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Class: XII
Subject: Maths
Topic: Limit and Continuity
No. of Questions: 25

Q1. If f(x) = $\lim_{n\to\infty} \frac{x^{2n}-1}{x^{2n}+1}$, then

A. f(x) = 1 for |x| =

B. f(x) = $\begin{cases} 1 & \text{for } |x| > 1 \\ -1 & \text{for } |x| < 1 \end{cases}$

C. f(x)= $\begin{cases} 1 & \text{for } |x| > 1 \\ -1 & \text{for } |x| \le 1 \end{cases}$

D. f is not defined for any value of x

Right Answer Explanation: B

For $|x| < 1$, $x^{2n} \to 0$ as $n \to \infty$ and for $|x| > 1$,
$\frac{1}{x^{2n}} \to 0$ as $n \to \infty$. For $|x| = 1$ we have $x^2 = 1$. Thus

$$\frac{x^{2n}-1}{x^{2n}+1} = \begin{cases} 0, & |x| = 1 \\ \frac{1-x^{-2n}}{1+x^{-2n}}, & |x| > 1 \\ \frac{x^{2n}-1}{x^{2n}+1}, & |x| < 1 \end{cases}$$

Hence $f(x) = \begin{cases} 0, & |x| = 1 \\ 1, & |x| > 1 \\ -1, & |x| < 1 \end{cases}$

Q2. Let f (y) = sin $\frac{y-a}{2}$ tan $\frac{\pi y}{2a}$, $y \neq a$ .The value of f(a) so that f is a continuous function is

A. π / a

B. $-a/\pi$

C. $\pi/2a$

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D. none of these

Right Answer Explanation: B

$$f(a) = \lim_{y \to a} \sin\frac{(y-a)}{2} \tan\frac{\pi}{2}\frac{y}{a}$$

$$= \lim_{y \to a} \frac{\sin\frac{y-a}{2}}{\cot\frac{\pi}{2}\frac{y}{a}} \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{y \to a} \frac{\left(\frac{1}{2}\right)\cos\frac{y-a}{2}}{\left(-\frac{\pi}{2a}\right)\operatorname{cosec}^2\frac{\pi}{2}\frac{y}{a}} = -\frac{a}{\pi}.$$

Q3. Let f(x) = $\begin{cases} \dfrac{\sqrt{1+ax}-\sqrt{1-ax}}{x}, & -1 \le x < 0 \\ \dfrac{2x+1}{x-2}, & 0 \le x \le 1 \end{cases}$ The value of a such that f is continuous on [– 1, 1] is

A. 1/2

B. –1/2

C. 1

D. –1

Right Answer Explanation: B

$$f(0) = -\frac{1}{2} \text{ and } \lim_{x \to 0-} f(x) = a$$

Q4. The value of k (k > 0) for which the function

$f(x) = \dfrac{(e^x - 1)^4}{\sin(x^2/k^2)\log(1 + (x^2/2))}, x \neq 0; f(0) = 8$ may be a continuous function is

A. 1

B. 4

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C. 2

D. 3

Right Answer Explanation: C

We have $\lim_{x\to 0} f(x) =$

$$\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)^4 \times \frac{x^2/k^2}{\sin\left(x^2/k^2\right)} \times \frac{x^2/2}{\log\left(1+x^2/2\right)} \times 2k^2$$

$= 2k^2$

So $2k^2 = 8 \Rightarrow k = \pm 2.$

Q5. If f be a function on [0, 1] defined by $f(x) = (1/2)^n$, $(1/2)^{n+1} \le x < (1/2)^n$, n = 0, 1, 2, …, then

A. fis a continuous function

B. f is continuous except for x = 1/2

C. f is continuous except for finitely many points

D. the set of points where f is not continuous is infinite

Right Answer Explanation: D

$\lim_{x\to(1/2)^n-} f(x) = \left(\frac{1}{2}\right)^n$ whereas

$\lim_{x\to(1/2)^n+} f(x) = (1/2)^{n-1}$ as $f(x) = (1/2)^{n-1}$ for $(1/2)^n \le x < (1/2)^{n-1}$. So f is discontinuous at 1/2, $(1/2)^2$…. Thus f has infinitely many discontinuities.

