

Ans 1. $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \quad 0 < \alpha < \frac{\pi}{2}$

$$A + A^T = \sqrt{2} I_2$$

$$\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = \sqrt{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

$$2 \cos \alpha = \sqrt{2}$$

$$\cos \alpha = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\alpha = \frac{\pi}{4}$$

Ans 2. $|3A| = k|A|$
 $|3A| = 27|A|$
 $k = 27$

Ans 3. for unique solution $|A| \neq 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

$$C_2 \rightarrow C_2 - C_1 ; C_3 \rightarrow C_3 - C_1$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & -1 & -3 \\ 3 & -1 & k-3 \end{vmatrix} \neq 0$$

expansion along R_1

$$-(k-3) - 3 \neq 0$$

$$-k + 3 - 3 \neq 0$$

$$k \neq 0$$

Ans 4. $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$

in Cartesian form

$$2x + y - z - 5 = 0$$

$$2x + y - z = 5$$

$$\frac{2x}{5} + \frac{y}{5} - \frac{z}{5} = 1$$

$$\frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$$

Intercept cutt of on the axes $\left(\frac{5}{2}, 5, -5\right)$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$a = \frac{5}{2} \quad b = 5 \quad c = -5$$

$$a + b + c = 5/2$$

Ans 5. $(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i} - \lambda \hat{j} + \mu \hat{k}) = \vec{0}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 9 \\ 3 & -\lambda & \mu \end{vmatrix} = \vec{0}$$

$$\hat{i}(3\mu + 9\lambda) - \hat{j}(\mu - 27) + \hat{k}(-\lambda - 9) = \vec{0}$$

$$3\mu + 9\lambda = 0 \text{ ---(1)} \quad 27 - \mu = 0 \text{ ---(2)}$$

$$\text{by eq}^n \text{(2) \& (3)} \quad \mu = 27$$

$$-\lambda - 9 = 0 \text{ ---(3)}$$

$$\lambda = -9$$

λ, μ value satisfy the eqⁿ (1)

So $\mu = 27, \lambda = -9$

Ans 6. $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$
 $\vec{a} + \vec{b} = (4\hat{i} - \hat{j} + \hat{k}) + (2\hat{i} - 2\hat{j} + \hat{k})$
 $= 6\hat{i} - 3\hat{j} + 2\hat{k}$

unit vector parallel to $(\vec{a} + \vec{b}) = \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|}$

$$= \frac{6\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{36 + 9 + 4}}$$

$$= \frac{6\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{49}}$$

$$= \frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}$$

Ans 7. $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$

$$\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1}3x - \tan^{-1}x$$

$$\tan^{-1}\left(\frac{x-1+x+1}{1-(x-1)(x+1)}\right) = \tan^{-1}\left(\frac{3x-x}{1+3x^2}\right)$$

$$\frac{2x}{1-(x^2-1)} = \frac{2x}{1+3x^2}$$

$$x(1+3x^2) = x(2-x^2)$$

$$x(1+3x^2-2+x^2) = 0$$

$$x(4x^2 - 1) = 0$$

$$x = 0 ; \quad 4x^2 - 1 = 0$$

$$x^2 = \frac{1}{4}$$

$$x = \pm \frac{1}{2}$$

$$x = 0, \pm \frac{1}{2}$$

OR

L.H.S.

$$\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) - \tan^{-1}\left(\frac{4x}{1-4x^2}\right)$$

$$\tan^{-1}\left(\frac{\frac{6x-8x^3}{1-12x^2} - \frac{4x}{1-4x^2}}{1 + \frac{6x-8x^3}{1-12x^2} \times \frac{4x}{1-4x^2}}\right)$$

$$\tan^{-1}\left(\frac{(6x-8x^3)(1-4x^2) - 4x(1-12x^2)}{(1-12x^2)(1-4x^2) + (6x-8x^3)4x}\right)$$

$$\tan^{-1}\left(\frac{6x - 24x^3 - 8x^3 + 32x^5 - 4x + 48x^3}{1 - 4x^2 - 12x^2 + 48x^4 + 24x^2 - 32x^4}\right)$$

$$\tan^{-1}\left(\frac{32x^5 + 16x^3 + 2x}{16x^4 + 8x^2 + 1}\right)$$

$$\tan^{-1}\left\{2x \frac{(16x^4 + 8x^2 + 1)}{16x^4 + 8x^2 + 1}\right\}$$

$$\tan^{-1} 2x$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Ans 8. $[10 \ 3] \begin{bmatrix} x \\ y \end{bmatrix} = [145]$

$$[3 \ 10] \begin{bmatrix} x \\ y \end{bmatrix} = [180]$$

$$10x + 3y = 145$$

$$3x + 10y = 180$$

by solving the equations we get

$$x = 10, y = 15$$

but Typist charge 2 Rs. Per Page from a Poor student shyam

amount taken by shyam = $2 \times 5 = 10$ Rs.

