

Class: VII
Subject: Math's
Topic: Properties of triangle
No. of Questions: 20

Q1. The sum of the lengths of any two sides of a triangle is always _____ (greater/lesser) than the length of the third side.

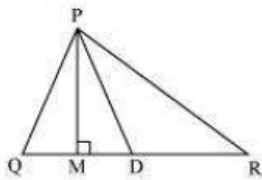
Sol. Greater

2. In $\triangle PQR$, D is the mid-point of \overline{QR} .

\overline{PM} is _____ .

PD is _____ .

Is $QM = MR$?



Sol.

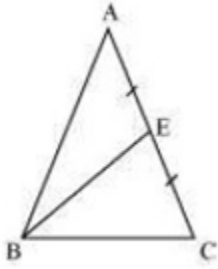
- (i) Altitude
- (ii) Median
- (iii) No.

Q3. Draw rough sketches for the following:

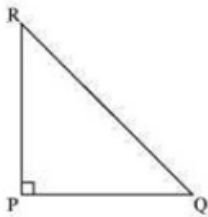
- (i) In $\triangle ABC$, BE is a median.
- (ii) In $\triangle PQR$, PQ and PR are altitudes of the triangle.
- (iii) In $\triangle XYZ$, YL is an altitude in the exterior of the triangle.

Sol.

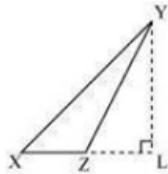
- (i)



(ii)



(iii)



Here, it can be observed that for ΔXYZ , YL is an altitude drawn exterior to side XZ which is extended up to point L .

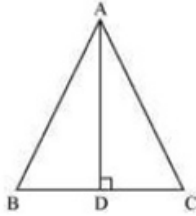
Q4. Q. Define parallel lines?

Sol. [Two lines are said to be in parallel if they don't intersect each other and maintain a equal distance through out.



Q5. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

Sol.



Draw a line segment AD perpendicular to BC. It is an altitude for this triangle. It can be observed that the length of BD and DC is also same. Therefore, AD is also a median of this triangle.

Q6. The difference between the measures two complementary angles is 18. What are the measures of the two angles

Sol. Let measure of angle be x° Thus, the complementary angle of this angle is $(90 - x)^\circ$

Q7. Each of the two angles of triangle is $\frac{3}{4}$ th of the third angle. Find the angles.

Sol. Let the third angle be x . Then other angles will be $\frac{3x}{4}$

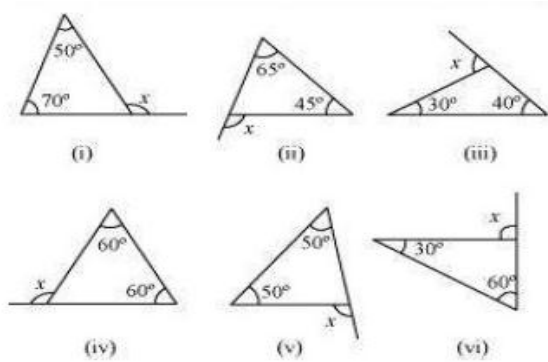
By angle sum property we have, $\frac{3x}{4} + \frac{3x}{4} + x = 180$

$$5x/2 = 180 \Rightarrow 5x = 360 \Rightarrow x = 72$$

Other angle $3 \times \frac{72}{4} = 54$

So the angles of the triangle are 72, 54 and 54

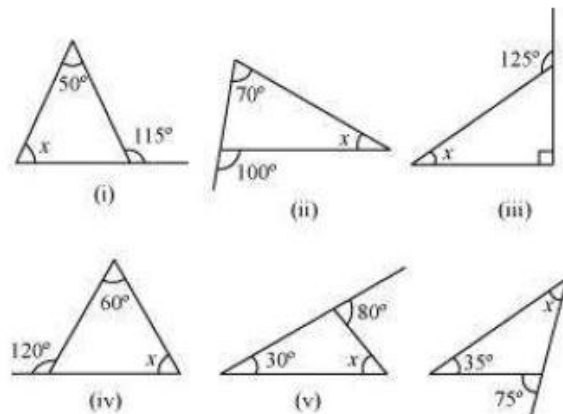
Q8. Find the value of the unknown exterior angle x in the following diagrams:



Sol.

