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Class: XII
Subject: Maths
Topic: Sets, Relations and Functions
No. of Questions: 25

Q1. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and f = {(1,4), (2,5)(3,6)} be a function from A to B. state whether f is one-one or not. [All India 2011]

Ans.1 Given that, A = {1. 2, 3} and B = {4, 5, 6, 7}
Now, f : A →B is defined a
f = {(1,4), (2,5), (3,6)}
therefore, f(1) = 4, f(2) = 5 and f(3) = 6.
It is seen that the images of distinct elements of a under f are distinct.
∴ f is one-one as for $x_1, x_2 \in A$.
$\Rightarrow x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Q2. Write fog, if f : R → R and g : R → R are given by f(x) = |x| and g(x) = |5x – 2| [Foreign 2011]

Ans.2 Given, f(x) = |x|, g(x) = |5x – 2|
Now, fog(x) = f[g(x)] = f{|5x – 2|}
=||5x – 2|| = |5x – 2|

Q3. The domain of $f(x) = \sin\left(\log\left(\sqrt{\frac{4 - x^2}{1 - x}}\right)\right)$ is

(A) $(-2,1) \cup (2, \infty)$

(B) $(-2, \infty)$

(C) $(2, \infty)$

(D) None of these.

Ans.3(A)

Given $f(x) = \sin\left(\log\sqrt{\frac{4 - x^2}{1 - x}}\right)$

Domain of sin x is R. But domain of log x is x > 0. Hence domain of given function is values of x such that $\sqrt{\frac{4-x^2}{1-x}} > 0 \Rightarrow \frac{4-x^2}{1-x} > 0$

$$\Rightarrow \frac{(x-2)(x+2)}{x-1} > 0$$

$$\Rightarrow x \in (-2,1) \cup (2,\ \infty)$$

Hence (A) is the correct answer.

Q4. The period of $f(x) = \sin 3® \{x\} + \tan ® [x]$ is :

(A) 0

(B) 1

(C) π

(D) None of these.

Ans.4(b)

tanπ[x] = 0 because π [x] will always be integral multiple of π
⇒*f(x)* = sin 3π {x}
Hence period of *f(x)* is 1
Hence (B) is the correct answer.

Q5. If f(x)=$(3-x^7)^{\frac{1}{7}}\ \forall x \in R$, then f(f(x)) =

(A) x

(B) x^2

(C) x^7

(D) $x - x^7$

Ans.5(a)

$f(x) = (3 - x^7)^{1/7}$

$$f(f(x)) = \left(3 - f(x)^7\right)^{1/7} = \left(3 - ((3-x^7)^{1/7})^7\right)^{1/7} = x$$

Hence (A) is the correct answer.

Q6. If R is a relation from a non-empty set A to a non-empty set B, then

A. $R = A \cup B$

B. $R = A \times B$

C. $R \subset A \times B$.

D. $R = A \cap B$

Ans.6 (b) By definition.

Q7. Let* be a binary operation on N given by a*b = LCM(a, b) for all a, b ∈ N. find 5 * 7

[Delhi 2012; Foreign 2008]

Ans. Given that, a*b = LCM (a, b), ∀ a, b ∈ N

$\therefore$ 5 * 7 = LCM (5, 7) = 35

Q8. *Let* $R\{(x,y) : x^2 + y^2 = 1, x, y \in R\}$ be a relation in R. The relation R is :

(A) reflexive
(B) symmetric
(C) transitive
(D) anti-symmetric

Ans. B

We have $R = \{(x, y) : x^2 + y^2 = 1; x, y \in R\}$.

4Î abd $(4)^2 + (4)^2 = 32 \neq 1$. $\therefore$ $(4,4) \notin R$.

\ R is not reflexive.

Let (x, y) Î R.\ $x^2 + y^2 = 1$

$\Rightarrow y^2 + x^2 = 1 \Rightarrow (y,x) \in R$

\ R is symmetric.

(0, 1), (1, 0) Î R because

$(0)^2 + (1)^2 = 1$ and $(1)^2 + (0)^2 = 1$.