but from another person, he take for

5 Pages = 15 × 5
= 75 Rs.
amount differ by = 75 – 10
= 65 Rs. Less.
sympathy are reflect this problem

Ans 9. at x = 0 function is continuous,

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = f(0)$$

R.H.L.

$$\begin{aligned} &= \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{\sqrt{1+bh}-1}{h} \times \frac{\sqrt{1+bh}+1}{\sqrt{1+bh}+1} \\ &= \lim_{x \rightarrow 0} \frac{1+bh-1}{h(\sqrt{1+bh}+1)} \\ &= \lim_{x \rightarrow 0} \frac{bh}{h(\sqrt{1+bh}+1)} \\ &= \lim_{x \rightarrow 0} \frac{b}{\sqrt{1+bh}+1} \\ &= \frac{b}{2} \end{aligned}$$

$$f(0) = 2$$

L.H.L.

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} f(0-h) \\ &= \lim_{h \rightarrow 0} \frac{\sin(a+1)(0-h) + \sin(0-h)}{0-h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(-(a+1)h) + \sin(-h)}{-h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(a+1)h}{h} + \frac{2\sin h}{h} \\ &= (a+1) + 2 \end{aligned}$$

$$\begin{aligned} a+3 &= 2 & \frac{b}{2} &= 2 \\ a &= -1 & b &= 4 \end{aligned}$$

Ans 10. x cos (a+y) = cos y

$$x = \frac{\cos y}{\cos(a+y)}$$

$$\frac{dx}{dy} = \frac{\cos(a+y) \times (-\sin y) - \cos y (-\sin(a+y))}{\cos^2(a+y)}$$

$$\frac{dx}{dy} = \frac{-\sin y \cos(a+y) + \cos y \sin(a+y)}{\cos^2(a+y)}$$

$$\therefore \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\frac{dx}{dy} = \frac{\sin a}{\cos^2(a+y)} \quad \text{So } \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

$$\sin a \frac{d^2y}{dx^2} = 2\cos(a+y) \times (-\sin(a+y)) \frac{dy}{dx}$$

$$\sin a \frac{d^2y}{dx^2} + 2\cos(a+y) \times (\sin(a+y)) \frac{dy}{dx} = 0$$

$$\sin a \frac{d^2y}{dx^2} + \sin 2(a+y) \frac{dy}{dx} = 0$$

OR

$$y = \sin^{-1} \left(\frac{6x - 4\sqrt{1-4x^2}}{5} \right)$$

$$y = \sin^{-1} \left(\frac{6x}{5} - \frac{4}{5}\sqrt{1-4x^2} \right)$$

$$y = \sin^{-1} \left(2x \times \frac{3}{5} - \frac{4}{5}\sqrt{1-(2x)^2} \right)$$

$$\sin^{-1} p - \sin^{-1} q = \sin^{-1} \left(p\sqrt{1-q^2} - q\sqrt{1-p^2} \right)$$

$$p = 2x \quad q = \frac{4}{5}$$

$$y = \sin^{-1} 2x - \sin^{-1} \frac{4}{5}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x)^2}} \times 2 \cdot 1 - 0$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$

Ans 11. $y = x^3 + 2x - 4$

$$\left(\frac{dy}{dx} \right)_{C_1} = 3x^2 + 2 \quad \dots\dots\dots(1)$$

eqⁿ of tangent : $y - y_1 = m(x - x_1) \dots\dots(2)$

is \perp to $x + 14y + 3 = 0$

$$m = \left(\frac{dy}{dx} \right)_{C_1} = 3x^2 + 2$$

$$14y = -x - 3$$

$$y = \frac{-x-3}{14}$$

$$m \times \frac{-1}{14} = -1$$

$$m = 14$$

$$3x^2 + 12 = 14$$

$$3x^2 = 12$$

$$x^2 = 4 \quad x = \pm 2$$

if $x = 2$ $x = -2$

$$y = 2^3 + 2 \cdot 2 - 4 \quad y = -8 - 4 - 4$$

$$y = 8 + 4 - 4 \quad y = -16$$

$$y = 8$$

$$P_1(2, 8) \quad P_2(-2, -16)$$

eqⁿ of tangent at $P_1(2, 8)$

$$y - 8 = 14(x - 2)$$

$$y - 8 = 14x - 28$$

$$14x - y = 20$$

eqⁿ of tangent at $P_2(-2, -16)$

$$y + 16 = 14(x + 2)$$

$$14x - y = 16 - 28$$

$$14x - y = -12$$

Ans 12.

