Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

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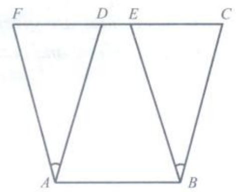
Class: 9
Subject: Mathematics
Topic: Area of Parallelogram & Triangles
No. of Questions: 20

Q1. Parallelograms on the same (or equal) base and between the same parallel lines are equal in area.

Sol. Given: two parallelograms ABCD and ABEF, which have the same base AB and which are between the same parallel lines AB and FC.
To Prove: Area of ∥ gm ABCD = area of ∥ gm ABEF



Proof: In ΔADF and ΔBCE
AD = BC [opp. Sides of a ∥ gm]
AF = BE [opp. Sides of a ∥ gm]
Also, AD ∥ BC and AF ∥ BE
So the angle between AD and AF is equal to the angle between BC and BE.
i.e. $\angle DAF = \angle CBE$

$\therefore \Delta ADF \cong \Delta BCE$ [By SAS congruent Rule]

$\therefore \Delta area\ \Delta ADF = area\ \Delta BCE$ [By congruent area Axiom] ...(i)

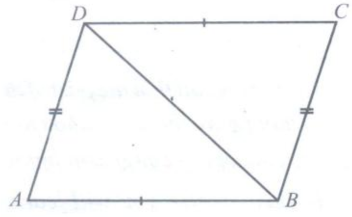
Now, area ∥ gm ABCD = area (▭ ABED) + (Δ BCE)

= area (▭ABED) + area (ΔADF) [Using (i)]

Area(∥ gm ABCD) = area (∥ gm ABEF)

Q2. A diagonal of parallelogram divides it into two triangles of equal area.

Sol. Given: A parallelogram ABCD in which BD is one of its diagonals, side AB ∥ CD and BC ∥ AD.
To Prove: Area of ΔABD = area of ΔBCD



Proof: In ΔABD and ΔBCD,

AD = BC [opp. Sides of a ∥ gm]

AD = CB [opp. Sides of a ∥ gm]

BD = BD [common side]

∴ ΔABD ≅ ΔCDB [By SSS congruent Rule]

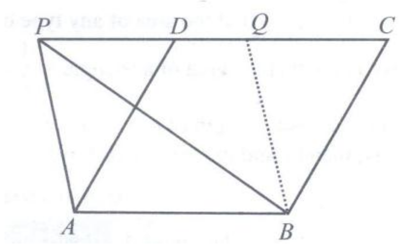
∴ Area of Δ ABD = area of ΔBCD [congruent area Axiom]

Q3. If a triangle and a parallelogram area on the same (on equal) base and between the same parallels lines, then prove that area of the triangle is equal to half the area of the parallelogram.

Sol. Given: ΔABP and ∥ gm ABCD be on the same base AB and between the same parallel lines AB and PC (See Fig).

To Prove: Area of ΔPAB = $\frac{1}{2}$ area of ∥ gm ABCD

Construction: From B, draw BQ ∥ AP



Proof: Since AB ∥ PQ and AP ∥ BQ, therefore ABQP is parallelogram.

Now parallelogram ABQP and ABCD are on the same base AB and between the same parallels AB and PC. N

∴ area of ∥ gm ABQP = area of ∥ gm ABCD ...(i)

Since diagonal PB divides ∥ gm ABQP into two triangles of equal area (Theorem)

∴ Area of ΔPAB = area of ΔPAB + area of ΔBQP ... (ii)

∴ Now area of ∥ gm ABQP = area of ΔPAB + area of ΔBQP = 2 ×area of ΔPAB [from (ii)]

∴ Area of ΔPAB = $\frac{1}{2}$ area of ∥ gm ABQP

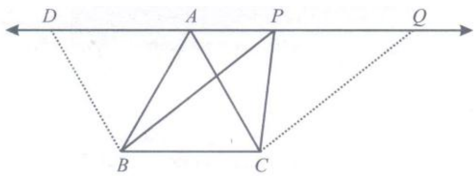
⇒ Area of ΔPAB = $\frac{1}{2}$ area of ∥ gm ABCD

Q4. Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

Sol. Given: Δ ABC and Δ PBC are on the same base BC and between the same parallel lines BC and AP.