Also $(0)^2 + (0)^2 = 0 \neq 1$. $\therefore$ $(0,0) \notin R$.

\ R is not transitive.

Hence (B) is the correct answer.

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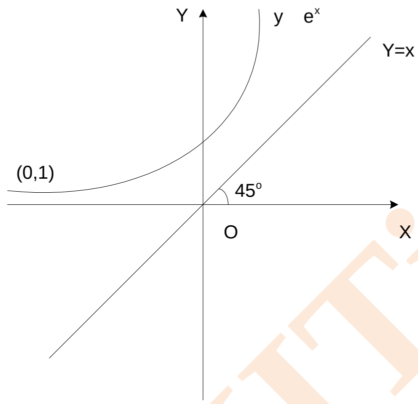
Q9. If $A = \{ (x, y) : y = e^x, x \in R \}$ and $B = \{ (x, y) : y = x, x \in R \}$ then

(A) $A \subseteq B$ (B) $A \subseteq B$

(C) $A \cap B = ¤$ (D) $A \cap B \neq ¤$.

Ans. C

A is the set of all points on the graph of $y = e^x$. B is the set of all points on the line y = x. Since the curves are non intersecting, we have $A \cap B = ¤$.



Hence (C) is the correct answer.

Q10 If $x = \dfrac{\left(7+5\sqrt{2}\right)^{\frac{1}{3}}}{2+2\sqrt{2}}$, then x belongs to:

(A) (2, 3) (B) (0,1)

(C) (– 1, 0) (D) (3,4)

Ans. Cube both the sides, we get

$$x^3 = \frac{7+5\sqrt{2}}{8(7+5\sqrt{2})} = \frac{1}{8}$$

$\Rightarrow x = \frac{1}{2}$. Cleary x lies between 0 and 1.

Hence (B) is the correct answer.

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Q11. Solution set of the inequality $5^{x+2} > \left(\frac{1}{25}\right)^{\frac{1}{x}}$ is:

(A) (– 2,0) (B) (0, ∞)
(C) (– 5,5) (D) (– 2,2)

Ans. We have $5^{x+2} > \left(\frac{1}{25}\right)^{\frac{1}{x}} \left(\begin{matrix}\text{If } a > 1, \text{ than} \\ a^m > a^n \Rightarrow m > n\end{matrix}\right)$

$$\Rightarrow 5^{x+2} > 5^{-\frac{2}{x}}$$

$$\Rightarrow x + 2 > -\frac{2}{x}$$

$$\Rightarrow \left(\frac{x^2 + 2x + 2}{x}\right) > 0$$

$$\Rightarrow \frac{1}{x} > 0 \Rightarrow x \in (0, \infty)$$

Hence (B) is the correct answer.

Q12. If $(\log_5 x)^2 + \log_5 x < 2$, then x belong to:

(A) $\left(\frac{1}{25}, 5\right)$ (B) $\left(\frac{1}{5}, \frac{1}{\sqrt{5}}\right)$
(C) (1, ∞) (D) none of these

Ans. A We have $(\log_5 x)^2 + \log_5 x < 2$
Put $\log_5 x = a$ then $a^2 + a < 2$
$\Rightarrow a^2 + a - 2 < 0 \Rightarrow (a + 2)(a - 1) < 0$
$\Rightarrow -2 < a < 1$ or $-2 < \log_5 x < 1$
$\therefore 5^{-2} < x < 5$ i.e. $1/25 < x < 5$
Hence (A) is the correct answer.

Q13. Let A = {p, q, r, s} and B ={1, 2, 3} . Which of the following relations from A to B is not a function?

(A) R_1 = {(p, 1), (q, 2), (r, 1), (s, 2)
(B) R_2 = {(p, 1), (q, 1), (r, 1), (s, 1)
(C) R_3 = {(p, 1), (q, 2), (r, 2), (s, 2)
(D) R_1 = {(p, 2), (q, 3), (r, 2), (s, 2)

Ans. C R_3 = {(p, 1), (p, 2), (r, 2), (s, 2)}
is not a function as p is having 2 images which is violating definition of function
Hence (C) is the correct answer.