Q6. If f : **R** $\rightarrow$ **R** is a function defined by f(x) = [x] cos $\left(\frac{2x-1}{2}\right)\pi,$ where [x] denotes the greatest integer function, then f is

A. discontinuous only at x = 0

B. discontinuous only at non-zero integral value of x

C. continuous only at x = 0

D. continuous for every real x

Right Answer Explanation: D

For $n \in \mathbf{I}$, we have $[x] = n$, for $n \le x < n + 1$.

Thus $f(x) = n\cos\left(\frac{2x-1}{2}\right)\pi$, for $n \le x < n+1$. Since $[x]$ is continuous for $x \notin I$ and $\cos\left(\frac{2x-1}{2}\right)\pi$ is a continuous.

So f is continuous at all $x \notin I$. For $x = n \in \mathbf{I}$,

$$\lim_{x \to n+} f(x) = \lim_{x \to n+} n\cos\left(\frac{2x-1}{2}\right)\pi = 0$$

$$\lim_{x \to n-} f(x) = \lim_{x \to} (n-1)\cos\left(\frac{2x-1}{2}\right)\pi = 0$$

Also $f(n) = 0$. So f is continuous on $\mathbf{R}$.

Q7. If f(x) = $\frac{\log(1+x^2)}{x^4 - 26x^2 + 25}$, then

A. f is continuous on [6, 10]

B. f is continuous on [– 2, 2]

C. f is continuous on [– 6, 6]

D. f is continuous on [1, 7]

Right Answer Explanation: A

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$$f(x) = \frac{\log\left(1+x^2\right)}{\left(x^2-25\right)\left(x^2-1\right)}$$

$$= \frac{\log\left(1+x^2\right)}{(x-5)(x+5)(x-1)(x+1)}$$

Since $\log(1+x^2)$ is continuous on $(-\infty, \infty)$, so f is continuous on $\mathbf{R} - \{-5, -1, 1, 5\}$ which clearly contains the interval $[6, 10]$.

Q8. The value of $\lim_{x\to 0} \frac{e^{nx} - \left(1 + nx + \frac{n^2}{2}x^2\right)}{x^3} (n > 0)$ is

A. $\frac{n^2}{6}$

B. $\frac{n^3}{3}$

C. $\frac{n^3}{6}$

D. $\frac{1}{6}$

Right Answer Explanation: C

Using series expansion of e^{nx} the required limit is equal to

$$\lim_{x\to 0} \frac{\frac{n^3}{6}x^3 + \frac{n^4}{24}x^4 + \cdots}{x^3}$$

$$= \frac{n^3}{6}.$$

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Q9. The value of $\lim_{x\to 0}\left(\frac{e^x + e^{-x} - 2}{x^2}\right)^{1/x^2}$ is

A. $e^{1/2}$

B. $e^{1/4}$

C. $e^{1/3}$

D. $e^{1/12}$

Right Answer Explanation: D

For $x \neq 0$, let $u = \frac{e^x + e^{-x} - 2}{x^2}$

$$= \frac{1}{x^2}\left[\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) + \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \cdots\right) - 2\right]$$

$$= \frac{2}{x^2}\left[\frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right] = \left[1 + \frac{x^2}{12} + \frac{2x^4}{6!} + \cdots\right]$$

$$\therefore \quad u^{1/x^2} = \left[1 + \frac{x^2}{12} + \cdots\right]^{\frac{1}{x^2}}$$

$= e^{(1/x^2)\ (1 + x^2/12 + \ldots\ -1)} = e^{1/12}$ (1^{∞} form)

Q10. $\lim_{x\to \pi/2} \frac{\cos x}{\sqrt[3]{(1-\sin x)^2}}$ is equal to

1. 1

2. 2

3. 21/3

4. None of these

Right Answer Explanation: D

The limit does not exist.