$$\begin{aligned} I &= \int \frac{(2x-5)e^{2x}}{(2x-3)^3} dx \\ &= \int e^{2x-3} \cdot e^3 \left[\frac{(2x-3)-2}{(2x-3)^3} \right] dx \\ &= \int e^{2x-3} \cdot e^3 \left[\frac{1}{(2x-3)^2} - \frac{2}{(2x-3)^3} \right] dx \end{aligned}$$

Let $f(x) = \frac{1}{(2x-3)^2}$ whose, $2x - 3 = t$

$$\Rightarrow 2dx = dt$$

$$\begin{aligned} I &= e^3 \int e^t \left[\frac{t-2}{t^3} \right] \frac{dt}{2} \\ &= \int e^3 e^t \left[f(t) + f'(t) \right] \frac{dt}{2} \\ &= \frac{e^3}{2} \cdot e^t \cdot f(t) + C \\ &= \frac{e^3}{2} e^{2x-3} \cdot \frac{1}{(2x-3)^2} + C \end{aligned}$$

$$= \frac{e^{2x}}{2(2x-3)^2} + C$$

OR

$$I = \int \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} dx$$

$$\begin{aligned} \text{Let } \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} &= \frac{A}{x + 2} + \frac{Bx + C}{x^2 + 1} \\ &= \frac{A(x^2 + 1) + (Bx + C)(x + 2)}{(x + 2)(x^2 + 1)} \end{aligned}$$

$$= \frac{(A + B)x^2 + (2B + C)x + (2C + A)}{(x + 2)(x^2 + 1)}$$

$$\therefore A + B = 1 \quad \dots\dots(1)$$

$$2B + C = 1 \quad \dots\dots(2)$$

$$2C + A = 1 \quad \dots\dots(3)$$

$$\Rightarrow A = 1 - 2C \quad \dots\dots(4)$$

From (2)

$$B = \frac{1 - C}{2} \quad \dots\dots(5)$$

Put (4) and (5) in (1) we get.

$$1 - 2C + \frac{1 - C}{2} = 1$$

$$\Rightarrow 2 - 4C + 1 - C = 2$$

$$\Rightarrow 5C = 1$$

$$\Rightarrow C = \frac{1}{5}$$

$$\therefore A = 1 - 2C = 1 - \frac{2}{5} = \frac{3}{5}$$

$$B = \frac{1 - C}{2} = \frac{1 - \frac{1}{5}}{2} = \frac{\frac{4}{5}}{2} = \frac{2}{5}$$

$$\therefore \frac{x^2 + x + 1}{(x + 2)(x^2 + 1)} = \frac{3/5}{x + 2} + \frac{\frac{2}{5}x + \frac{1}{5}}{x^2 + 1}$$

$$\therefore I = \int \frac{x^2 + x + 1}{(x + 2)(x^2 + 1)} dx$$

$$\Rightarrow I = \int \frac{3/5}{x + 2} + \frac{\frac{2}{5}x + \frac{1}{5}}{x^2 + 1} dx$$

$$\Rightarrow I = \frac{3}{5} \log(x+2) + \frac{1}{5} \left[\int \frac{2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx \right]$$

$$\Rightarrow I = \frac{3}{5} \log(x+2) + \frac{1}{5} \log(x^2+1) + \tan^{-1} x + C$$

Ans 13. $I = \int_{-2}^2 \frac{x^2}{1+5^x} dx \dots\dots\dots(1)$

Using Property $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$$I = \int_{-2}^2 \frac{(-x)^2}{1+5^{-x}} dx$$