Q14. If the binary operation*, defined on Q, is defined as a*b = 2a + b –ab, for all as b ∈ Q find th value of 3*4.

Sol. Given that, a*b = 2a + b – ab, ∀ a, b ∈ Q.
On putting a = 3 and b = 4, we get
3*4 = 2 . 3 + 4 – 3 . 4
= 6 + 4 -12
= -2

Q15. Let* is the binary operation on N given by a*b = HCF (a, b) where, a, b ∈ N. write the value of 22*4.

Sol. Given binary operation is a*b = HCF of a and b, where a and b ∈ N.
∴ 22*4 = HCF of 22 and 4
= HCF of (2 × 11) and (2 × 2) = 2
∴ 22*4 = 2

Q16. Let *: R × R → R is defined as a*b = 2a + b. find (2*3)*4.

Ans. Given that, *: R × R → R
Such that a*b = 2a + b.
On putting a = 2 and b = 3, we get
(2*3) = 2 (2) + 3 = 4 + 3 = 7
∴ (2*3)*4=7*4 = 2 (7)+4 =14 +4 =18

Q17. If f : R is defined by f(x) = 3x + 2, then define f[f(x)].

Ans. We are given that, f(x) =3x + 2
Now, f[f(x)] = f(3x + 2)
=3(3x+2) + 2
= 9x + 6 +2 = 9x + 8

Q18. What is the range of the function $f(\text{x}) = \frac{|x-1|}{x-1}, x \neq 1$?

[Hots Delhi 2010]

Ans. Given function is, $f(\text{x}) = \frac{|x-1|}{x-1}, x \neq 1$

The above function may be written as

$$f(x) = \begin{cases} \frac{x-1}{x-1}, & if \ x > 1 \\ -\frac{(x-1)}{x-1}, & if \ x < 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 1 & if \ x > 1 \\ -1 & if \ x < 1 \end{cases}$$

∴ Range of f(x) is the set {-1, 1}.

Q19. If f : R → R be defined by $f(x) = (\ -x^3)^{1/3}$, then find fof(x).

Ans. Given function is f : R → R such that $f(x) = (3 - x^3)^{1/3}$.

Now, fof (x) = $f[f(x)] = f[3 - x^3)^{1/3}]$

$=[3 - (3 - x^3)]^{1/3} = (x^3)^{1/3} = x$

∴ fof(x) = x

Q20. If f is an invertible function, defined as $f(x) = \frac{3x-4}{5}$, then write $f^{-1}(x)$.

Ans. Given that, $f(x) = \frac{3x-4}{5}$ and is invertible.

Let $y = \frac{3x-4}{5} \Rightarrow 5y = 3x - 4$

$$\Rightarrow 3x = 5y + 4 \Rightarrow x = \frac{5y+4}{3}$$

$$\therefore f^{-1}(y) = \frac{5y+4}{3} \Rightarrow f^{-1}(x) = \frac{5x+4}{3}$$

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Q21. Prove that the relation R in Set A = {1, 2, 3, 4, 5,} Given by R = {(a , b) : $|a-b|$ is even} is an equivalence relation. [Delhi 2009]

Sol. The given relation is R = {[a , b) : $|a-b|$ is even} defined on set A = {1, 2, 3, 4, 5}.
To show that R is an equivalence relation, we show that it is reflexive, symmetric and transitive.

(i) Reflexive As $|x-x| = 0$ is even, $\forall X \in A$.
$\Rightarrow$ (x, x) ∈ R, $\forall X \in A$
Hence, R is reflexive.

(ii) Symmetric As (x, y) $\in R$
$\Rightarrow$ $|x-y|$ is even
[By the definition of given relation]
$\Rightarrow$ $|y-x|$ is also even
[$\because |a| = |-a|, \forall a \in R$]
$\Rightarrow$ (y, x)$\in R, \forall X, Y \in R$
Hence, R is symmetric.