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Q11. Let f(x) = $\begin{cases} x+a & ; x<0 \\ |x-1| & ; x \ge 0 \end{cases}$ and $g(x) = \begin{cases} x+1 & ; \text{if } x<0 \\ (x-1)^2+b & ; x \ge 0 \end{cases}$ If gof is continuous, then

A. a = 2, b = 0

B. a = 0, b = 1

C. a = 1, b = 0

D. b = 1, a = 1

Right Answer Explanation: C

$$gof(x) = \begin{cases} x+a+1 & : \quad x<-a \\ (x+a-1)^2+b & : \quad -a \le x < 0 \\ x^2+b & : \quad 0 \le x < 1 \\ (x-2)^2+b & : \quad x \ge 1 \end{cases}$$

gof is continuous except possibly at $x = -a, 0, 1$.

$$\lim_{x \to -a-} f(x) = -a + a + 1 = 1, \; gof(-a) = 1 + b$$

Therefore, $b + 1 = 1 \Rightarrow b = 0.$

$$\lim_{x \to 0-} f(x) = (a-1)^2 + b$$

$$= (a-1)^2, \; gof(0) = b = 0.$$

So $(a-1)^2 = 0 \Rightarrow a = 1.$

Q12. The function f(x) = $(Sin2x)^{tan^2 2x}$ is not defined at x = π/4. The value of f(π/4) so that f is continuous at x = π/4 is

1. $\sqrt{e}$

2. 1

3. 2

4. None of these

Right Answer Explanation: D

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For f to be continuous at $x = \pi/4$,

$$f(\pi/4) = \lim_{x \to \pi/4} f(x) = \lim_{x \to \pi/4} (1 - \cos^2 2x)^{(1/2)\tan^2 2x}$$

$$= \lim_{x \to \pi/4} \left\{(1 - \cos^2 x)^{-(1/\cos^2 x)}\right\}^{-(\sin^2 2x)/2} = \frac{1}{\sqrt{e}}.$$

Q13. If $f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \dfrac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}}, & x > 0 \end{cases}$ is continuous at $x = 0$, then

1. a = – 3/2, b = 0, c = 1/2
2. a = – 3/2, b = 1, c = 1/2
3. a = – 3/2, b $\in$ **R**, c = 1/2
4. none of these

Right Answer Explanation: C

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0} \frac{\sin(a+1)x + \sin x}{x}$$

$$= \lim_{x \to 0} \left[(a+1)\frac{\sin(a+1)x}{(a+1)x} + \frac{\sin x}{x}x^2\right] = a + 2$$

Also $$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}}$$

$$= \lim_{x \to 0} \frac{\sqrt{1 + bx} - 1}{bx} = \left(\frac{0}{0}\text{form}\right)$$

$$= \lim_{x \to 0} \frac{b}{2\sqrt{1 + bx}} \cdot \frac{1}{b}\frac{1}{2}$$

Thus we must have $a + 2 = c = 1/2$.
So $a = -3/2$, $c = 1/2$ and b is any real number.

Q14. Let f : **R** $\rightarrow$ [0, ∞] be such that $\lim_{x\to 5}\frac{(f(x)^2-9)}{\sqrt{|x-5|}}=0$ Then $\lim_{x\to 5} f(x)$ equals

A. 0

B. 1

C. 2

D. 3

Right Answer Explanation: D

Since, $\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$ So

for $\epsilon = 1$ there is $\delta > 0$ such that

$$-1 < \frac{(f(x)^2-9)}{\sqrt{x-5}} < 1 \text{ whenever } 0 < |x-5| < \delta$$

$$\Rightarrow -\frac{\sqrt{|x-5|}}{f(x)+3} < f(x)-3 < \frac{\sqrt{|x-5|}}{f(x)+3} \text{ whenever } 0 < |x-5| < \delta$$

