Class: XII
Subject: Mathematics
Topic: Indefinite Integration
No. of Questions: 26

1. If $\int \frac{e^{4x}-1}{e^{2x}} \log\left(\frac{e^{2x}+1}{e^{2x}-1}\right) dx = \frac{t^2}{2}\log t - \frac{t^2}{4} + \frac{u^2}{2}\log u - \frac{u^2}{4} + C$, then

(a) $t = e^{-x} - e^{x}$ $u = e^{x} + e^{-x}$
(b) $t = e^{x} - e^{-x}$, $u = e^{x} + e^{-x}$
(c) $t = e^{x} + e^{-x}$, $u = e^{x} - e^{-x}$
(d) none of these

Sol. C

The integrand can be written as

$$\left(e^{2x} - e^{-2x}\right)\log\left(e^{x} + e^{-x}\right) - \left(e^{2x} - e^{-2x}\right)\log\left(e^{x} - e^{-x}\right)$$

$$= \left(e^{x} + e^{-x}\right)\left(e^{x} - e^{-x}\right)\log\left(e^{x} + e^{-x}\right)$$

$$- \left(e^{x} + e^{-x}\right)\left(e^{x} - e^{-x}\right)\log\left(e^{x} - e^{-x}\right).$$

Hence the given integral is equal to

$$\int t \log t \, dt - \int u \log u \, du$$

$$(t = e^{x} + e^{-x}, u = e^{x} - e^{-x})$$

$$= \frac{t^2}{2}\log t - \frac{1}{2}\int t\, dt + \frac{u^2}{2}\log u - \frac{1}{2}\int u\, du$$

(integration by parts)

$$= \frac{t^2}{2}\log t - \frac{t^2}{4} + \frac{u^2}{2}\log u - \frac{u^2}{4} + C,$$

where $t = e^{x} + e^{-x}$ and $u = e^{x} - e^{-x}$.

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2. If $\int \frac{\sqrt{1+\sqrt[3]{x}}}{x^{2/3}} dx = 2f(x)^{3/2} + C$, then f(x) is equal to

(a) $1 + x^{2/3}$
(b) $1 + x^{1/3}$
(c) $1 - x^{1/3}$
(d) $1 - x^{2/3}$

Sol. B

$I = \int x^{-2/3} (1 + x^{1/3})^{1/2}\, dx.$

Here $m = -2/3$, $n = 1/3$, $p = 1/2$; $\frac{m+1}{n} = \frac{(-2/3+1)}{1/3} = 1$

Put $1 + x^{1/3} = t^2$, $(1/3)x^{-2/3}dx = 2t\, dt.$

Hence $I = 6\int t^2 dt = 2t^3 + C = 2(1+x^{1/3})^{3/2} + C.$

3. Let f and g be two polynomials. Then $\int f(x)g''(x) - f''(x)g(x)$ dx (ignoring the constant of integration) is equal to

(a) $\frac{f(x)}{g'(x)}$
(b) f '(x) g(x) – f(x) g'(x)
(c) f(x) g'(x) – f '(x) g(x)
(d) f'(x) g'(x) + f '(x) g(x)

Sol. C

$\int (f(x)g''(x) - f''(x)g(x))\, dx$

$= = f(x)\, g'(x) - \int f'(x)\, g'(x)\, dx$

$-\left[g(x)f'(x) - \int f'(x)g'(x)dx\right]$

$= f(x)\, g'(x) - g(x) f'(x).$

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4. Let $f(x) = \int e^x (x - 1)(x - 2)\,dx$. Then f decreases in the interval

(a) $(-\infty, -2)$
(b) $(-2, -1)$
(c) $(1, 2)$
(d) $(2, \infty)$

Sol. C

We have $f'(x) = e^x (x-1)(x-2)$. Since $e^x > 0$ for $x \in \mathbf{R}$ so $f'(x) < 0$ if and only if $(x-1)(x-2) < 0$ i.e. if $1 < x < 2$.