(iii) Transitive As(X, Y) ∈ R and (Y, Z) $\in R$
$\Rightarrow$ $|x-y|$ is even and $|Y-Z|$ is even
[By using definition of given relation]
Now, $|X-Y|$ is even
$\Rightarrow$ X and Y both are even of odd
Two cases arise
Case I When Y is even.
Now, (X, Y)$\in R$ and (y, z) $\in R$.
$\Rightarrow |x-y|$ *is even*
And $|y-z|$ is even
$\Rightarrow$ x is even and z is even
$\Rightarrow$ $|x-z|$ is even
[$\therefore$ Difference of two even number is also even]
$\Rightarrow$ (X, Z) $\in R$ (1/2)

Case II When y is odd.
Now, (X, Y) $\in R$
And (y, z) $\in R$
$\Rightarrow$ $|x-y|$ is even
And $|y-z|$ is even
$\Rightarrow$ x is odd and z is odd
$\Rightarrow$ $|x-z|$ is even
[$\therefore$ difference of two odd numbers is even]
$\Rightarrow$ (x, z)$\in R$
So, we have shown that
(x, z)$\in R$ *and* $(y, z) \in R$
$\Rightarrow$ (x, z) $\in R, \forall z, y, z \in A$

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Hence, R is transitive. (1/2)
Since, R is reflexive, symmetric and transitive. So, it is and equivalence relation.

Q22. Consider $f : R_+ \longrightarrow [4, \infty)$ given by $f(x) = x^2+4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, *where* R_+ is the set of all non-negative real numbers.
[Hots: All India 2013; Foreign 2011]

Ans. Here, function $f : R_+ \longrightarrow [4, \infty)$ is given as $f(x) = X^2 + 4$.
Let $x, y \in R_+$, Such that $f(x) = f(y)$.
...(i) $\Rightarrow$ $x^2 + 4 = y^2 + 4$
$\Rightarrow$ $x^2 = y^2$ $\Rightarrow$ $x = y$
[$\therefore$ We take only positive sign as $x, y \in R_+$]
Therefore, f is a one-one function. (1)
[$\therefore x = f^{-1}(y)$] For $y \in [4, \infty)$
Let $y = x^2 + 4$ (1)
$\Rightarrow$ $x^2 = y - 4 \geq 0$ [As $y \geq 4$]
$\Rightarrow$ $x = \sqrt{y-4} \geq 0$
[We take positive sigh, as $X \in R_+$]
Therefore, for any $y \in R_+$, there exists
$x = \sqrt{y-4} \in R_+$ Such that
$f(x) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4$
$= y - 4 + 4 = y$
Therefore, f is onto. Thus, F is one-one and onto and therefore, F^{-1} exists.
Let us define $g : [4, \infty) \rightarrow R_+$, by $g(y) = \sqrt{y-4}$.
Now, go $f(x) = g(f(x)) = g(x^2 + 4)$
$= \sqrt{(x^2+4) - 4} = \sqrt{x^2} = x$
And $fog(y) = f[g(y)] = f(\sqrt{y-4})$

$= (\sqrt{y-4})^2 + 4$

$= (y - 4) + 4 = y$

Therefore, $gcf = I_{R+}$ and $fog = I_{[4, \infty)}$

$\Rightarrow f^{-1}(y) = g(y) = \sqrt{y-4}$

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23. Show that the function $f : W \longrightarrow W$ defined by $f(n) = \begin{Bmatrix} n+1, if\ n\ is\ even \\ n-1, if\ n\ is\ odd \end{Bmatrix}$ is a bijective function. [All India 2011C]

Ans. (i) one- one Let

$f(x_1) = f(x_2), \forall x_1, x_2 \in w$

Case I

When x_1 *and* x_2 *both are even.*

Let $f(x_1) = f(x_2), \forall x_1, x_2 \in W$

$\therefore \quad f(x_1) = f(x_2)$

$\Rightarrow x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$

$\therefore f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2, \forall x_1, x_2 \in W$

case II

When both x_1 *and* x_2 *are odd.*

Let $f(x_1) = f(x_2) \forall x_1\ x_2 W$

$\therefore \quad f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 - 1 = x_2 - 1 \Rightarrow x_1 = x_2$

$\therefore \quad f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 = x_2, \quad \forall x_1, x_2 \in W$

Hence, from case I and case II, we observe that

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2, \forall x_1, x_2 \in W$

Hence, $f(x)$ is a one– one function.