Since the range of f is $[0, \infty]$ So $\lim_{x\to 5}(f(x)+3)\neq 0$ and exists. Thus

$$0 \le \lim_{x\to 5}(f(x)-3) \quad \le 0 \Rightarrow \lim_{x\to 5} f(x) = 3.$$

Q15. Let f(x) = g(x) $\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}$, where g is a continuous function. Then $\lim_{x\to 0} f(x)$ exists if

A. g(x) = x + 2

B. g(x) = x^2 + 4

C. g(x) = xh(x), where h(x) is a polynomial

D. g(x) is a constant function

Right Answer Explanation: C

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$$\lim_{x\to 0+}\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}=\lim_{x\to 0+}\frac{1-e^{-2/x}}{1+e^{-2/x}}=1$$

$$\lim_{x\to 0-}\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}=\lim_{x\to 0-}\frac{e^{2/x}-1}{e^{2/x}+1}=-1$$

Hence $\lim_{x\to 0} f(x)$ exists if $g(x) = xh(x)$ where $h(x)$ is a continuous function. If $g(x) = a$ $(a \neq 0)$ then $\lim_{x\to 0+} f(x) = a$ and $\lim_{x\to 0-} f(x) = -a$. Thus $\lim_{x\to 0} f(x)$ does not exist if $f(x) = x + 2$ or $x^2 + 4$ or is a constant function.

Q16. Let f(x) = $\dfrac{\sqrt{1-\cos 2(x-2)}}{x-2}, x \neq 2.$ Then $\lim_{x\to 2} f(x)$

A. exists and is equal to $\sqrt{2}$

B. does not exist because $\lim_{x\to 2+} f(x)$ does not exist

C. equal to 1

D. does not exist because $\lim_{x\to 2+} f(x) \neq \lim_{x\to 2-} f(x)$

Right Answer Explanation: D

$f(x) = \dfrac{\sqrt{2}|\sin(x-2)|}{(x-2)}$ so $\lim_{x\to 2+} f(x) = 1$

and $\lim_{x\to 2-} f(x) = -1$.

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Q17.

Let f(x) = $\begin{cases} \dfrac{e^{\alpha x} - e^{x} - x}{x^2} & x \neq 0 \\ \dfrac{3}{2} & x = 0 \end{cases}$, The value of α, so that f is a continuous function, is

A. 2

B. 0

C. 3/2

D. None of these

Right Answer Explanation: A

$$\frac{3}{2} = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{e^{\alpha x} - e^{x} - x}{x^2} \left(\frac{0}{0}\text{form}\right)$$

$$= \lim_{x\to 0} \frac{\alpha e^{\alpha x} - e^{x} - 1}{2x}$$

For the last limit to exist, we must have

$$\lim_{x\to 0} (\alpha e^{\alpha x} - e^{x} - 1) = 0$$

$\therefore\ \alpha - 1 - 1 = 0 \Rightarrow \alpha = 2$. In this case, the last limit is equal to

$$\lim_{x\to 0} \frac{\alpha^2 e^{\alpha x} - e^{x}}{2} = \frac{\alpha^2 - 1}{2} = \frac{3}{2}.$$

Q18. $\lim_{x\to \pi/2} \dfrac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

A. $\frac{1}{16}$

B. $\frac{1}{8}$

C. $\frac{1}{4}$

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D. $\frac{\pi}{2}$

Right Answer Explanation: A

Put $x - \pi/2 = \theta$, so that

$$\lim_{x\to\pi/2}\frac{\cot x - \cos x}{(\pi - 2x)^3} = \lim_{\theta\to 0}\frac{\cot(\pi/2+\theta) - \cos(\pi/2+\theta)}{(-2\theta)^3}$$