5. If the primitive of $\sin^{-3/2} x \sin^{-1/2}(x + \theta)$ is $-2 \operatorname{cosec} \theta \sqrt{f(x)} + C$, then

(a) $f(x) = \dfrac{\sin x}{\sin(x+\theta)}$
(b) $f(x) = \tan(x+\theta)$
(c) $f(x) = \dfrac{\sin(x+\theta)}{\sin x}$
(d) $f(x) = \dfrac{\tan(x+\theta)}{\tan x}$

Sol. C

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The primitive of the given function is

$$\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\theta)}}$$

$$= \int \frac{dx}{\sqrt{\sin^3 x (\sin x \cos\theta + \cos x \sin\theta)}}$$

$$= \int \frac{\operatorname{cosec}^2 x \, dx}{\sqrt{\cos\theta + \cot x \sin\theta}}$$

$$= -\frac{1}{\sqrt{\sin\theta}} \int \frac{dt}{\sqrt{\cot\theta + t}} \quad (t = \cot x)$$

$$= -\frac{2}{\sqrt{\sin\theta}} (\cot\theta + t)^{1/2} + C$$

$$= -\frac{2}{\sin\theta} (\cos\theta + \sin\theta\, t)^{1/2} + C$$

$$= \frac{-2 \operatorname{cosec}\theta\, (\sin x \cos\theta + \sin\theta \cos x)^{1/2}}{\sqrt{\sin x}} + C$$

$$= -2 \operatorname{cosec}\theta \left(\frac{\sin(\theta + x)}{\sin x} \right)^{1/2} + C.$$

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6. If f(x) = $\lim_{n\to\infty} n^2(x^{1/n} - x^{1/(n+1)})$, x > 0, then $\int^x$ f(x) dx is equal to

(a) $x^2/2$
(b) 0
(c) $x^2 \log x - \frac{1}{2}x^2 + C$
(d) none of these

Sol. D

$$f(x) = \lim_{n\to\infty} n^2 x^{1/(n+1)}\left[x^{\frac{1}{n}-\frac{1}{n+1}} - 1\right]$$

$$= \lim_{n\to\infty} \frac{x^{1(n+1)}\left(x^{\frac{1}{n(n+1)}} - 1\right)}{\frac{1}{n(n+1)} \times \frac{n(n+1)}{n^2}} = \log x.$$

Hence $\int x\, f(x)\, dx = \int x \log x\, dx = \frac{x^2}{2} \log x - \frac{1}{4} x^2 + C.$

7. If the primitive of $\frac{\sin x}{\sqrt{1+\sin x}}$ is $-2\sqrt{f(x)} + \sqrt{2}$ log |tan g(x)| + C, then

(a) f(x) = 1 + sin x
(b) g(x) = ($3\pi/8$) – (x/4)
(c) f(x) = 2(1 - sin x)
(d) none of these

Sol. B

$$\int \frac{\sin x}{\sqrt{1+\sin x}}\, dx$$

$$= -\int \frac{\cos u}{\sqrt{1+\cos u}}\, du \left(x = \left(\frac{\pi}{2}\right) - u\right)$$

$$= -\int \frac{2\cos^2(u/2) - 1}{\sqrt{2}\cos(u/2)}$$

$$= -\sqrt{2}\int \cos(u/2)\, du + \frac{1}{\sqrt{2}}\int \sec(u/2)\, du$$

$$= -2\sqrt{2}\sin(u/2)$$

$$= +\sqrt{2}\log|\tan(\pi/4) + (u/4)| + C.$$

$$= -2\sqrt{2}\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

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$$= +\sqrt{2}\log\left|\tan\left(\frac{3\pi}{8}-\frac{x}{4}\right)\right|+C.$$

8. If $\int \frac{(x^2-1)\,dx}{(x^4+3x^2+1)\tan^{-1}\frac{x^2+1}{x}}$ = log |tan^{-1} f(x)| + C, then

(a) $f(x) = x^2 + 1$

(b) $f(x) = \frac{x^2+1}{2x}$

(c) $f(x) = \frac{1}{2}(x^2+1)$

(d) $f(x) = \frac{x^2+1}{x}$

Sol. D

Dividing the numerator and denominator by x^2, the given integral transforms into

$$\int \frac{\left(1-1/x^2\right)dx}{\left((x+1/x)^2+1\right)\tan^{-1}(x+1/x)}$$

$$= \int \frac{dt}{\left(t^2+1\right)\tan^{-1}t}\ (t=(x+1/x))$$

$$= \int \frac{du}{u}, \qquad (u=\tan^{-1}t)$$

$$= \log\left|\tan^{-1}\frac{x^2+1}{x}\right| + C.$$

Thus $f(x) = \frac{x^2+1}{x}$.