(ii) onto

Case I

Let $n = 2m$

$\Rightarrow f(2m) = 2m + 1 = n + 1$

Case II

Let $n = 2m + 1$

$f(2m+1) = (2m+1) - 1 = 2m$

$\therefore f(2m+1) = 2m = n$

Also, $f(1) = 0$

So, $f : W \longrightarrow W$ is an onto function.

Since, f(x) is both one-one and onto functions, so it is a bijective function.

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24. Let Z be the set of all integers and R is the relation on Z defined as R = {(a, b): a, b ∈ Z and a– b is divisible by 5 } Prove that R is an equivalence relation. [HOTS; Delhi 2010]

Ans. The given relation is R = {(a, b) : a, b ∈ Z and a – b is divisible by 5}. We shall prove that R is reflexive, symmetric and transitive. [Hots: Delhi 2010]

(i) Reflexive As for any x ∈ Z, we have

x – x = 0 and 0 is divisible by 5 ⟹ (x - x) is divisible by 5. ⟹ (x, x) ∈ R, ∀ x ∈ Z

Hence, R is reflexive.

(ii) Symmetric As (x, y) ∈ R, Where (x, y) ∈ Z ⟹ (x - y) is divisible by 5. [by definition of R]

⟹ x – y = 5 A for some A ∈ Z

⟹ Y – X = 5 (- A)

⟹ (y - x) is also divisible by 5

⟹ (y, x) ∈ R

Hence, R is symmetric.

(iii) Transitive As (x , y) ∈ R, Where x, y ∈ Z

⟹ (x - y) is divisible by 5.

⟹ x – y = 5 A for some A ∈ Z

Again, for (y, z)∈ R, Where y, z ∈ Z

⟹ (y - z) is divisible by 5.

⟹ y – z = 5 B for some B ∈ Z

$Now, (x-y)+(y-z) = 5A + 5B$

⟹ x- z = 5 (A + B)

⟹ (x - z) is divisible by 5 for some A + B ∈ Z

Hence, R is transitive.

Since, R is reflexive, symmetric and transitive. Therefore, It is an equivalence relation.

25. Show that relation R in the set of real numbers, defined as R = {(a, b): a ≤ b^2} is neither reflexive, nor symmetric nor transitive. [Foreign 2009]

Sol. The given relation is R = {(a, b): a ≤ b^2}

We need to verify that R is reflexive, symmetric and transitive.

(i) Reflexive Since, $\frac{1}{2} \leq \left(\frac{1}{2}\right)^2$ is not ture

$\Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right) \notin R$

Hence, R is not reflexive.

[*R is said to be reflexive, if* (x, x) ∈ R, ∀ x ∈ domain]

(ii) Symmetric Since, $\frac{1}{2} \leq (1)^2$ is true but

$1 \leq \left(\frac{1}{2}\right)^2$ *is not true.*

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$\Rightarrow \left(\frac{1}{2}, 1\right) \in R$ *but* $\left(1, \frac{1}{2}\right) \notin R$

Hence, R is not symmetric

[∴ R is said to be symmetric, if $(x, y) \in R$
And $(y, x) \in R$, $\forall$ x, y ∈ domain]

(iii) Transitive Since, $2 \leq (-3)^2$ and $(-3) \leq (1)^2$ are both true. But $2 \leq (1)^2$ is not true.

∴ $(2, -3) \in R$ and $(-3, 1) \in R \Rightarrow (2, 1) \notin R$

∴ R is not transitive.

[∴R is said to be transitive, if $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$, $\forall$ x, y , z ∈ domain]

Hence, R is none of these i.e., reflexive, Symmetric and transitive.