$$\Rightarrow I = \int_{-2}^2 \frac{5^x x^2}{1+5^x} dx \dots\dots\dots(2)$$

Add (1) and (2), we get

$$2I = \int_{-2}^2 \frac{1+5^x}{1+5^x} \cdot x^2 dx$$

$$= \int_{-2}^2 x^2 dx$$

$$= \left[\frac{x^3}{3} \right]_{-2}^2$$

$$= \frac{1}{3} [2^3 - (-2)^3]$$

$$= \frac{1}{3} [8 - (-8)]$$

$$2I = \frac{16}{3} \quad \therefore I = \frac{8}{3}$$

Ans 14. $I = \int (x+3)\sqrt{3-4x-x^2} dx$

Let $x+3 = \lambda \frac{d}{dx}(3-4x-x^2) + \mu$

$$\Rightarrow x+3 = \lambda(-2x-4) + \mu$$

$$\Rightarrow x+3 = -2\lambda x - 4\lambda + \mu$$

$$\therefore -2\lambda = 1$$

$$\Rightarrow \lambda = -\frac{1}{2}$$

$$-4\lambda + \mu = 3$$

$$\Rightarrow -4 \left(-\frac{1}{2} \right) + \mu = 3$$

$$\Rightarrow 2 + \mu = 3$$

$$\Rightarrow \mu = 1$$

$$\begin{aligned} \therefore I &= \int \left[-\frac{1}{2} \frac{d}{dx} (3 - 4x - x^2) + 1 \right] \sqrt{3 - 4x - x^2} dx \\ &= -\frac{1}{2} \int \frac{d}{dx} (3 - 4x - x^2) \sqrt{3 - 4x - x^2} dx + \int \sqrt{3 - 4x - x^2} dx \\ &= -\frac{1}{2} \frac{(3 - 4x - x^2)^{3/2}}{\frac{3}{2}} + \int \sqrt{3 - x^2 - 4x - 4 + 4} dx \\ &= -\frac{(3 - 4x - x^2)^{3/2}}{3} + \int \sqrt{7 - (x+2)^2} dx \\ &= -\frac{(3 - 4x - x^2)^{3/2}}{3} + \frac{x+2}{2} \sqrt{7 - (x+2)^2} + \frac{7}{2} \sin^{-1} \left(\frac{x+2}{\sqrt{7}} \right) + C \end{aligned}$$

Ans 15. $\frac{dy}{dx} = -\frac{x + y \cos x}{1 + \sin x}$

$$\Rightarrow \frac{dy}{dx} + \frac{\cos x}{1 + \sin x} y = -\frac{x}{1 + \sin x} \dots(1)$$

This is a linear differential equation with

$$P = \frac{\cos x}{1 + \sin x}, Q = \frac{-x}{1 + \sin x}$$

$$\begin{aligned} \therefore \text{I.F.} &= e^{\int \frac{\cos x}{1 + \sin x} dx} \\ &= e^{\log(1 + \sin x)} \\ &= (1 + \sin x) \end{aligned}$$

Multiplying both sides of (i) by I.F. = $1 + \sin x$, we get

$$(1 + \sin x) \frac{dy}{dx} + y \cos x = -x$$

Integrating with respect to x , we get

$$\begin{aligned} y(1 + \sin x) &= \int -x dx + C \\ \Rightarrow y &= \frac{2C - x^2}{2(1 + \sin x)} \dots\dots(2) \end{aligned}$$

Given that $y = 1$ when $x = 0$

$$\therefore 1 = \frac{2C}{2(1+0)}$$

$$\Rightarrow C = 1 \quad \dots\dots\dots (3)$$

\therefore Put (3) in (2), we get

$$y = \frac{2 - x^2}{2(1 + \sin x)}$$

Ans 16. $2ye^{x/y} dx + (y - 2x e^{x/y}) dy = 0$

$$\Rightarrow \frac{dx}{dy} = \frac{2x e^{x/y} - y}{2y e^{x/y}}$$

The given D.E. is a homogeneous differential equation.

$$\therefore \text{Put } x = vy$$

$$\frac{dx}{dy} = v + y \frac{dv}{dy}$$

$$v + y \frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$$

$$\Rightarrow y \frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v$$

$$\Rightarrow y \frac{dv}{dy} = -\frac{1}{2e^v}$$

$$\Rightarrow 2ye^v dv = -dy$$

$$\Rightarrow 2e^v dv = -\frac{1}{y} dy$$

$$\Rightarrow 2 \int e^v dv = - \int \frac{1}{y} dy$$

$$\Rightarrow 2e^v = -\log |y| + \log C$$

$$\Rightarrow 2e^v = \log \left| \frac{C}{y} \right|$$

$$\Rightarrow 2e^{x/y} = \log \left| \frac{C}{y} \right|$$

Given that at $x = 0, y = 1$.

