askIITians
ENGINEERING | MEDICAL | FOUNDATION

Class: XII
Subject: Maths
Topic: Inverse Trigonometric Functions
No. of Questions: 25

Q1. If $\sin^{-1}x = \frac{\pi}{5}$, $x \in (-1, 1)$, then $\cos^{-1}x =$

A. $\frac{3\pi}{10}$

B. $\frac{5\pi}{10}$

C. $-\frac{3\pi}{10}$

D. $\frac{9\pi}{10}$

Ans. A $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ Or $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

Q2. The domain of $\sin^{-1} x$ is

[Foreign 2010]

A. $[-\pi, \pi]$

B. $[-1, 1]$

C. $(2, 2\pi]$

D. $[-\infty, \infty]$

Right Answer Explanation: B
Fact

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Q3. $\tan(\cos^{-1} x)$ is equal to

A. $\frac{\sqrt{1-x^2}}{x}$

B. $\frac{x}{\sqrt{1+x^2}}$

C. $\frac{\sqrt{1+x^2}}{x}$

D. $\sqrt{1-x^2}$

Ans. A

Let $\cos^{-1} x = \theta \therefore x = \cos\theta \therefore \tan\theta = \frac{\sqrt{1-x^2}}{x} \therefore \tan(\cos^{-1} x) = \tan\theta = \frac{\sqrt{1-x^2}}{x}$

Q4. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to [Hots; Delhi 2011, 2009; All India 2009]

A. $\frac{7\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Ans. B

Range is [0, π]. So first convert 7 π/6 into that range.

Q5. If $\sec^{-1}\sqrt{1+x^2} + \operatorname{cosec}^{-1}\frac{\sqrt{1+y^2}}{y} + \cot^{-1}\frac{1}{z} = \pi$, then x + y + z is equal to

A. xyz

B. 2xyz

C. xyz^2

D. x^2yz

Ans. A

$\sec^{-1}\sqrt{1+x^2} + \operatorname{cosec}^{-1}\frac{\sqrt{1+y^2}}{y} + \cot^{-1}\frac{1}{z} = \pi$ $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ $x + y + z = xyz$

Q6. $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$ holds true if

A. $|x| \le 1$

B. $0 \le x \le 1$

C. $|x| < \frac{1}{2}$

D. None of these

[Ans. B The result holds true only if $0 \le 2\cos^{-1}x \le \pi$ i.e. if $0 \le \cos^{-1}x \le \frac{\pi}{2}$, i.e. if $1 \ge x \ge 0$

Q7. $\tan(\sin^{-1}x)$ is equal to

A. $\frac{x}{\sqrt{1-x^2}}$

B. $\frac{-x}{\sqrt{1-x^2}}$

C. $\frac{|x|}{\sqrt{1-x^2}}$

D. None of these

Ans. A $\tan(\sin^{-1}x) = \dfrac{\sin(\sin^{-1}x)}{\cos(\sin^{-1}x)} = \dfrac{x}{\sqrt{1-x^2}}$

Q8. $\cos^{-1}\left(\cos\left(-\dfrac{\pi}{3}\right)\right)$ is equal to

A. $-\dfrac{\pi}{3}$

B. $\dfrac{\pi}{3}$

C. $\dfrac{2\pi}{3}$

D. None of these

Ans. B $\cos^{-1}\left(\cos\left(-\dfrac{\pi}{3}\right)\right) = \cos^{-1}\left(\cos\dfrac{\pi}{3}\right) = \dfrac{\pi}{3}$

Q9. If $\sin^{-1}x = \pi/6$, then $\cos^{-1}x$ is equal to

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\dfrac{5\pi}{4}$

D. None of these

Ans. B $\cos^{-1}x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Q10. If $\theta = \tan^{-1} x$, then sin 2θ is equal to

A. $\frac{2x}{1+x^2}$

B. $\frac{2x}{1-x^2}$

C. $\frac{1-x^2}{1+x^2}$

D. None of these

Ans. A Given $\theta = \tan^{-1} x$, Therefore, sin 2θ = $\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2x}{1+x^2}$

Q11. The range of $\tan^{-1} x$ is

A. (π, -π)

B. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

C. (–π, π)

D. $\left(\frac{\pi}{2}, \frac{-\pi}{2}\right)$

Ans. B Range of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q12. cot $(\cos^{-1}x)$ is equal to