- (i) $X = 50^\circ + 70^\circ$ (Exterior angle theorem)
 $X = 120^\circ$
- (ii) $X = 65^\circ + 45^\circ$ (Exterior angle theorem)
 $= 110^\circ$
- (iii) $X = 40^\circ + 30^\circ$ (Exterior angle theorem)
 $= 70^\circ$
- (iv) $X = 60^\circ + 60^\circ$ (Exterior angle theorem)
 $= 120^\circ$
- (v) $X = 50^\circ + 50^\circ$ (Exterior angle theorem)
 $= 100^\circ$
- (vi) $X = 30^\circ + 60^\circ$ (Exterior angle theorem)

Q9. Find the value of the unknown interior angle x in the following figures:



Sol.

- (i) $x + 50^\circ = 115^\circ$ (Exterior angle theorem)
 $x = 115^\circ - 50^\circ = 65^\circ$
- (ii) $70^\circ + x = 100^\circ$ (Exterior angle theorem)
 $x = 100^\circ - 70^\circ = 30^\circ$
- (iii) $x + 90^\circ = 125^\circ$ (Exterior angle theorem)
 $x = 125^\circ - 90^\circ = 35^\circ$
- (iv) $x + 60^\circ = 120^\circ$ (Exterior angle theorem)
 $x = 120^\circ - 60^\circ = 60^\circ$
- (v) $x + 30^\circ = 80^\circ$ (Exterior angle theorem)
 $x = 80^\circ - 30^\circ = 50^\circ$
- (vi) $x + 35^\circ = 75^\circ$ (Exterior angle theorem)
 $x = 75^\circ - 35^\circ = 40^\circ$

Q10. The hypotenuse of a right triangle is 2 cm more than the longer side of the triangle. The shorter side of the triangle is 7 cm less than the longer side. Find the length of hypotenuse.

Sol. Hypotenuse = Longer side + 2 cm Shorter side = Longer side - 7 cm

In right triangle,

(Hypotenuse)² = (Longer side)² + (Shorter side)² [from Pythagoras theorem]

$$\therefore (x + 2)^2 = (x)^2 + (x - 7)^2$$

$$\Rightarrow x^2 + 4x + 4 = x^2 + x^2 - 14x + 49$$

$$\Rightarrow x^2 + 4x + 4 = 2x^2 - 14x + 49$$

$$\Rightarrow 2x^2 - x^2 - 14x - 4x + 49 - 4 = 0$$

$$\Rightarrow x^2 - 18x + 45 = 0$$

$$\Rightarrow x^2 - 15x - 3x + 45 = 0$$

$$\Rightarrow x - 15 = 0 \text{ or } x - 3 = 0$$

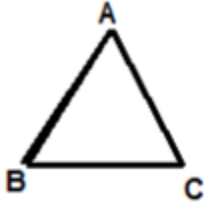
$$\Rightarrow x = 15 \text{ or } x = 3$$

$\therefore x = 15$ (When $x = 3$, length of shorter side is negative which is not possible)

Length of hypotenuse of the triangle = $(x + 2)$ cm = $(15 + 2)$ cm = 17 cm

Q11. If the angles of a triangle are in the ratio 3:4:5 determine the three angles

Sol.



Given, $\angle A : \angle B : \angle C = 3 : 4 : 5$

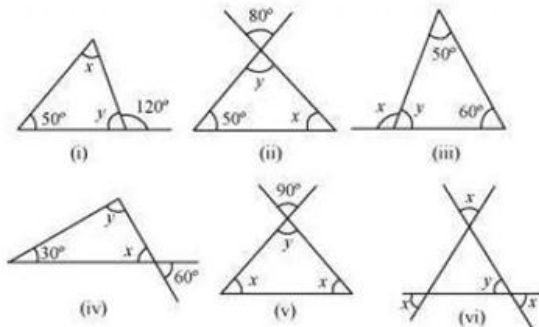
$\therefore \angle A = 3x, \angle B = 4x$ and $\angle C = 5x$, Where x is some constant.

In $\triangle ABC$, $\angle A + \angle B + \angle C = 180^\circ$ (Angle Sum Property)

$$\Rightarrow 3x + 4x + 5x = 180^\circ \Rightarrow 12x = 180^\circ \Rightarrow x = \frac{180^\circ}{12} = 15^\circ$$

$$\therefore \angle A = 3 \times 15^\circ = 45^\circ \quad \angle B = 4 \times 15^\circ = 60^\circ \quad \angle C = 5 \times 15^\circ = 75^\circ$$

Q12. Find the value of the unknowns x and y in the following diagrams:



Sol.