$$\therefore 2 \cdot e^0 = \log \left| \frac{C}{1} \right|$$

$$\Rightarrow C = e^2$$

$$\therefore 2e^{x/y} = \log \frac{e^2}{y}$$

$$\Rightarrow \log y = -2e^{x/y} + 2$$

$$\Rightarrow y = e^{2-2e^{x/y}}$$

Ans 17. A (4, 5, 1) B (0, -1, -1) C (3, 9, 4) D (-4, 4, 4)

$$\therefore \vec{AB} = (-4\hat{i} - 6\hat{j} - 2\hat{k})$$

$$\vec{AC} = (-\hat{i} + 4\hat{j} + 3\hat{k})$$

$$\vec{AD} = (-8\hat{i} + \hat{j} + 3\hat{k})$$

$$\therefore \begin{bmatrix} \vec{AB} & \vec{AC} & \vec{AD} \end{bmatrix} = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

$$= -4 \begin{vmatrix} 4 & 3 \\ -1 & 3 \end{vmatrix} + 6 \begin{vmatrix} -1 & 3 \\ -8 & 3 \end{vmatrix} - 2 \begin{vmatrix} -1 & 4 \\ -8 & -1 \end{vmatrix}$$

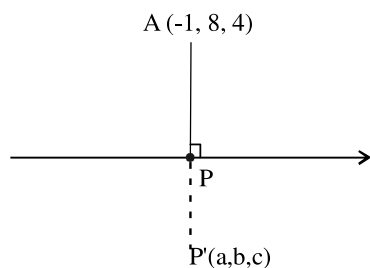
$$= -4(12 + 3) + 6(-3 + 24) - 2(1 + 32)$$

$$= -60 + 126 - 66$$

$$= 0$$

\therefore Four points A, B, C, D are coplanar.

Ans 18.



$$\text{eq}^n \text{ of Line BC} \Rightarrow \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x - 0}{2} = \frac{y + 1}{-2} = \frac{z - 3}{-4} = \lambda$$

general coordinates of P

$$P(2\lambda, -2\lambda - 1, -4\lambda + 3)$$

$$\text{D.R of AP } (2\lambda + 1, -2\lambda - 9, -4\lambda - 1)$$

AP \perp BC

$$2(2\lambda + 1) - 2(-2\lambda - 9) - 4(-4\lambda - 1) = 0$$

$$4\lambda + 2 + 4\lambda + 18 + 16\lambda + 4 = 0$$

$$24 + 24\lambda = 0$$

$$\lambda = -1$$

P (-2, 1, 7)

Coordinates of foot of \perp (-2, 1, 7)

Coordinates of image of A is P' (a, b, c) is

$$\frac{a-1}{2} = -2, a = -3$$

$$\frac{b+8}{2} = 1, b = -6$$

$$\frac{c+4}{2} = 7, c = 10$$

P' (-3, -6, 10)

Ans 19. bag A = 4 white, 2 black

bag y = 3 white, 3 black

E_1 = first bag selected

E_2 = second bag selected

$$P(E_1) = \frac{1}{2} \quad P(E_2) = \frac{1}{2}$$

$$P(A / E_1) = \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} = \frac{16}{30}$$

$$P(A / E_2) = \frac{3}{6} \times \frac{3}{5} + \frac{3}{6} \times \frac{3}{5} = \frac{18}{30}$$

$$P(E_2 / A) = \frac{P(E_2) P(A / E_2)}{P(E_1) \cdot P(A / E_1) + P(E_2) P(A / E_2)}$$

$$P(E_2 / A) = \frac{\frac{1}{2} \times \frac{18}{30}}{\frac{1}{2} \times \frac{16}{30} + \frac{1}{2} \times \frac{18}{30}}$$

$$= \frac{18}{16+18} = \frac{18}{34} = \frac{9}{17}$$

OR

$$P(\text{win}) = \frac{3}{36} = \frac{1}{12}$$

$$P(\text{lose}) = \frac{11}{12}$$

$$P(\text{A wins}) = \frac{1}{12} + \frac{11}{12} \times \frac{11}{12} \times \frac{1}{12} + \left(\frac{11}{12}\right)^4 \times \frac{1}{12} + \dots$$

$$a = \frac{1}{12} \quad r = \frac{121}{144}$$

by using formula of infinite G.P.

$$P(\text{A wins}) = \frac{\frac{1}{12}}{1 - \frac{1}{144}} = \frac{12}{23}$$

Ans 20. X = larger of three numbers
X = 3, 4, 5, 6

$$P(x = 3) = 6 \times \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$$

$$P(x = 4) = 18 \times \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} = \frac{3}{20}$$

$$P(x = 5) = 36 \times \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} = \frac{6}{20}$$

$$P(x = 6) = 60 \times \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} = \frac{10}{20}$$

