

Class: XII
Subject: Maths
Topic: Limit and Continuity
No. of Questions: 25

Q1. If $f(x) = \lim_{n \rightarrow \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$, then

- A. $f(x) = 1$ for $|x| =$
- B. $f(x) = \begin{cases} 1 & \text{for } |x| > 1 \\ -1 & \text{for } |x| < 1 \end{cases}$
- C. $f(x) = \begin{cases} 1 & \text{for } |x| > 1 \\ -1 & \text{for } |x| \leq 1 \end{cases}$
- D. f is not defined for any value of x

Q2. Let $f(y) = \sin \frac{y-a}{2} \tan \frac{\pi y}{2a}$, $y \neq a$. The value of $f(a)$ so that f is a continuous function is

- A. π/a
- B. $-a/\pi$
- C. $\pi/2a$
- D. none of these

Q3. Let $f(x) = \begin{cases} \frac{\sqrt{1+ax} - \sqrt{1-ax}}{x}, & -1 \leq x < 0 \\ \frac{2x+1}{x-2}, & 0 \leq x \leq 1 \end{cases}$

The value of a such that f is continuous on $[-1, 1]$ is

- A. $1/2$
- B. $-1/2$
- C. 1

D. -1

Q4. The value of k ($k > 0$) for which the function

$f(x) = \frac{(e^x - 1)^4}{\sin(x^2/k^2) \log(1 + (x^2/2))}$, $x \neq 0$; $f(0) = 8$ may be a continuous function is

- A. 1
- B. 4
- C. 2
- D. 3

Q5. If f be a function on $[0, 1]$ defined by $f(x) = (1/2)^n$, $(1/2)^{n+1} \leq x < (1/2)^n$, $n = 0, 1, 2, \dots$, then

- A. f is a continuous function
- B. f is continuous except for $x = 1/2$
- C. f is continuous except for finitely many points
- D. the set of points where f is not continuous is infinite

Q6. If $f : \mathbf{R} \rightarrow \mathbf{R}$ is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\pi\right)$, where $[x]$ denotes the greatest integer function, then f is

- A. discontinuous only at $x = 0$
- B. discontinuous only at non-zero integral value of x
- C. continuous only at $x = 0$
- D. continuous for every real x

Q7. If $f(x) = \frac{\log(1+x^2)}{x^4 - 26x^2 + 25}$, then

- A. f is continuous on $[6, 10]$
- B. f is continuous on $[-2, 2]$
- C. f is continuous on $[-6, 6]$
- D. f is continuous on $[1, 7]$

Q8. The value of $\lim_{x \rightarrow 0} \frac{e^{nx} - \left(1 + nx + \frac{n^2}{2}x^2\right)}{x^3}$ ($n > 0$) is

- A. $\frac{n^2}{6}$
- B. $\frac{n^3}{3}$
- C. $\frac{n^3}{6}$
- D. $\frac{1}{6}$

Q9. The value of $\lim_{x \rightarrow 0} \left(\frac{e^x + e^{-x} - 2}{x^2} \right)^{1/x^2}$ is

- A. $e^{1/2}$
- B. $e^{1/4}$
- C. $e^{1/3}$
- D. $e^{1/12}$

Q10. $\lim_{x \rightarrow \pi/2} \frac{\cos x}{\sqrt[3]{(1 - \sin x)^2}}$ is equal to

- 1
- 2
- 21/3
- None of these

Q11. Let $f(x) = \begin{cases} x + a & ; x < 0 \\ |x - 1| & ; x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x + 1 & ; \text{if } x < 0 \\ (x - 1)^2 + b & ; x \geq 0 \end{cases}$ If $g \circ f$ is continuous, then

- a = 2, b = 0
- a = 0, b = 1
- a = 1, b = 0
- b = 1, a = 1

Q12. The function $f(x) = (\sin 2x)^{\tan^2 2x}$ is not defined at $x = \pi/4$. The value of $f(\pi/4)$ so that f is continuous at $x = \pi/4$ is

- \sqrt{e}
- 1
- 2
- None of these

Q13. If $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & x > 0 \end{cases}$ is continuous at $x = 0$, then

1. $a = -3/2, b = 0, c = 1/2$
2. $a = -3/2, b = 1, c = 1/2$
3. $a = -3/2, b \in \mathbf{R}, c = 1/2$
4. none of these

Q14. Let $f : \mathbf{R} \rightarrow [0, \infty]$ be such that $\lim_{x \rightarrow 5} \frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$. Then $\lim_{x \rightarrow 5} f(x)$ equals

- A. 0
- B. 1
- C. 2
- D. 3

Q15. Let $f(x) = g(x) \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$, where g is a continuous function. Then $\lim_{x \rightarrow 0} f(x)$ exists if

- A. $g(x) = x + 2$
- B. $g(x) = x^2 + 4$
- C. $g(x) = xh(x)$, where $h(x)$ is a polynomial
- D. $g(x)$ is a constant function

Q16. Let $f(x) = \frac{\sqrt{1 - \cos 2(x-2)}}{x-2}, x \neq 2$. Then $\lim_{x \rightarrow 2} f(x)$

- A. exists and is equal to $\sqrt{2}$
- B. does not exist because $\lim_{x \rightarrow 2^+} f(x)$ does not exist
- C. equal to 1
- D. does not exist because $\lim_{x \rightarrow 2^+} f(x) \neq \lim_{x \rightarrow 2^-} f(x)$

Q17. Let $f(x) = \begin{cases} \frac{e^{\alpha x} - e^x - x}{x^2} & x \neq 0 \\ \frac{3}{2} & x = 0 \end{cases}$. The value of α , so that f is a continuous function, is

- A. 2
- B. 0
- C. 3/2
- D. None of these

Q18. $\lim_{x \rightarrow \pi/2} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

- A. $\frac{1}{16}$
- B. $\frac{1}{8}$
- C. $\frac{1}{4}$
- D. $\frac{\pi}{2}$

Q19. Let $f(x) = \frac{x(1 + a \cos x) - b \sin x}{x^3}$, $x \neq 0$ and $f(0) = 1$. The values of a and b , respectively, so that f is a continuous function, are

- A. 5/2 and 3/2
- B. 5/2 and - 3/2
- C. -5/2 and - 3/2
- D. None of these

Q20. Let f be a continuous function on \mathbf{R} such that $f(1/2^n) = (\sin e^n) e^{-n^2} + \frac{2n^2}{n^2 + 1}$. Then the value of $f(0)$ is

- A. 1
- B. 1/2
- C. 2
- D. None of these

Q21. if $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & \text{when } x < 0 \\ \frac{a}{\sqrt{x}}, & \text{when } x = 0 \\ \sqrt{16 + \sqrt{x}} - 4, & \text{when } x > 0 \end{cases}$ And f is continuous at $x = 0$, then find the value of a .

[Delhi 2013C]

Q22. Find the value of k , so that the function f defined by $\begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$

[Hots; Delhi 2012C, Foreign 2011]

Q23. For what values of λ , is the function $f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$?

[Foreign 2001]

Q24. Find all points of discontinuity of f, where f is defined as follows $F(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$
[Delhi 2010]

Q25. For what value of k is the following function continuous at $x = 2$? $f(x) = \begin{cases} 2x + 1, & x < 2 \\ k, & x = 2 \\ 3x - 1, & x > 2 \end{cases}$
[Delhi 2008]

askITians