9. If $f(x) = \lim_{n\to\infty} e^{x\tan(1/n)\log(1/n)}$ and $\int \frac{f(x)}{\sqrt[3]{\sin^{11} x \cos x}} dx = g(x) + C$ (C being the constant of integration), then

(a) $g(\pi/4) = 3/2$
(b) $g(x)$ is continuous for all $x \in \mathbf{R}$
(c) $g(\pi/4) = -15/8$
(d) $g(\pi/4) = 1/2$

Sol. C

$f(x) = \lim_{n\to\infty} e^{x\tan(1/n)\cdot\log(1/n)}$.

But $\lim_{n\to\infty} \tan(1/n)\log(1/n) = \lim_{n\to\infty} -\frac{\log n}{n}\frac{\tan(1/n)}{1/n} = 0$

So $f(x) = e^0 = 1$. Hence

$$\int \frac{f(x)}{\sqrt[3]{\sin^{11} x \cos x}} dx = \int \frac{dx}{\sqrt[3]{\sin^{11} x \cos x}}$$

$$= \int \frac{1+t^2}{t^{11/3}} dt \ (t = \tan x)$$

$$= \int (t^{-11/3} + t^{-5/3})\, dt$$

$$= -\frac{3}{8} t^{-8/3} - \frac{3}{2} t^{-2/3} + C$$

$$= -\frac{3}{8}\frac{(1+4\tan^2 x)}{\tan^2 x \sqrt[3]{\tan^2 x}} + C$$

Thus $g(x) = -\frac{3}{8}\frac{(1+4\tan^2 x)}{\tan^2 x \sqrt[3]{\tan^2 x}}$, $g(\pi/4) = -\frac{15}{8}$.

Clearly g is not defined at $x = 0$.

10. If f (x) = $\lim_{n\to\infty} \frac{x^n - x^{-n}}{x^n + x^{-n}}$, 0 < x < 1, n ∈ **N,** then $\int (\sin^{-1} x)$ f(x) dx is equal to

(a) $-[x \sin^{-1} x + \sqrt{1-x^2}] + C$
(b) $x \sin^{-1} x + \sqrt{1-x^2} + C$
(c) a constant
(d) none of these

Sol. A

$$f(x) = \lim_{n\to\infty} \frac{x^{2n}-1}{x^{2n}+1} = -1 \ (0 < x < 1), \text{ so}$$

$$\int \sin^{-1} x \,(f(x))\, dx = -\int \sin^{-1} x \, dx$$

$$= -[x \sin^{-1} x + \sqrt{1-x^2}\,] + C$$

11. If f (x) = $\lim_{n\to\infty}$ [2x + 4x^3 +…+ 2nx^{2n-1}] (0 < x < 1), then $\int f(X)$ dx is equal to

(a) $-\sqrt{1-x^2}$
(b) $\frac{1}{\sqrt{1-x^2}}$
(c) $\frac{1}{x^2-1}$
(d) $\frac{1}{1-x^2}$

Sol. D

$$\text{Let } g_n(x) = 1 + x^2 + x^4 + \cdots + x^{2n} = \frac{x^{2n+2}-1}{x^2-1}$$

$$\text{so} \quad h_n(x) = g'_n(x) = \frac{2x\left(n x^{2n+2} - (n+1)x^{2n} + 1\right)}{\left(x^2-1\right)^2}$$

$$\text{Now} \quad f(x) = \lim_{n\to\infty} h_n(x) = \frac{2x}{\left(x^2-1\right)^2} \quad \text{as } 0 < x < 1.$$

$$\text{Thus} \quad \int f(x)\, dx = \int \frac{2x}{\left(x^2-1\right)^2}\, dx = -\frac{1}{x^2-1} = \frac{1}{1-x^2}.$$

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12. $\int \tan^2 x \, dx =$

(a) $2 \tan x \sec^2 x + c$
(b) $\tan x + x + c$
(c) $\tan x - x + c$
(d) None of these

Sol. C

$I = \int (\sec^2 x - 1) \, dx = \tan x - x + c$

13. $\int \sqrt{1 + \cos (x/4)} \, dx =$

(a) $8 \sqrt{2} \sin (x/8) + C$
(b) $- 8 \sqrt{2} \cos (x/8) + C$
(c) $8 \left(1 + \cos \frac{x}{4}\right)^{3/4} + C$
(d) None of these

Sol. C

$I = \int \sqrt{2 \cos^2 (x/8)} \, dx$

$= \sqrt{2} \int \cos (x/8) dx = \sqrt{2} \frac{\sin (x/8)}{1/8} + c$

$= 8\sqrt{2} \sin (x/8) + c$

14. $\int e^{-\log x} \, dx =$

(a) $e^{-\log x} + c$
(b) $-x e^{-\log x} + c$
(c) $\log | x | + c$
(d) None of these