To prove: Area of Δ ABC = area of ΔPBC

Construction: Through B, draw BD ∥ CA intersecting PA produced at D and through C, draw CQ ∥ BP intersecting AP produced in Q.



Proof: BCQP is a parallelogram.

Now, ∥ gm BCQP and BCAD are on the same base BC and lie between the same parallel lines BC and DQ,

∴ area of (∥ gm BCQP) = AREA OF (∥ gm BCAD)

We know that the diagonals of a ∥ gm divide it into two triangles of equal area. [By theorem]

∴ Area of (Δ PBC) = $\frac{1}{2}$ are of (∥ gm BCQP) ....(i)

And area of (Δ ABC)= $\frac{1}{2}$ area of (∥ gm BCAD) ....(ii)

Now area of (∥ gm BCQP) = area of (∥ gm BCAD) [from (i)]

∴ $\frac{1}{2}$ area of (∥ gm BCQP) = $\frac{1}{2}$ area of (∥ gm BCAD)

∴ area of ΔABC= area of ΔPBC [From (ii) and (iii)]

Q5. Find the area of a triangle, two of its sides are of length 6 cm and 12 cm and the perimeter is = 26 cm.

Sol. Here, a = 6 cm

b = 12 cm

And Perimeter = 26 cm

Let c be the length of third side

Now, Perimeter = a + b c + c

$26 = 6 + 12 + c$

$C = 26 - 18 = 8$ cm

∴ $s = \frac{a+b+c}{2} = \frac{6+12+18}{2} = 13$ cm

∴ Area of Δ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{13(13-6)(13-8)(13-12)} \quad = \sqrt{13 \times 7 \times 5 \times 1}$

$= \sqrt{13 \times 35} \quad = \sqrt{455}$ cm^2

Q6. The sides of a triangular plot are in the ration 3 : 5 : 7 and its perimeter is 300 m. Find its area.

Sol. The sides are in the ratio 3 : 5 : 7. So let the siders are 3x, 5x, and 7x respectively.

Now perimeter = 300 m

$\Rightarrow \quad 3x + 5x + 7x = 300$

$15x = 300$

$x = \frac{300}{15} = 20$

So the sides are 60m, 100m and 140m

$a = 60\ m$

$b = 100\ m$

$c = 140\ m$

$\therefore \quad s = \frac{a+b+c}{2} = \frac{300}{2} = 150\ m$

$\therefore$ Area of $\Delta\ ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150(150-60)}(150-100)(150-140)$

$= \sqrt{150 \times 90 \times 50 \times 10} = 1500\sqrt{3}\ m^2$

7. A traffic signal board, indicating 'SCHOOL AHEAD' is an equilateral triangle with side's length a. Find the area of the signal board, using Heron's Formula. If its perimeter is 180 cm.

Sol. The traffic signal is in the form of an equilateral triangle having side's length 'a' cm.

$\therefore$ Area of triangle using Heron's Formula

$\therefore \quad = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{3}{2}a\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)}$

$= \sqrt{\frac{3}{2}a \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \sqrt{\frac{3}{4}a^2}$

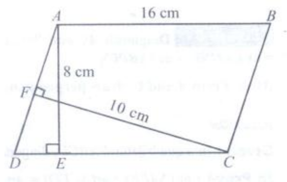
If perimeter of triangle is 180 cm i.e. a + a + a = 180

$\therefore \quad a = \frac{180}{3} = 60cm$

$\therefore$ Area of triangle $= \frac{\sqrt{3}}{4} \times 60 \times 60$

$= \sqrt{3} \times 15 \times 60 = 900 = 900\sqrt{3}\ cm^2$

Q8. In figure, ABCD is a parallelogram AE ⊥ DC and FC ⊥ AD. If AB = 16 cm, AE = 8 CM and CF = 10 cm. Find AD.



Sol. We have AB = 16 cm, AE = 8 cm, CF = 10 cm

We know that, Area of parallelogram = base × height

ABCD = CD × AE = 16 × 8 = 128 cm^2

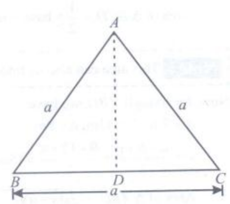
Again, Area of parallelogram = base × height = AD ×CF

128 = AD ×10

AD = $\frac{128}{10} = 12.8$ cm.