Xi	Pi	PiXi	PiXi ²
3	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{9}{20}$
4	$\frac{3}{20}$	$\frac{12}{20}$	$\frac{48}{20}$
5	$\frac{6}{20}$	$\frac{30}{20}$	$\frac{150}{20}$
6	$\frac{10}{20}$	$\frac{60}{20}$	$\frac{360}{20}$

$$\text{Mean} = \sum P_i X_i = \left(\frac{105}{20}\right) = 5.25$$

$$\sum P_i X_i^2 = \frac{567}{20}$$

$$\text{Var}(X) = \sum P_i X_i^2 - (\sum P_i X_i)^2$$

$$= \frac{567}{20} - \left(\frac{105}{20}\right)^2 = 0.787$$

Ans 21. (a, b) * (c, d) = (a + c, b + d)

(i) Commutative

$$(a, b) * (c, d) = (a+c, b+d)$$

$$(c, d) * (a, b) = (c+a, d+b)$$

for all, a, b, c, d ∈ R

* is commulative on A

(ii) Associative : _____
 $(a, b), (c, d), (e, f) \in A$
 $\{ (a, b) * (c, d) \} * (e, f)$
 $= (a + c, b + d) * (e, f)$
 $= ((a + c) + e, (b + d) + f)$
 $= (a + (c + e), b + (d + f))$
 $= (a * b) * (c + d, d + f)$
 $= (a * b) \{ (c, a) * (e, f) \}$
 is associative on A

Let (x, y) be the identity element in A.

then,

$$\begin{aligned} (a, b) * (x, y) &= (a, b) \quad \text{for all } (a, b) \in A \\ (a + x, b + y) &= (a, b) \quad \text{for all } (a, b) \in A \\ a + x = a, b + y &= b \quad \text{for all } (a, b) \in A \\ x = 0, y &= 0 \\ (0, 0) &\in A \end{aligned}$$

$(0, 0)$ is the identity element in A.

Let (a, b) be an invertible element of A.

$$\begin{aligned} (a, b) * (c, d) &= (0, 0) = (c, d) * (a, b) \\ (a + c, b + d) &= (0, 0) = (c + a, d + b) \\ a + c = 0 \quad b + d &= 0 \\ a = -c \quad b = -d \\ c = -a \quad d = -b \end{aligned}$$

(a, b) is an invertible element of A, in such a case the inverse of (a, b) is $(-a, -b)$

Ans 22. $y = \frac{4 \sin \theta}{2 + \cos \theta} - \theta$

$$\begin{aligned} \frac{dy}{d\theta} &= \frac{(2 + \cos \theta)(4 \cos \theta) + 4 \sin^2 \theta}{(2 + \cos \theta)^2} - 1 \\ &= \frac{8 \cos \theta + 4 \cos^2 \theta + 4 \sin^2 \theta}{(2 + \cos \theta)^2} - 1 \\ &= \frac{8 \cos \theta + 4}{(2 + \cos \theta)^2} - 1 \\ &= \frac{4 \cos \theta - \cos^2 \theta}{(2 + \cos \theta)^2} \end{aligned}$$

$$\frac{dy}{d\theta} = \frac{\cos \theta (4 - \cos \theta)}{(2 + \cos \theta)^2}$$

for increasing $\frac{dy}{d\theta} > 0$

$$\theta \in \left(0, \frac{\pi}{2} \right)$$

$$0 \leq \cos \leq 1$$

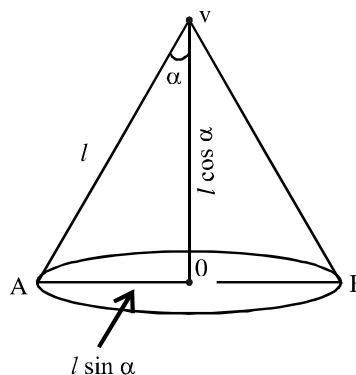
$(2 + \cos \theta)^2$ always greater than 0

