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Class: XII
Subject: Maths
Topic: Matrices
No. of Questions: 26

Q1. If A is a diagonal matrix, what is the value of A^T ?

1. I
2. A
3. A^{-1}
4. A^2

Sol. B

Let A be the diagonal matrix.

$A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$A^T = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$A = A^T$

If the A is a diagonal matrix, then $A^T = A$.

Q2. If $a_{ij} = 0$ for all value of i and j, then it is a

1. null matrix
2. row matrix
3. identity matrix
4. none of these

Sol: A

$a_{ij} = 0$ for all values of i & j

$a_{11} = 0$, $a_{12} = 0$, $a_{21} = 0$, $a_{22} = 0$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

A will be a null matrix.

Q3. If the rank of matrix is 3, then its order could be

1. 3 × 3
2. 3 × 2
3. 2 × 2
4. 2 × 1

Sol: A

The rank of the matrix is given by the lowest value of m + n.

If the rank is 3, i.e. either m could be 3 or n could be 3.

Order can be 3 x 3.

Q4. The inverse of a symmetric matrix is:

1. Symmetric
2. Skew symmetric
3. Diagonal matrix
4. None of these

Sol: A

Fact

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Q5. If a matrix is of order 2 × 7, the number of elements in it is

1. 14
2. 7
3. 2
4. 9

Sol: A

If a matrix is of order 2 × 7, then the number of elements = 2 (7) = 14.

Q6. If A = $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}$, then

1. AB = BA = 0
2. AB = 0, BA $\neq$ 0
3. BA = 0, AB $\neq$ 0
4. none of the above

Sol: B

A = $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ B = $\begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}$

AB = $\begin{bmatrix} -2+2 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

BA = $\begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -2+0 & -4+0 \\ 1+0 & 2+0 \end{bmatrix}$

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$= \begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix}$

= so AB = 0, BA $\neq$ 0

Q7. If A = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then for all natural numbers n, A^n is equal to

1. $\begin{bmatrix} n & 0 \\ 1 & 1 \end{bmatrix}$
2. $\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 0 \\ 1 & n \end{bmatrix}$
4. None of these

Sol: B

$A^2 = AA = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $A^3 = A^2 A$

$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ & so on.

Hence $A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$ $n \in N$

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Q8. If $f(x) = x^2 + 4x - 5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, then f(A) =

1. $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$
2. $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$
4. $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

Sol: D

$f(A) = A^2 + 4A - 5I$

$= \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

Q9. Let A be $\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ is equal to

1. 5^2
2. 1
3. $\frac{1}{5}$
4. 5

Sol: C

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$$\text{Given } A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \text{ and } |A^2| = 25$$

$$\therefore \quad A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$$

$$\therefore |A^2| = 25\ (25\alpha^2)$$

$$\therefore 25 = 25\ (25\alpha^2) \Rightarrow |\alpha| = \frac{1}{5}$$

Q10. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, a, b ∈ N, then

1. there exist more than one but finite number of B's such that AB = BA
2. there exists exactly one B such that AB = BA
3. there exists infinitely many B's such that AB = BA
4. there cannot exist any B such that AB = BA

Sol: B

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

$$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$$

AB = BA only when a = b

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Q11. If A and B are square matrices of size n x n such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true?

1. AB = BA
2. either A or B is a zero matrix
3. either A or B is an identity matrix
4. A = B

Sol: A

$$A^2 - B^2 = (A - B)(A + B)$$
$$A^2 - B^2 = A^2 + AB - BA - B^2$$
$$\Rightarrow AB = BA.$$

Q12. If $A = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then |AB| is equal to

1. 80
2. 100
3. –110
4. 92

Sol: B

Find the matrix AB using multiplication rule and then find determinant.

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Q13. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

1. a = 1, b = 1
2. a = sin 2θ, b = cos 2θ
3. a = cos 2θ, b = sin 2θ
4. none of these

Sol: D

$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix} = \begin{bmatrix} 1+tan^2\theta & 0 \\ 0 & 1+tan^2\theta \end{bmatrix}$

so clearly b=0 which follows answer is D

Q14. The matrix $\begin{bmatrix} 0 & -3 & 4 \\ 3 & 0 & -5 \\ -4 & 5 & 0 \end{bmatrix}$ is:

1. symmetric
2. skew-symmetric
3. non-singular
4. singular

Sol: B

Since A^t= -A

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Q15. Find the inverse of a matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

1. $\begin{bmatrix} -2 & 1 \\ \frac{-3}{2} & \frac{-1}{2} \end{bmatrix}$
2. $\begin{bmatrix} 2 & -1 \\ \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$
3. $\begin{bmatrix} 2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$
4. $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$

Sol: A

Definition - Inverse of a matrix is defined as the matrix when multiplied with the original matrix yields an identity matrix.