Q9. Prove that the area of an equilateral triangle is equal to $\frac{\sqrt{3}}{4}a^2$, where a is the side of the triangle.

Sol. Draw AD ⊥ BC



⟹ Δ ABD ≅ Δ ACD [By R.H.S.]

∴ BD = DC

∵ BC = a [CPCT]

∴ BD = DC = $\frac{a}{2}$

In right angled Δ ABD

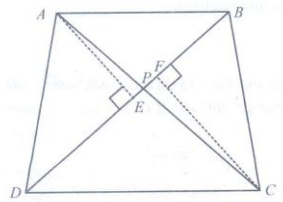
$AD^2 = AB^2 - BD^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4} \Longrightarrow AD = \frac{\sqrt{3}a}{2}$

Area of ΔABC = $\frac{1}{2}$ BC ×AD = $\frac{1}{2}a\frac{\sqrt{3}}{2} = \frac{\sqrt{3}\,a^2}{4}$

Q10. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar(ΔAPB) × ar(ΔCPD) = ar(ΔAPD) × ar(ΔBPC). [Hint. From A and C, draw perpendiculars to BD.]

Sol. Given: In a quadrilateral ABCD, diagonals AC and BD intersect each other at P.

To Prove: ar(Δ APM) × ar(Δ CPD) = ar (ΔAPD) × ar (ΔBPC)



Proof: From A and C, draw perpendiculars AE and CF respectively to BD.

ar(Δ APD) × ar (ΔCPD) $\frac{(PB)(AE)}{2} \times \left(\frac{DP \times CF}{2}\right) = \frac{1}{4} = (PB)(AE)(DP)(CF)$ ...(i)

and ar(ΔAPD) × ar(ΔBPC) $\frac{(DP(AE)}{2} \times \frac{(DP)(CF)}{2} = \frac{1}{4} = (PB)(AE)(DP)(CF)$ ...(ii)

From (i) and (ii),

Ar (ΔAPB) × Ar (ΔCPF) = Ar(ΔAPD) ×Ar (ΔBPC)

Hence Proved.

Q11. Area if the triangle whose sides are 13 cm, 9 cm and 6 cm is

(a) 23.6 cm^2
(b) 26.3 cm^2
(c) 36.34 cm^2
(d) 23.66 cm^2

Sol. (d)

The sides of the triangle are a= 13 cm, b = 9 cm and c = 6 cm

S = $\frac{a+b+c}{2} = \frac{13+9+6}{2} = 14$ cm.

∴ Area of the triangle = $\sqrt{S\,(S-a)(S-b)(S-c)}$ [Heron's formula]

= $\sqrt{14(14-13)(14-9)(14-6)}\ cm^2$

= $\sqrt{14 \times 1 \times 5 \times 8}\ cm^2$ = $\sqrt{560}\ cm^2$

= 23.66 cm^2

Q12. The sides of a triangle plot are in the ratio of 3: 5: 7 and its perimeter is 300 m. Its area is

(a) $1500\sqrt{2}\ cm^2$
(b) $1500\sqrt{3}\ cm^3$
(c) $1425\sqrt{2}\ cm^2$
(d) 1500 cm^2

Sol. (b)

Suppose that the sides, in metres, are 3x, 5x and 7x.

Then, we know that 3x + 5x + 7x = 300 (Perimeter of the triangle)

Therefore, 15x = 300, which gives x =20.

So the sides of the triangle are 3 ×20 m, 5 ×20m and 7 × 20m
i.e., 60 m, 100m and 140m
We have s
= $\frac{60+100+140}{2}\ m = 150\ m, and\ area\ will\ be\ \sqrt{150(150-60)(150-100)(150-140)}\ m^2$

= $\sqrt{150 \times 90 \times 50 \times 10}\ m^2 = 1500\sqrt{3}\ m^2$

Q13. The lengths of two adjacent sides of a parallelogram are 5 cm and 3.5 cm. One of its diagonals is 6.5 cm long. Area of the parallelogram is
(a) $13\sqrt{10}\ cm^2$
(b) $23\sqrt{5}\ cm^2$
(c) $10\sqrt{5}\ cm^2$
(d) $10\sqrt{3}\ cm^2$