Sol. C

$I = \int e^{\log (x^{-1})} dx = \int x^{-1} dx = \int \frac{dx}{x}$

$= \log | x |.$

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15. $\int |x|^3$ dx =

(a) $\frac{x^4}{4}$ + C
(b) - $\frac{x^4}{4}$ + C
(c) $\frac{|x^4|}{4}$
(d) None of these

Sol. D

$I = \int (|x|^3 .1)\, dx$

$= |x|^3 . x - \int \frac{x^3}{|x|^3} . 3x^2 . x\, dx$

or $I = |x|^3 .x - 3 \int |x|^3\, dx$

or $4I = |x|^3 . x + c$

or $I = \frac{1}{4} |x|^3 . x + c$

16. $\int \log x\, dx$ =

(a) x (1 - log x) + C
(b) x (log x - 1) + C
(c) (1 + x) log x + C
(d) (1 - x) log x + C

Sol. B

$I = \int 1 . \log x\, dx$

$= (\log x)\, x - \int \frac{1}{x} . x\, dx + c$

$= x \log x - x + c$

$= x (\log x - 1) + c.$

17. $\int \sin^{-1} x \, dx =$

(a) $x \sin^{-1} x + \sqrt{1-x^2} + C$
(b) $x \cos^{-1} x + C$
(c) $x \cos^{-1} x + \sqrt{1-x^2} + C$
(d) $-\cos^{-1} x + C$

Sol. A

$I = \int 1 \,.\, \sin^{-1} x \, dx$

$= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$

$= x \sin^{-1} x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}} dx$

$= x \sin^{-1} x + \sqrt{1-x^2} + c$

18. $\int \sin x \sin 2x \sin 3x \, dx =$

(a) $\frac{\cos 6x}{24} - \frac{\cos 4x}{16} - \frac{\cos 2x}{8} + C$

(b) $\frac{\cos 4x}{16} + \frac{\cos 2x}{8} - \frac{\cos 6x}{8} + C$

(c) $\frac{\sin 6x}{8} - \frac{\sin 4x}{16} - \frac{\sin 2x}{8}$

(d) none of these

Sol. A

$(\sin x \sin 3x) \sin 2x$

$= \frac{1}{2} (2 \sin x \sin 3x) \sin 2x$

$= \frac{1}{2} (\cos 2x - \cos 4x) \sin 2x = \frac{1}{4} (2 \sin 2x \cos 2x) - \frac{1}{4} (2 \cos 4x \sin 2x)$

$= \frac{1}{4} \sin 4x - \frac{1}{4} (\sin 6x - \sin 2x)$

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$= \frac{1}{4}\sin 4x - \frac{1}{4}\sin 6x + \frac{1}{4}\sin 2x.$

$I = -\frac{\cos 4x}{16} + \frac{\cos 6x}{24} - \frac{\cos 2x}{8} + c.$

19. $\int \frac{dx}{e^x + e^{-x}} =$

(a) $\log(e^x + 1) + C$
(b) $\log(e^x + e^{-x}) + C$
(c) $\tan^{-1} e^x + C$
(d) $\sin^{-1} e^x + C$

Sol. C

$I = \int \frac{dx}{e^{-x}(1+e^{2x})} = \int \frac{e^x dx}{(1+e^{2x})} = \int \frac{dt}{1+t^2}$

$= \tan^{-1} t + c = \tan^{-1} e^x + c.$

20. $\int \frac{e^{\tan^{-1}x} dx}{1+x^2} =$

(a) $(\tan^{-1}x)\, e^{\tan^{-1}x} + C$
(b) $\tan^{-1} x + C$
(c) $e^{\tan^{-1}x} + C$
(d) $e^{\tan^{-1}x}\log(1+x^2) + C$

Sol. C

Putting $\tan^{-1} x = t$ and $\frac{dx}{(1+x^2)} = dt$, we get,

$I = \int e^t\, dt = e^t + C = e^{\tan^{-1}x} + C$

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21. If $\frac{d}{dx}(g(x)) = g(x)$, then $\int g(x)(f(x) + f'(x))$ is equal to

(a) $g(x) f(x)$
(b) $1/2\, g(x) (f(x))^2$
(c) $f(x) (g(x) + g(x))$
(d) None of these

Sol. A

Since $\frac{d}{dx}(g(x)) = g(x)$,

Therefore, $\int g(x)\, dx = g(x)$.