$$= \frac{1}{8}\lim_{\theta\to 0}\frac{\tan\theta - \sin\theta}{\theta^3}$$

$$= \frac{1}{8}\lim_{\theta\to 0}\frac{\sec^2\theta - \cos\theta}{3\theta^2}$$

$$= \frac{1}{24}\lim_{\theta\to 0}\frac{2\sec^2\theta\tan\theta + \sin\theta}{2\theta}$$

$$= \frac{1}{24}\left(\frac{2+1}{2}\right) = \frac{1}{16}.$$

Q19. Let f(x) = $\frac{x(1 + a\cos x) - b\sin x}{x^3}, x \neq 0$ and f(0) = 1. The values of a and b, respectively, so that f is a continuous function, are

A. 5/2 and 3/2

B. 5/2 and – 3/2

C. -5/2 and – 3/2

D. None of these

Right Answer Explanation: C

Note that f is continuous everywhere except possibly at $x = 0$. For f to be continuous at $x = 0$,

$$1 = f(0) = \lim_{x\to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3}\left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 0} \frac{(1 + a\cos x) - x\,a\sin x - b\cos x}{3x^2}$$

$$= \lim_{x\to 0} \frac{1 + (a - b)\cos x - x\,a\sin x}{3x^2}$$

This limit will exist only if $1 + (a - b) = 0$. In this case limit is equal to

$$= \lim_{x\to 0} \frac{-(a-b)\sin x - a\,x\cos x - a\sin x}{6x}$$

(using L'Hôpital Rule)

$$= \frac{1}{6}\lim_{x\to 0}\left[(b - 2a)\frac{\sin x}{x} - a\cos x\right]$$

$$= \frac{1}{6}[b - 3a] \Rightarrow b - 3a = 6$$

Solving $a - b = -1$ and $b - 3a = 6$, we get $b = -3/2$ and $a = -5/2$.

Q20. Let f be a continuous function on **R** such that $f(1/2^n) = (\sin e^n)\,e^{-n^2} + \frac{2n^2}{n^2+1}$. Then the value of f(0) is

A. 1

B. 1/2

C. 2

D. None of these

Right Answer Explanation: C

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As f is continuous so $f(0) = \lim_{x \to 0} f(x) =$

$\lim_{n \to \infty} f(x_n)$ for any sequence x_n such that $\lim_{n \to \infty} x_n = 0$.

Thus

$$f(0) = \lim_{n \to \infty} f\left(1/2^n\right) = \lim_{n \to \infty}\left(\left(\sin e^n\right)e^{-n^2} + \frac{2n^2}{n^2+1}\right)$$

$$= \lim_{n \to \infty}\left(\left(\sin e^n\right)e^{-n^2} + \frac{2}{1+1/n^2}\right) = 0 + 2 = 2.$$

Q21. $if\ f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & when\ x < 0 \\ a, & when\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & when\ x > 0 \end{cases}$ And f is continuous at x = 0, then find the value of a.

[Delhi 2013C]

Right Answer Explanation:

$$f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & when\ x < 0 \\ a, & when\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & when\ x > 0 \end{cases}$$

Since, f (x) is continuous at x = 0

$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

$\Rightarrow \lim_{x \to 0^-} \frac{1-\cos 4x}{x^2}$

$= \lim_{x \to 0^+} \frac{\sqrt{x}}{\sqrt{16+\sqrt{(x)}-4}} = f(0) \qquad (1)$

$\Rightarrow \lim_{h \to 0^-} \frac{1-\cos 4(0-h)}{(0-h)^2}$

$= \lim_{h \to 0} \frac{\sqrt{0+h}}{\sqrt{16+\sqrt{(0+h-4)}}} = f(0)$

$\Rightarrow \lim_{h \to 0} \frac{1-\cos 4h}{h^2}$

$\lim_{h \to 0} \frac{\sqrt{h}}{\sqrt{16+\sqrt{4}-4}} \times \frac{\sqrt{16+\sqrt{h+4}}}{\sqrt{16+\sqrt{h}-4}} = a$ (1)