A. $\frac{|x|}{\sqrt{1-x^2}}$

B. $\frac{x}{\sqrt{1-x^2}}$

C. $\frac{-x}{\sqrt{1-x^2}}$

D. None of these

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Ans. B $\cot(\cos^{-1}x) = \frac{\cos(\cos^{-1}x)}{\sin(\cos^{-1}x)} = \frac{x}{\sqrt{1-x^2}}$

Q13. $\cos(\tan^{-1}x)$ is equal to

A. $\frac{1}{\sqrt{1+x^2}}$

B. $-\frac{1}{\sqrt{1+x^2}}$

C. $\frac{\sqrt{1+x^2}}{x}$

D. None of these

Ans. A Let $\tan^{-1}x = \theta \Rightarrow x = \tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ $\therefore \cos(\tan^{-1}x) = \cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+x^2}}$

Q14. If $x > 0$, then $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)$ is equal to

A. 1

B. tan 1

C. $\frac{\pi}{2}$

D. None of these

Right Answer Explanation: C

$\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)$

$= \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Q15. If $x \in [-1, 1]$ and $\sin^{-1} x = \frac{3\pi}{13}$, then $\cos^{-1} x$ is equal to

A. $\frac{10\pi}{13}$

B. $\frac{\pi}{2}$

C. $\frac{19\pi}{26}$

D. None of these

Ans. D $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x = \frac{\pi}{2} - \frac{3\pi}{13} = \frac{7\pi}{26}$

Q16. A solution of the equation $\tan^{-1}(1 + x) + \tan^{1}(1 - x) = \frac{\pi}{2}$ is

A. $x = 1$

B. $x = -1$

C. $x = 0$

D. $x = \pi$

Ans. C

$\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1} \frac{1+x+1-x}{1-(1+x)(1-x)} = \frac{\pi}{2}$

$\Rightarrow \tan^{-1} \frac{2}{1-1+x^2} = \frac{\pi}{2}$

$\Rightarrow \frac{2}{x^2} = \tan \frac{\pi}{2} = \infty$

$\Rightarrow x^2 = 0 \Rightarrow x = 0$

Q17. The value of $\sin^{-1}\left[\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\left(\frac{\sqrt{12}}{4}\right)+\sec^{-1}(\sqrt{2})\right)\right]$ is

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Ans. A

$\sin^{-1}\left[\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\left(\frac{\sqrt{12}}{4}\right)+\sec^{-1}(\sqrt{2})\right)\right]$

$= \sin^{-1}\left[\cot\left(\sin^{-1}\left(\sqrt{\frac{2-\sqrt{3}}{4}}\right)+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)\right]$

$= \sin^{-1}\left[\cot\left(\sin^{-1}\left(\sqrt{\frac{2-\sqrt{3}}{4}}\right)+\cos^{-1}\left(\sqrt{\frac{2-\sqrt{3}}{4}}\right)\right)\right]$

$= \sin^{-1}\left(\cot\frac{\pi}{2}\right) = 0$

So, first option is the answer.

Q18. **Directions:** The following question has four choices, out of which ONLY ONE is correct.

A function f: R $\rightarrow$ R describes a curve y = f(x). A point P(x, y) lies on the curve and satisfies the equation $\tan^{-1}x+\tan^{-1}\left(\frac{1}{y}\right)=\tan^{-1}3$ The number of asymptote(s) of the curve is (are)

A. 0

B. 1

C. 2

D. None of these

Ans. C

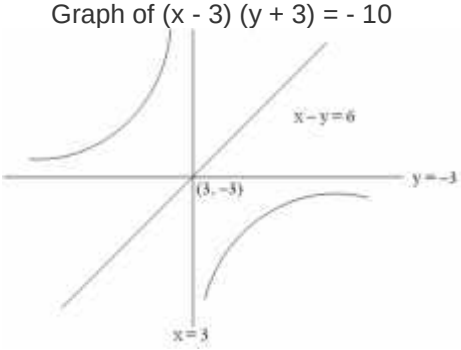
The given equation is

$$\tan^{-1}x+\tan^{-1}\left(\frac{1}{y}\right)=\tan^{-1}3$$

$$\frac{x+\frac{1}{y}}{1-\frac{x}{y}}=3,\ y\neq 0$$

(x - 3) (y + 3) = - 10

Graph of (x - 3) (y + 3) = - 10



By graph The number of asymptotes = 2

Q19. **Directions:** The answer to the following question is a single digit integer, ranging from 0 to 9. A polygon is obtained by joining the points (taken in order) on the curve $|y|=\left|\sin^{-1}\left(\frac{1+x^2}{2|x|}\right)\right|$ and its area is recorded. Now another polygon is obtained by joining the midpoints of the sides of the above polygon taken in the same order and its area is also recorded. The process continued indefinitely. If A is the sum of all such areas, the value of $\left[\frac{A}{2}\right]$, where [.] denotes g.i.f., is