Now, $\int g(x)(f(x) + f'(x))\, dx = \int g(x) f(x)\, dx + \int g(x) f'(x)\, dx$

(Evaluate the first integral by parts taking f(x) as the first function)

$= f(x) g(x) - \int f'(x) g(x)\, dx + \int g(x) f'(x)\, dx = f(x) g(x)$

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22. Evaluate $\int \frac{x^2}{(x^2+4)(x^2+9)}dx.$ [Delhi 2013]

Sol.

Let $I = \int \frac{x^2}{(x^2+4)(x^2+9)}dx$

$$= \frac{1}{2}\int \frac{2x^2+4+9-4-9}{(x^2+4)(x^2+9)}dx$$

$$= \frac{1}{2}\int \frac{x^2+4}{(x^2+4)(x^2+9)}dx + \frac{1}{2}\int \frac{x^2+9}{(x^2+4)(x^2+9)}dx - \frac{1}{2}\int \frac{13}{(x^2+4)(x^2+9)}dx$$

$$= \frac{1}{2}\int \frac{dx}{x^2+9} + \frac{1}{2}\int \frac{dx}{x^2+4} - \frac{1}{2}\int \frac{13}{(x^2+4)(x^2+9)}dx$$

$$= \frac{1}{2}\cdot\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}\cdot\frac{13}{5}\int\left(\frac{1}{(x^2+4)} - \frac{1}{(x^2+9)}\right)dx$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{x}{3}\right) + \frac{1}{4}\tan^{-1}\left(\frac{x}{2}\right) - \frac{13}{10}\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + \frac{13}{10}\cdot\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$\left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{x}{3}\right) + \frac{1}{4}\tan^{-1}\left(\frac{x}{2}\right) - \frac{13}{20}\tan^{-1}\left(\frac{x}{2}\right) + \frac{13}{30}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$= \tan^{-1}\left(\frac{x}{3}\right)\left(\frac{1}{6}+\frac{13}{30}\right) + \tan^{-1}\left(\frac{x}{2}\right)\left(\frac{1}{4}-\frac{13}{20}\right) + C$$

$$= \tan^{-1}\left(\frac{x}{3}\right)\left(\frac{5+13}{30}\right) + \tan^{-1}\left(\frac{x}{2}\right)\left(\frac{5-13}{20}\right) + C$$

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$$= \frac{18}{30} \tan^{-1}\left(\frac{x}{3}\right) - \frac{8}{20} \tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= \frac{3}{5} \tan^{-1}\left(\frac{x}{3}\right) - \frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + C$$

Let $I = \int \frac{2x^2+1}{x^2(x^2+4)} dx$

Put $x^2 = t$ and then using partial fraction, we get

$$\frac{2t+1}{t(t+4)} = \frac{A}{t} + \frac{B}{t+4}$$

$$\Rightarrow \quad 2t+1 = A(t+4) + Bt$$

23. Evaluate $\int \frac{x^2+1}{(x^2+4)(x^2+25)} dx.$ [Delhi 2013]

Sol.

Let $I = \int \frac{x^2+1}{(x^2+4)(x^2+25)} dx$

Let $\frac{x^2+1}{(x^2+4)(x^2+25)} = \frac{A}{x^2+4} + \frac{B}{x^2+25}$

[By partial fraction] **(1)**

At $x = 0$, $\frac{A}{4} + \frac{B}{25} = \frac{1}{100}$

$\Rightarrow \quad 25A + 4B = 1$ ...(i) **(1)**

At $x = 1$, $\frac{2}{5 \times 26} = \frac{A}{5} + \frac{B}{26}$

$\Rightarrow \quad \frac{A}{5}+\frac{B}{26}=\frac{1}{65}$

$\Rightarrow \quad 13A+\frac{5}{2}B=1$

$\Rightarrow \quad 26A+5B=2$ ...(ii) **(1)**

On solving Eqs. (i) and (ii), we get

$$A=-\frac{1}{7},\ B=\frac{8}{7}$$

$$\therefore\ I=-\frac{1}{7}\int\frac{dx}{x^2+4}+\frac{8}{7}\int\frac{dx}{x^2+25}$$

$$=-\frac{1}{7}\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\frac{8}{7}\cdot\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right)+C \quad \textbf{(1)}$$

$$\left[\because\ \int\frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

24. Evaluate $\int\frac{2x}{(x^2+1)(x^2+3)}dx$ [Delhi 2011]

Sol.