$\Rightarrow \lim_{h \to 0} \frac{2\sin^2 2h}{h^2} \lim_{h \to 0} \frac{\sqrt{h}\sqrt{16+\sqrt{h+4}}}{(16+\sqrt{h})-16} = a$

[$\therefore$ 1– $\cos 2\theta = 2\sin^2\theta$ and (a + b) (a – b)= $a^2 - b^2$]

$\Rightarrow \lim_{h \to 0} \left(\frac{\sin 2h}{2h}\right)^2 \times 4$

$= \lim_{h \to 0} \frac{\sqrt{h}\sqrt{16+\sqrt{h+4}}}{\sqrt{h}} = a$ (1)

$\left[\therefore \lim_{h \to 0} \frac{\sin h}{h} = 1\right]$

$\Rightarrow 2 \times (1)^2 \times = 4\sqrt{16+\sqrt{0}} + 4 = a$

$\Rightarrow$ 8 = 4 + 4 = a

$\therefore$ a = 8 (1)

Q22. Find the value of k, so that the function f defined by $\begin{cases} \frac{k \cos x}{\pi - 2x}, & if \times \neq \frac{\pi}{2} \\ 3, & if \times = \frac{\pi}{2} \end{cases}$ is continuous at x = $\frac{\pi}{2}$

[Hots; Delhi 2012C, Foreign 2011]

Right Answer Explanation:

$\begin{cases} \frac{k \cos x}{\pi - 2x}, & if \times \neq \frac{\pi}{2} \\ 3, & if \times = \frac{\pi}{2} \end{cases}$

Given that function is continuous at X= $\frac{\pi}{2}$.

$\therefore$ LHL = RHL = $f\left(\frac{\pi}{2}\right)$ ....(i) (1)

Now, LHL = $\lim_{x \to \frac{\pi^-}{2}}$ f (x) = $\lim_{x \to \frac{\pi^-}{2}} \frac{k \cos x}{\pi - 2x}$

Put $x = \frac{\pi}{2} + h$, when $x \to \frac{\pi}{2}, h \to 0$

LHL $= \lim_{h \to 0} \frac{k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = \lim_{h \to 0} \frac{k \sin h}{\pi - \pi + 2h}$

$\left[\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right]$

$= \lim_{h \to 0} \frac{k \sin h}{2h}$

$= \frac{k}{2} \lim_{h \to 0} \frac{\sin h}{h} = \frac{k}{2}$ $\quad \left[\because \lim_{h \to 0} \frac{\sin h}{h} = 1\right]$

$\therefore$ LHL $= \frac{k}{2}$ (1$^{1/2}$)

Also, from the given function, we get

$f\left(\frac{\pi}{2}\right) = 3$ (1/2)

Now, from Eq. (i), we get

LHL $= f\left(\frac{\pi}{2}\right)$

$\therefore \quad = \frac{k}{2} = 3$

Hence, k = 6. (1)

Q23. For what values of λ, is the function f (x)= $\begin{cases} \lambda(x^2 - 2x), & if\ x \le 0 \\ 4x + 1, & if\ x > 0 \end{cases}$ is continuous at x = 0?

[Foreign 2001]

Right Answer Explanation:

The given function is

f (x)= $\begin{cases} \lambda(x^2 - 2x), & if\ x \le 0 \\ 4x + 1, & if\ x > 0 \end{cases}$

Given that f (x) is continuous at x = 0.

$\therefore$ LHL = RHL = f (0) ....(1)

Now, LHL $= \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \lambda(x^2 - 2x)$ (1)

Put x= $0 - h = -h$, when x $\to 0, h \to 0$

LHL = $\lim_{h \to 0} \lambda(h^2 + 2h)$

$\lambda = (0)$ [Put h = 0] (1)

= 0

Now, RHL = $\lim_{h \to 0^+} f(x) = \lim_{h \to 0^+} (4x+1)$

Put x = 0 + h = h, when x ⟶ 0, $h \longrightarrow 0$

RHL = $\lim_{h \to 0} (4h + 1)$

RHL = 1 [Put h = 0]

∴ We get, LHL = 0 and RHL = 1

∴ We get, LHL ≠ RHL

But given that LHL = RHL [From Eq. (i)]

Hence, we get a contradiction.