So, $\frac{dy}{d\theta}$ is increasing on $\left[0, \frac{\pi}{2}\right]$

OR

Volume of cone

$$\begin{aligned} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi (\ell \sin \alpha)^2 (\ell \cos \alpha) \\ &= \frac{1}{3} \pi \ell^3 \sin^2 \alpha \cos \alpha \\ \frac{dv}{d\alpha} &= \frac{\pi \ell^3}{3} [-\sin^3 \alpha + 2 \sin \alpha \cos \alpha \times \cos \alpha] \\ &= \frac{\pi \ell^3 \sin \alpha}{3} (-\sin^2 \alpha + 2 \cos^2 \alpha) \end{aligned}$$



for maximum or minimum

$$\begin{aligned} \frac{dv}{d\alpha} &= 0 \\ \frac{\pi \ell^3 \sin \alpha}{3} (-\sin^2 \alpha + 2 \cos^2 \alpha) &= 0 \end{aligned}$$

$$\sin \alpha \neq 0$$

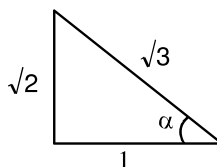
$$2 \cos^2 \alpha = \sin^2 \alpha$$

$$\tan^2 \alpha = 2$$

$$\tan \alpha = \sqrt{2}$$

$$\cos \alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = \cos^{-1} \frac{1}{\sqrt{3}}$$



again diff. w.r.t. α , we get

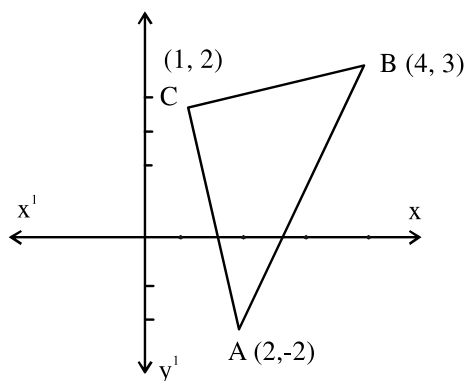
$$\frac{d^2v}{d\alpha^2} = \frac{1}{3} \pi \ell^3 \cos^2 \alpha (2 - 7 \tan^2 \alpha)$$

$$\text{at } \cos \alpha = \frac{1}{\sqrt{3}}$$

$$\frac{d^2v}{d\alpha^2} < 0$$

V is maximum when $\cos \alpha = \frac{1}{\sqrt{3}}$ or $\alpha = \cos^{-1} \frac{1}{\sqrt{3}}$

Ans 23.



Equation of line AB :-

$$y + 2 = \frac{3+2}{2}(x-2)$$

$$\Rightarrow 2y = 5x - 14$$

Equation of line BC :-

$$y - 3 = \frac{1}{3}(x-4)$$

$$\Rightarrow 3y = x + 5$$

Equation of line CA :-

$$(y-2) = -4(x-1)$$

$$4x + y = 6$$

\therefore ar (ΔABC)

$$= \int_{-2}^3 \frac{2y+14}{5} dy - \int_{-2}^3 3y-5 dy - \int_{-2}^2 \frac{6-y}{4} dy$$

$$= \frac{75}{5} - \frac{5}{2} - \frac{24}{4}$$

$$= \frac{300-120-50}{20} = \frac{130}{20}$$

$$= \frac{13}{2} \text{ sq.units}$$

Ans 24.

$$\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) - 4 = 0$$

$$\vec{r} \cdot (-2\hat{i} + \hat{j} + \hat{k}) + 5 = 0$$

$$\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda \{ \vec{r} \cdot (-2\hat{i} + \hat{j} + \hat{k}) \} - 4 + 5\lambda = 0$$

$$\Rightarrow \vec{r} \cdot [(1-2\lambda)\hat{i} + (-2+\lambda)\hat{j} + (3+\lambda)\hat{k}] - 4 + 5\lambda = 0$$

$$\Rightarrow (1-2\lambda)x + (-2+\lambda)y + (3+\lambda)z = -5\lambda + 4$$

$$\Rightarrow \frac{x}{\frac{-5\lambda+4}{1-2\lambda}} + \frac{y}{\frac{-5\lambda+4}{-2+\lambda}} + \frac{z}{\frac{-5\lambda+4}{3+\lambda}} = 1$$