Right Answer Explanation: 6

$y=\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is well defined, if $\left|\frac{1+x^2}{2x}\right|\leq 1$

$\Rightarrow |x|=1$

$\Rightarrow x=1 \text{ or } -1$

∴ The points are $A_1\left(1, \frac{\pi}{2}\right), A_2\left(-1, \frac{\pi}{2}\right), A_3\left(-1, -\frac{\pi}{2}\right), A_4\left(1, -\frac{\pi}{2}\right)$

If Δ_1 is the area of the polygon $A_1A_2A_3A_4$, then $\Delta_1 = 2\pi$

If Δ_2 is the area of the polygon obtained by joining the mid points of the sides of the above polygon, then $\Delta_2 = \pi$.

Hence, some of all such areas A = $2\pi + \pi + \frac{\pi}{2} + \ldots\ldots = 4\pi$

$\Rightarrow \left[\frac{A}{2}\right] = 6$

Q20. Write the principal value of $\left[\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(-\frac{1}{2}\right)\right]$. [Delhi 2013C]

Ans.

$\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(\frac{-1}{2}\right)$

$= \cos^{-1}\frac{\sqrt{3}}{2} + \left[\pi - \cos^{-1}\left(\frac{1}{2}\right)\right]$

$[\because \cos^{-1}(-x) = \pi - \cos^{-1}x]$

$= \frac{\pi}{6} + \pi - \frac{\pi}{3} = \frac{\pi + 6\pi - 2\pi}{6} = \frac{5\pi}{6}$ (1)

Q21. $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$. [All India 2013]

Ans.

$\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$

$= \tan^{-1}(\sqrt{3}) - \{\pi - co^{-1}(\sqrt{3})\}$

[∴ Principal value of $\cot^{-1}$ is] 0, π [$\because \cot^{-1}(-x) = \pi - \cot^{-1}x$]

$= \tan^{-1}(\sqrt{3}) - \pi + \cot^{-1}\sqrt{3}$

$= \frac{\pi}{2} - \pi = -\frac{\pi}{2}\left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Q22. $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$. [Delhi 2012]

Ans.

$\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$

$= \cos^{-1}\left(\cos\frac{\pi}{3}\right) - 2\sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right]$

$[\because Principal\ value\ for \cos^{-1} x is\ (0,\pi)]$

And that of $\sin^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$= \frac{\pi}{3} - 2\left(-\frac{\pi}{6}\right)$

$[\because \cos^{-1}(\cos\theta) = \theta\ and\ \sin^{-1}(\sin\theta) = \theta]$

$= \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

Q23. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$. [All India 2012]

Ans.

We know that, the principal value for $\tan^{-1}$ x is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and that of $\sec^{-1}$ x is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.

So, $\tan^{-1}\sqrt{3} - \sec^{-1}(12)$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \sec^{-1}\left(\sec\frac{2\pi}{3}\right)$

$\left[\because \tan\frac{\pi}{3} = \sqrt{3}\ and\ \sec = -2\right]$

$= \frac{\pi}{3} - \frac{2\pi}{3}$

$= \frac{-\pi}{3}$

$[\because \tan^{-1}(\tan\theta) = \theta\ and\ \sec^{-1}(\sec\theta) = \theta]$

Q24. $\sin^{-1}\cdot\frac{\sqrt{3}}{2}$

Ans.

We know that, the principal value of $\sin^{-1}\theta$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{3}\right)$

$= \sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right]$ $\left[\because \sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} \; as \; \sin(-\theta) = -\sin\theta\right]$

$= -\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $[\because \sin^{-1}(\sin\theta) = \theta]$

$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ (1)

Q25. If $\tan^{-1}\left(\sqrt{3}\right) + \cot^{-1}x = \frac{\pi}{2}$ then find the value of x. [All India 2010C]

Ans.

Given that $\tan^{-1}\left(\sqrt{3}\right) + \cot^{-1}x = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\sqrt{3} = \frac{\pi}{2} - \cot^{-1}x$

$\Rightarrow \tan^{-1}\sqrt{3} = \tan^{-1}x$ $\left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

On equating, we get

$x = \sqrt{3}$ (1)