Inverse of a matrix is defined only when certain conditions are satisfied out of which one is that it should be a square matrix.

Let the inverse of the given matrix be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

By definition, $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Hence, on comparing the elements, we get

a + 2c = 1, b + 2d = 0, 3a + 4c = 0 and 3b + 4d = 1

Solving for 'a' and c, we get

a = 1 - 2c and 3 - 6c + 4c = 0

c = $\frac{3}{2}$, a = - 2

Similarly, solving for b and d, we get

b = - 2d, - 6d + 4d = 1

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d = $\frac{-1}{2}$ and b = - 1

Hence, the matrix is $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$.

Q16. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ satisfies the relation $A^n = kA + mI$, the values of k and m are

1. (k, m) = (n, n - 1)
2. (k, m) = (n, n)
3. (k, m) = (n, n + 1)
4. (k, m) = (n, 1 - n)

Sol: A

Substituting the value of A and I, we get

$$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$$

$$kA + mI = k\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k+m & 0 \\ k & k+m \end{bmatrix}$$

$$A^n = kA + mI$$

$$\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = \begin{bmatrix} k+m & 0 \\ k & k+m \end{bmatrix}$$

Comparing the elements on both sides, we get

k = n and m = 1 - n

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Q17. For any two 3 x 3 matrices P and Q, if PQ = P and QP = Q, find the value of $P^2 + Q^2$.

1. PQ
2. QP
3. P - Q
4. P + Q

Sol: D

$$P^2 = P \times P = (PQ)(PQ) = PQPQ = PQQ = PQ = P$$

From the definition, $Q^2 = Q \times Q = (QP)(QP) = QPQP = QPP = QP = Q$

From the above two relations, we get

$P^2 + Q^2 = P + Q$

Q18. In trigonometry,

$$\sin(A+B) = \sin A\cos B + \cos A\sin B, \cos(A+B) = \cos A\cos B - \sin A\sin B$$

If $A = \begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix}$ and $B = \begin{bmatrix} \cos B & -\sin B \\ \sin B & \cos B \end{bmatrix}$, find the value of $A^n + B^n$.

1. $\begin{bmatrix} \cos nA - \cos nB & -(\sin nA + \sin nB) \\ \sin nA + \sin nB & \cos nA - \cos nB \end{bmatrix}$
2. $\begin{bmatrix} \cos nA + \cos nB & -(\sin nA + \sin nB) \\ \sin nA + \sin nB & \cos nA + \cos nB \end{bmatrix}$
3. $\begin{bmatrix} \cos nA + \cos nB & \sin nA + \sin nB \\ \sin nA + \sin nB & \cos nA + \cos nB \end{bmatrix}$
4. $\begin{bmatrix} \cos nA + \cos nB & -(\sin nA - \sin nB) \\ \sin nA - \sin nB & \cos nA + \cos nB \end{bmatrix}$

Sol: B

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$$A^2 = \begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix}\begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix}$$

$$= \begin{bmatrix} \cos A\cos A - \sin A\sin A & -(\cos A\sin A + cisA\sin A) \\ \cos A\sin A + \cos A\sin A & \cos A\cos A - \sin A\sin A \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2A & -\sin 2A \\ \sin 2A & \cos 2A \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2A & -\sin 2A \\ \sin 2A & \cos 2A \end{bmatrix}\begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2A\cos A - \sin 2A\sin A & -(\cos 2A\sin A + \sin 2A\cos A) \\ \sin 2A\cos A + \cos 2A\sin A & \cos 2A\cos A - \sin 2A\sin A \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3A & -\sin 3A \\ \sin 3A & \cos 2A \end{bmatrix}$$