- (i) $y + 120^\circ = 180^\circ$ (Linear pair)
 $y = 180^\circ - 120^\circ = 60^\circ$
 $x + y + 50^\circ = 180^\circ$ (Angle sum property)
 $x + 60^\circ + 50^\circ = 180^\circ$
 $x + 110^\circ = 180^\circ$
 $x = 180^\circ - 110^\circ = 70^\circ$
- (ii) $y = 80^\circ$ (Vertically opposite angles)
 $y + x + 50^\circ = 180^\circ$ (Angle sum property)

$$80^\circ + x + 50^\circ = 180^\circ$$

$$x + 130^\circ = 180^\circ$$

$$x = 180^\circ - 130^\circ = 50^\circ$$

(iii) $y + 50^\circ + 60^\circ = 180^\circ$ (Angle sum property)

$$y = 180^\circ - 60^\circ - 50^\circ = 70^\circ$$

$$x + y = 180^\circ \text{ (Linear pair)}$$

$$x = 180^\circ - y = 180^\circ - 70^\circ = 110^\circ$$

(iv) $x = 60^\circ$ (Vertically opposite angles)

$$30^\circ + x + y = 180^\circ$$

$$30^\circ + 60^\circ + y = 180^\circ$$

$$Y = 180^\circ - 30^\circ - 60^\circ = 90^\circ$$

(v) $y = 90^\circ$ (Vertically opposite angles)

$$x + x + y = 180^\circ \text{ (Angle sum property)}$$

$$2x + y = 180^\circ$$

$$2x + 90^\circ = 180^\circ$$

$$2x = 180^\circ - 90^\circ = 90^\circ$$

$$X = \frac{90^\circ}{2} = 45^\circ$$

(vi) $y = x$ (Vertically opposite angles)

$$a = x \text{ (Vertically opposite angles)}$$

$$b = x \text{ (Vertically opposite angles)}$$

$$a + b + y = 180^\circ \text{ (Angle sum property)}$$

$$x + x + x = 180^\circ$$

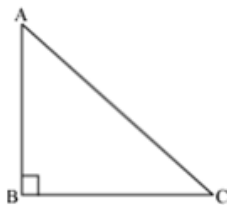
$$3x = 180^\circ$$

$$x = \frac{180^\circ}{3} = 60^\circ$$

$$Y = x = 60^\circ$$

Q13. $\triangle ABC$ is a right triangle right angled at B. If $AC = 25$ cm and $AB = 15$ cm, then find the side BC.

Sol.

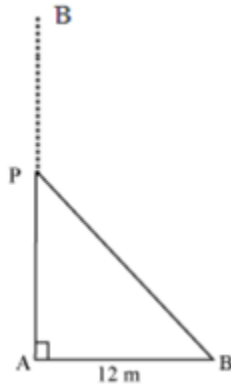


$\triangle ABC$ is a right-angled triangle, right-angled at B.

$$\begin{aligned} \therefore AC^2 &= AB^2 + BC^2 \text{ (Pythagoras theorem)} \\ \Rightarrow 15^2 + BC^2 &= 25^2 & \Rightarrow BC^2 &= 252 - 152 \\ \Rightarrow BC^2 &= 625 - 225 & \Rightarrow BC^2 &= 400 \\ \Rightarrow BC &= \sqrt{400} & \therefore BC &= 20 \text{ cm} \end{aligned}$$

Q14. A tree height 36m broke at a point P, but it did not separate. The top of the tree touched the ground at a distance of 12m from the base. Find the distance of the point P from the base of the tree.

Sol.



Height of the tree = 36 m

$$AP + PB = 36 \text{ m}$$

$$PB = 36 - AP$$

DAPB is a right-angled triangle, so by Pythagoras theorem

$$PB^2 = AP^2 + AB^2$$

$$(36 - AP)^2 = AP^2 + (12)^2$$

$$1296 + AP^2 - 72AP = AP^2 + 144$$

$$1296 + AP^2 - 72AP - AP^2 - 144 = 0$$

$$1152 - 72AP = 0 \quad AP = -\frac{1152}{-72} \quad AP = 16$$

$$-72AP = -1152$$

Distance of point P from the base = 16 m

Q15. In a right angled isosceles triangle, find the ratio of their sides.

Sol. We have $AB = BC$

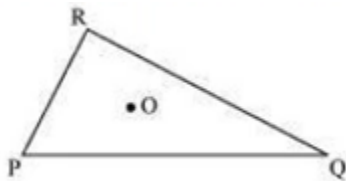
By Pythagorean Theorem

$$AC^2 = AB^2 + BC^2 \quad AC^2 = AB^2 + AB^2 \quad AC^2 = 2AB^2 \quad AC = \sqrt{2}AB$$

$$AC = \sqrt{2} : AB \quad AC = AB\sqrt{2}$$

$$AB : BC : AC = AB : AB : AB\sqrt{2} = 1 : 1 : \sqrt{2}$$

Q16. Take any point o in the interior of a triangle PQR. Is



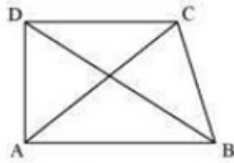
- (i) $OP + OQ > PQ$?
- (ii) $OQ + OR > QR$?
- (iii) $OR + OP > RP$?