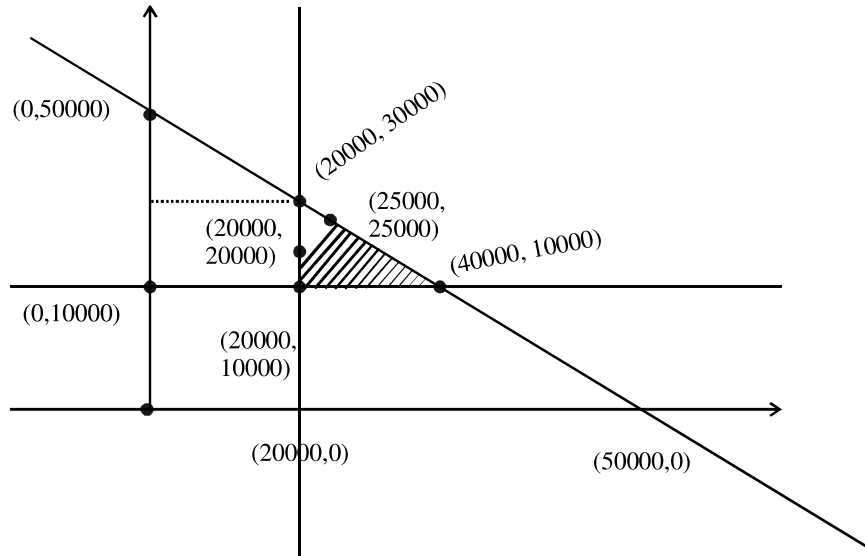
$$\therefore \frac{-5\lambda+4}{1-2\lambda} = \frac{-5\lambda+4}{-2+\lambda}$$

$$\begin{aligned} \Rightarrow 1 - 2\lambda &= -2 + \lambda \\ \Rightarrow -3\lambda &= -3 \\ \Rightarrow \lambda &= 1 \\ \therefore \text{Equation of the required plane} \\ -x - y + 4z &= -1 \\ x + y - 4z - 1 &= 0 \end{aligned}$$

Vector eqⁿ of the required Plane

$$\Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} - 4\hat{k}) - 1 = 0$$

Ans 25. $Z = 0.1x + 0.09y$
 $x + y \leq 50000$
 $x \geq 20000$
 $y \geq 10000$
 $y \leq x$



	$z = 0.1x + 0.09y$
$P_1 (20000, 10000)$	2900
$P_2 (40000, 10000)$	4900
$P_3 (25000, 25000)$	4750
$P_4 (20000, 20000)$	3800

When A invest 40000 & B invest 10000

Ans 26.
$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

L.H.S.

Multiplying R_1, R_2 and R_3 by z, x, y respectively

$$= \frac{1}{xyz} \begin{vmatrix} z(x+y)^2 & z^2x & z^2y \\ x^2z & x(z+y)^2 & x^2y \\ y^2z & xy^2 & y(z+x)^2 \end{vmatrix}$$

take common z, x, y from $C_1, C_2,$ & C_3

$$= \frac{xyz}{xyz} \begin{vmatrix} (x+y)^2 & z^2 & z^2 \\ x^2 & (z+y)^2 & x^2 \\ y^2 & y^2 & (z+x)^2 \end{vmatrix}$$

$C_1 \rightarrow C_1 - C_3$ and $C_2 \rightarrow C_2 - C_3$

$$= \frac{c+4}{2} = 7$$

taking common $x+y+z$ from C_1 & C_2

$$= (x+y+z)^2 \begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ y-z-x & y-z-x & (z+x)^2 \end{vmatrix}$$

$R_3 \rightarrow R_3 - (R_1 + R_2)$

$$= (x+y+z)^2 \begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ -2x & -2z & 2xz \end{vmatrix}$$

$C_1 \rightarrow zC_1, C_2 \rightarrow xC_3$

$$= \frac{(x+y+z)^2}{xz} \begin{vmatrix} z(x+y-z) & 0 & z^2 \\ 0 & x(z+y-x) & x^2 \\ -2xz & -2xz & 2xz \end{vmatrix}$$

$C_1 \rightarrow C_1 + C_3$ $C_2 \rightarrow C_2 + C_3$

$$= \frac{(x+y+z)^2}{xz} \begin{vmatrix} z(x+y) & z^2 & z^2 \\ x^2 & x(z+y) & x^2 \\ 0 & 0 & 2xz \end{vmatrix}$$

taking z and x common from R_1 & R_2

$$= \frac{(x+y+z)^2}{xz} \times zx \begin{vmatrix} x+y & z & z \\ x & z+y & x \\ 0 & 0 & 2xz \end{vmatrix}$$

expansion along R_3

$$\begin{aligned} &= (x+y+z)^2 \times 2xz ((x+y)(z+y) - xz) \\ &= (x+y+z)^2 \times 2xz (xz + xy + yz + y^2 - xz) \\ &= (x+y+z)^2 \times 2xz (xy + yz + y^2) \\ &= 2xyz (x + y + z)^3 \end{aligned}$$

OR

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A^2 = AA = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$\therefore A^3 - 6A^2 + 7A + KI_3 = 0$$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow k = 2$$