From the above two relations, we can generalize that

$$A^n = \begin{bmatrix} \cos nA & -\sin nA \\ \sin nA & \cos nA \end{bmatrix} \text{ and } B^n = \begin{bmatrix} \cos nB & -\sin nB \\ \sin nB & \cos nB \end{bmatrix}.$$

Adding the two expressions, we get

$$A^n + B^n = \begin{bmatrix} \cos nA & -\sin nA \\ \sin nA & \cos nA \end{bmatrix} + \begin{bmatrix} \cos nB & -\sin nB \\ \sin nB & \cos nB \end{bmatrix}$$

$$A^n + B^n = \begin{bmatrix} \cos nA + \cos nB & -(\sin nA + \sin nB) \\ \sin nA + \sin nB & \cos nA + \cos nB \end{bmatrix}$$

Q19. In trigonometry, sin (A+B) = SinA cosB + cosAsinB, cos(A+B) = cosA cosB – sinAsinB

If $A = \begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix}$ and $B = \begin{bmatrix} \cos B & -\sin B \\ \sin B & \cos B \end{bmatrix}$, what is the value of $A^n \times B^n$?

1. $\begin{bmatrix} \cos n(A-B) & -\sin n(A+B) \\ \sin n(A+B) & \cos n(A-B) \end{bmatrix}$
2. $\begin{bmatrix} \cos n(A+B) & -\sin n(A-B) \\ \sin n(A-B) & \cos n(A+B) \end{bmatrix}$
3. $\begin{bmatrix} \cos n(A+B) & -\sin n(A+B) \\ \sin n(A+B) & \cos n(A+B) \end{bmatrix}$
4. $\begin{bmatrix} \cos n(A+B) & \sin n(A+B) \\ \sin n(A+B) & \cos n(A+B) \end{bmatrix}$

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Sol: C

$$A^2 = \begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix}\begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix}$$

$$= \begin{bmatrix} \cos A\cos A - \sin A\sin A & -(\cos A\sin A + cisA\sin A) \\ \cos A\sin A + \cos A\sin A & \cos A\cos A - \sin A\sin A \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2A & -\sin 2A \\ \sin 2A & \cos 2A \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2A & -\sin 2A \\ \sin 2A & \cos 2A \end{bmatrix}\begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2A\cos A - \sin 2A\sin A & -(\cos 2A\sin A + \sin 2A\cos A) \\ \sin 2A\cos A + \cos 2A\sin A & \cos 2A\cos A - \sin 2A\sin A \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3A & -\sin 3A \\ \sin 3A & \cos 2A \end{bmatrix}$$

From the above two relations, we can generalize that

$$A^n = \begin{bmatrix} \cos nA & -\sin nA \\ \sin nA & \cos nA \end{bmatrix} \text{ and } B^n = \begin{bmatrix} \cos nB & -\sin nB \\ \sin nB & \cos nB \end{bmatrix}.$$

Multiplying these two expressions, we get

$$A^n \times B^n = \begin{bmatrix} \cos nA & -\sin nA \\ \sin nA & \cos nA \end{bmatrix} \times \begin{bmatrix} \cos nB & -\sin nB \\ \sin nB & \cos nB \end{bmatrix}$$

$$= \begin{bmatrix} \cos nA\cos nB - \sin nA\sin nB & -(\sin nA\cos nB + \cos nA\sin nB) \\ \cos nA\sin nB + \sin nA\cos nB & \cos nA\cos nB - \sin nA\sin nB \end{bmatrix}$$

$$= \begin{bmatrix} \cos n(A+B) & -\sin n(A+B) \\ \sin n(A+B) & \cos n(A+B) \end{bmatrix}$$

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Q20. The matrix product $\begin{bmatrix}1\\-2\\3\end{bmatrix}[4\ \ 5\ \ 2]\begin{bmatrix}2\\-3\\5\end{bmatrix}$ equals

1. $\begin{bmatrix}3\\-6\\9\end{bmatrix}$
2. $\begin{bmatrix}3\\6\\9\end{bmatrix}$
3. $\begin{bmatrix}3\\6\\-9\end{bmatrix}$
4. none of these

Sol: A

Follow simple product rule.

Q21. If $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$, then find the value of (x + y), [Delhi 2013C; All India 2012]

Sol:

Given, $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

On comparing corresponding elements, we get

$2 + y = 5$ and $2x + 2 = 8$

$\Rightarrow$ $y = 3$ and $2x = 6$

$\Rightarrow$` $y = 3$ and $x = 3$

$\therefore$ $x + y = 3 + 3 = 6$

Q22. If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$, then find the value of y.