Sol. If O is a point in the interior of a given triangle, then three triangles ΔOPQ , ΔOQR , and ΔORP can be constructed. In a triangle, the sum of the lengths of either two sides is always greater than the third side.

- (i) Yes, as ΔOPQ is a triangle with sides OP, OQ, and PQ.
 $OP + OQ > PQ$
- (ii) Yes as ΔOQR is a triangle with sides OR, OQ, and QR.
 $OQ + OR > QR$
- (iii) Yes, as ΔORP is a triangle with sides OR, OP, and PR.
 $OR + OP > PR$

Q17. ABCD is quadrilateral.

Is $AB + BC + CD + DA > AC + BD$?



Sol. In a triangle, the sum of the lengths of either two sides is always greater than the third side.

Considering $\triangle ABC$,

$$AB + BC > CA \text{ (I)}$$

In $\triangle BCD$,

$$BC + CD > DB \text{ (ii)}$$

In $\triangle CDA$,

$$CD + DA > AC \text{ (iii)}$$

In $\triangle DAB$,

$$DA + AB > DB \text{ (iv)}$$

Adding equations (i), (ii), (iii) and (iv), we obtain

$$AB + BC + BC + CD + CD + DA + DA + AB > AC + BD + AC + BD$$

$$2AB + 2BC + 2CD + 2DA > 2AC + 2BD$$

$$2(AB + BC + CD + DA) > 2(AC + BD)$$

$$(AB + BC + CD + DA) > (AC + BD)$$

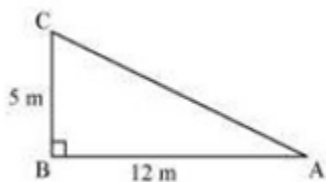
Yes, the given expression is true.

Q18. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Sol. In a triangle, the sum of the lengths of either two sides is always greater than the third side and also, the difference of the lengths of either two sides is always lesser than the third side. Here, the third side will be lesser than the sum of these two (i.e., $12 + 15 = 27$) and also, it will be greater than the difference of these two (i.e., $15 - 12 = 3$) therefore, those two measure are 27 cm and 3 cm.

Q19. A tree is broken at a height of 5 m from the ground and its top touché the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Sol.



In the given figure, BC represents the unbroken part of the tree. Point C represents the point where the tree broke and CA represents the broke part of the tree. Triangle ABC, thus formed, is right-angled at B.

$$AC^2 = BC^2 + AB^2$$

$$AC^2 = (5 \text{ m})^2 + (12 \text{ m})^2$$

$$AC^2 = 25 \text{ m}^2 + 144 \text{ m}^2 = 169 \text{ m}^2$$

$$AC = 13 \text{ m}$$

Q20. Which of the following can be the sides of a right triangle?

- (i) 2.5 cm, 6.5 cm, 6 cm
- (ii) 2 cm, 2 cm, 5 cm
- (iii) 1.5 cm, 2 cm, 2.5 cm

In the case of right-angled triangles, identify the right angles.

Sol.

- (i) 2.5 cm, 6.5 cm, 6 cm

$$(2.5)^2 = 6.25$$

$$(6.5)^2 = 42.25$$

$$(6)^2 = 36$$

It can be observed that,

The square of the length of one side is the sum of the squares of the lengths of the remaining two sides. Hence, these are the sides of a right-angled triangle. Right angle will be in front of the side of 6.5 cm measure.

(ii) 2 cm, 2 cm, 5 cm

$$(2)^2 = 4$$

$$(2)^2 = 4$$

$$(5)^2 = 25$$

$$\text{Here, } (2)^2 + (2)^2 \neq (5)^2$$

The square of the length of one side is not equal to the sum of the squares of the lengths of the remaining two sides. Hence, these sides are not of a right-angled triangle.

(iii) 1.5 cm, 2 cm, 2.5 cm

$$(1.5)^2 = 2.25$$

$$(2)^2 = 4$$

$$(2.5)^2 = 6.25$$

Here,

$$2.25 + 4 = 6.25$$

$$(1.5)^2 + (2)^2 = (2.5)^2$$

The square of the length of one side is the sum of the squares of the lengths of the remaining two sides. Hence, these are the sides of a right-angled triangle. Right angle will be in front of the side of 2.5 cm measure.

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