Hence, there doesn't exist any real value of λ

For which f (x) is continuous at X = 0 (1)

Q24. Find all points of discontinuity of f, where f is defined as follows F (x)= $\begin{cases} |x| + 3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \ge 3 \end{cases}$ [Delhi 2010]

Right Answer Explanation:

The given function is f (x) = $\begin{cases} |x| + 3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \ge 3 \end{cases}$

First, we verify continuity at x = –3 and then at x = 3.

Continuity at x = −3

LHL = $\lim_{x \to (-3)^-} f(x) = \lim_{x \to (-3)^-} (|x| + 3)$

Put x = -3– h, when X ⟶ −3, $h \longrightarrow 0$

= $\lim_{h \to 0} (|-3 - h| + 3) = |-3| + 3$ [Put h = 0]

$= 3 + 3$ $[\because |-x| = x, \forall\ x \in R]$

$= 6$ (1/2)

RHL $= \lim_{x \to (-3)^+} f(x) = \lim_{x \to (-3)^+} (-2x)$

Put X = -3 + h, when $x \to 3, h \to 0$

RHL $= \lim_{h \to 0} [-2(-3+h)] = \lim_{h \to 0} (6 - 2h)$

RHL = 6 [Put h = 0] (1/2)

Also, $f(-3)$ = value of f(x) at X = –3

$= |-3| +$

$= 2 + 3 = 6$ $[\because |-x| = x, \forall \in R]$(1/2)

$\therefore$ LHL = RHL = F(–3)

$\therefore$ f(x) is continuous at x = –3. So, x = –3 is the point of continuity.

Continuity at X = 3

LHL $= \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} -2x$

Put x = 3 – h, when $x \to 3, h \to 0$

LHL $= \lim_{h \to 0} - 2(3 - h) = \lim_{h \to 0} (-6 + 2h)$

LHL = – 6 [Put h = 0] (1/2)

Now, RHL $= \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (6x+2)$

Put x = 3 + h, when $x \to 3, h \to 0$

RHL $= \lim_{h \to 0}$ [(6 (3+h) + 2]

$= \lim_{h \to 0} (18 + 6h + 2)$

RHL = 20 [Put h = 0] (1/2)

$\therefore$ LHL $\neq$ RHL

As f (x) is a polynomial function in a given interval, so it is continuous in a given interval.

$\therefore$ f (x) is not continuous at x = 3. So, x = 3 is the point of discontinuous of f(x). (1)

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Q25. For what value of k is the following function continuous at x = 2? f (x)= $\begin{cases} 2x+1 , & x<2 \\ k , & x=2 \\ 3x-1 , & x>2 \end{cases}$

[Delhi 2008]

Right Answer Explanation:

The given function is f (x)= $\begin{cases} 2x+1 , & x<2 \\ k , & x=2 \\ 3x-1 , & x>2 \end{cases}$

Given that, f (x) is continuous at x = 2.

$\therefore$ $(LHL)_{x=2} = (RHL)_{x=2} = f(2)$ ....(i) (1)

Now, LHL = $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x+1)$

Put x = 2−h, when x→ 2, $h \to 0$

LHL = $\lim_{h \to 0}[2(2-4)+1] = \lim_{h \to 0}(4-2h+1)$

$= \lim_{h \to 0}(5-2h)$

LHL = 5 [Put h=0] (1.5)

Also, from f(x), we get at x=2, f(x) = K

$\therefore$ f(2) = k (1/2)

Also, from Eq. (i), we get

LHL = f(2) $\Rightarrow 5 = k$

$\therefore$ K = 5 (1)