[Foreign 2009]

Sol:

Given, $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$

On equating the corresponding elements, we get

$y + 2x = 7$ ….(i)

And $-x = -2 \Rightarrow x = 2$

On substituting x = 2 in Eq. (i). we get

$y + 4 = 7 \Rightarrow y = 3$

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Q23. If A = $\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$, then find value of

$A^2 - 3A + 2I.$ [All India 2010]

Sol:

Given that, $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

We have to find the value of $A^2 - 3A + 2I$.

Now, $A^2 = A \cdot A$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1-0 & 1-3+0 \end{bmatrix}$$

[Multiplying row by column]

$$\therefore \quad A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} \qquad \textbf{(1½)}$$

$$\text{Now, } 3A = 3\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix} \qquad \textbf{(1/2)}$$

$$\text{and} \quad 2I = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \textbf{(1/2)}$$

$\therefore A^2 - 3A + 2I$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\text{Hence, } A^2 - 3A + 2I = \begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & -4 \\ -3 & 2 & 0 \end{bmatrix} \qquad \textbf{(1½)}$$

Q24. For the following matrices A and B, verify that

$$[AB]' = B'A'; \;\; A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix},$$

$$B = [-1 \; 2 \; 1]$$ [All India 2010]

Sol:

Given that, $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = [-1 \;\; 2 \;\; 1]$

To verify $(AB)' = B'A'$

$\text{LHS} = (AB)'$

Now, $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3\times 1} [-1 \;\; 2 \;\; 1]_{1\times 3}$

$\therefore \quad AB = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

[Multiplying row by column]

$\therefore \quad (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ ...(i)

[Interchanging rows and columns] **(1½)**

$\text{RHS} = B'A'$

Now, $B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $A' = [1 \quad -4 \quad 3]$ **(1)**

$\therefore \quad B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} [1 \quad -4 \quad 3]$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} \quad \ldots(ii)$$

[Multiplying row by column]**(1)**

From Eqs. (i) and (ii), we get

$(AB)' = B'A'$

$\therefore$ LHS = RHS **(1/2)**

Hence verified.

Q25. Express the following matrix as a sum of a symmetric and a skew-symmetric matrix and

Verify your result $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$. [Hots; All India 2010]

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Let $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Let us introduce two matrices P and Q, such that

$$P = \frac{1}{2}(A + A')$$

and $$Q = \frac{1}{2}(A - A')$$

We will show that $A = (P + Q)$

Firstly, we find the matrices P and Q and check whether they are symmetric and skew-symmetric matrices.

$\therefore P = \frac{1}{2}(A + A')$

$$\Rightarrow P = \frac{1}{2}\left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} \quad \textbf{(1)}$$

Now, $P' = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = P$

$\because \quad P' = P$

$\therefore$ P is a symmetric matrix. **(1)**

Now, $Q = \frac{1}{2}(A - A')$

$$= \frac{1}{2}\left\{\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

Now, $Q' = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$

$= -Q$

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$\because \quad Q' = -Q$

$\therefore$ Q is a skew-symmetric matrix. **(1)**

Now,

$$P + Q = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & -4 & -8 \\ 6 & -4 & -10 \\ -2 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$\therefore$ We have proved that $P + Q = A$ **(1)**

Hence proved.

Q26. Using elementary row transformation, find inverse of following matrices

$$\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$ [Delhi 2012]

Sol:

Given matrix is $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

(1) **Let** $A = IA$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

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Applying $R_2 \to R_2 + R_1, R_3 \to R_3 + 3R_1$, we get

$$\begin{bmatrix} -1 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A \quad \textbf{(1)}$$

Applying $R_1 \to (-1)\,R_1$, we get

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_3$, we get

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & -1 & -2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} A \quad \textbf{(1)}$$

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 + 4R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & -1 \\ -5 & 4 & -3 \end{bmatrix} A \quad \textbf{(1)}$$

Applying $R_2 \to (-1)\,R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 2\,R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ -5 & 4 & -3 \end{bmatrix} A \quad \textbf{(1)}$$

Applying $R_3 \to (-1)\,R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

Hence, $$A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \quad \textbf{(1)}$$