Let $\quad I=\int\frac{2x}{(1+x^2)(x^2+3)}dx$

Put $\quad x^2=t$

$\Rightarrow \quad 2x\,dx=dt$

$\therefore \quad I=\int\frac{1}{(t+1)(3+t)}dt$

Let $\quad \frac{1}{(t+1)(3+t)}=\frac{A}{(1+t)}+\frac{B}{3+t}$

$\Rightarrow \quad 1=A(3+t)+B(1+t)$

On putting $t=-3$, we get

$$1=-2B$$

$\Rightarrow \quad B=-\frac{1}{2}$

Now, on putting $t=-1$, we get

$$1=2A$$

$\therefore \quad A=\frac{1}{2}$

On putting $A = \frac{1}{2}$ and $B = -\frac{1}{2}$ in Eq. (i), we get

$$\frac{1}{(1+t)(3+t)} = \frac{1/2}{1+t} + \frac{-1/2}{3+t}$$

$$\therefore \quad I = \int \frac{1}{(1+t)(3+t)} dt$$

$$= \frac{1}{2}\int \frac{1}{1+t} dt - \frac{1}{2}\int \frac{1}{3+t} dt$$

$$= \frac{1}{2}\log|1+t| - \frac{1}{2}\log|3+t|$$

$$\left[\because \int \frac{dx}{x} = \log|x|\right]$$

$$= \frac{1}{2}\log|1+x^2| - \frac{1}{2}\log|3+x^2| + C$$

$$\therefore \quad I = \frac{1}{2}\log\left|\frac{1+x^2}{3+x^2}\right| + C$$

$$\left[\because \log m - \log n = \log\frac{m}{n}\right]$$

25. Evaluate $\int\left[\log(\log x) + \frac{1}{(\log x^2)}\right]dx.$ [Hots; Delhi 2010C]

Sol.

$$\text{Let} \quad I = \int\left[\log(\log x) + \frac{1}{(\log x)^2}\right]dx$$

$$= \int \underset{\text{I}}{\log(\log x)} \cdot \underset{\text{II}}{1}\, dx + \int \frac{1}{(\log x)^2} dx \quad \mathbf{(1/2)}$$

Using integration by parts in first integral, we get

$$I = \log(\log x)\int 1dx - \int\left[\frac{d}{dx}\log(\log x)\int 1dx\right]dx + \int \frac{1}{(\log x)^2} dx + C \quad \mathbf{(1/2)}$$

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$$= \log(\log x)\cdot x - \int \frac{1}{\log x}\cdot\frac{1}{x}\cdot x\,dx + \int \frac{1}{(\log x)^2}dx + C$$

$$= x\log(\log x) - \int \underset{\text{I}}{(\log x)^{-1}}\ \underset{\text{II}}{1}\,dx + \int \frac{1}{(\log x)^2}dx + C \quad \textbf{(1)}$$

Again, applying integration by parts in the middle integral

$$= x\log(\log x) - \left[(\log x)^{-1}\int 1dx - \int\left\{\frac{d}{dx}(\log x)^{-1}\int 1dx\right\}dx\right] + \int\frac{1}{(\log x)^2}dx + C$$

(1)

$$= x\log(\log x) - \left[\frac{x}{\log x} - \int -(\log x)^{-2}\cdot\frac{1}{x}\cdot x\,dx\right] + \int\frac{1}{(\log x)^2}dx + C$$

$$= x\log(\log x) - \frac{x}{\log x} - \int\frac{1}{(\log x)^2}dx + \int\frac{1}{(\log x)^2}dx + C$$

$$= x\log(\log x) - \frac{x}{\log x} + C \quad \textbf{(1)}$$

26. Evaluate $\int \frac{dx}{\sqrt{5-4x-2x^2}}dx.$ [All India 2009]

Sol.

$$\text{Let } I = \int\frac{dx}{\sqrt{5-4x-2x^2}}$$

$$= \int\frac{dx}{\sqrt{-2\left(x^2+2x-\frac{5}{2}\right)}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{-\left(x^2 + 2x - \frac{5}{2} + 1 - 1\right)}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{-\left[(x^2 + 2x + 1) - \frac{5}{2} - 1\right]}} \quad \textbf{(1)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{-\left[(x+1)^2 - \left(\frac{5}{2} + 1\right)\right]}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{-\left[(x+1)^2 - \frac{7}{2}\right]}} \quad \textbf{(1)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2 - (x+1)^2}}$$