Sol. (d)
Area of the parallelogram ABCD = 2 × (Area of ΔABC)
In ΔABC, AB = 5 cm, BC = 3.5 cm and AC = 6.5 cm
So its perimeter S = $\frac{5+3.5+6.5}{2} = 7.5$
∵ Area of ΔABC (By Heron's formula)
= $\sqrt{S(S-a)(S-b)(S-c)}$
= $\sqrt{7.5\,(7.5-5)(7.5-3.5)(7.5-6.5)}$ sq. cm.
= $\sqrt{7.5 \times 2.5 \times 4 \times 1}$ sq. cm = $\sqrt{75}$ sq. cm = $5\sqrt{3}$ sq. cm.
Area of parallelogram = 2 ×5 $\sqrt{3}$ sq.cm = $10\sqrt{3}$ sq. cm.

Q14. A floral design on a floor is made up of 16 tiles which are triangular, the sides of a triangle being 9 cm, 28 cm, 35 cm. Cost of polishing the tiles at the rate of 50p. Per cm^2 is
(a) Rs. 45 (approx.)
(b) Rs. 405 (approx.)
(c) RS. 450 (approx.)
(d) RS. 706 (approx.)

Sol. (d)
First we find the area of 16 triangular tiles.
For area of 1 triangular tile
S = $\frac{9+28+35}{2} = \frac{72}{2} = 36\ cm$
∴ Area = $\sqrt{36(36-9)(36-28)(36-35)}\ cm^2$
= $\sqrt{36 \times 27 \times 8 \times 1}\ cm^2 = 36\sqrt{6}\ cm^2$

$\therefore$ Area of 16 tiles $= 16 \times 36\sqrt{6}\ cm^2$

$= 576\sqrt{6}\ cm^2$

Cost of polishing = Rs. $\left(\frac{50}{100} \times 576\sqrt{6}\right)$

= Rs. 704.45 = RS. 706 (approx.)

Q15. A regular hexagon has a side 6 cm. Its perimeter and area are

(a) 35 cm, 8 $\sqrt{3}\ cm^2$

(b) 38 cm, 10 $\sqrt{2}\ cm^2$

(c) 40 cm, 11$\sqrt{2}\ cm^2$

(d) 36 cm, 54 $\sqrt{3}\ cm^2$

Sol. (d)

Side = 6 cm

$\therefore$ Perimeter of regular hexagon $= 6 \times 6 = 36$ cm

ABCDEF is a regular hexagon. Join diagonals AD, BE and CF.

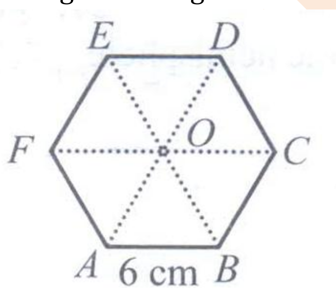
The three diagonal divides the hexagonal in six.

Congruent equilateral triangle with side 6 cm.

Area of one such triangle = $\sqrt{S\,(S-a)(S-b)(S-c)}$

$= \sqrt{9(9-6)(9-6)(9-6)} = \sqrt{9 \times 3 \times 3 \times 3} = 9\sqrt{3}$

$\therefore$ Area of the regular hexagon $= 6 \times$ Area of equilateral triangle OAB



= 6 ×9$\sqrt{3}$ = 54$\sqrt{3}\ cm^2$.

Q16. If the radius of a right circular cylinder, open at both the ends, is decreased by 25% and the height of the cylinder is increased by 25%, then the surface area of the cylinder thus formed.

(a) Remains unaltered

(b) Is increased by 25%

(c) Is decreased by 25%

(d) Is decreased by 6.25%

Sol. (d)

Q17. A triangular park ABC has sides 120m, 80m and 50m. A gardener has to put a fence all around it and also plant grass inside. Area of garden and cost of fencing the garden with barbed wire at the rate of Rs. 20 per metre leaving a space 3m wide for a gate on one side are

(a) $375\sqrt{15}\ m^2$, Rs. 4940

(b) $357\sqrt{10}\,m^2$, Rs. 9440

(c) $573\sqrt{8}\,m^2$, Rs. 4944

(d) $683\sqrt{10}\,m^2$ Rs. 5490

Sol. (a)

For area of the park, we have

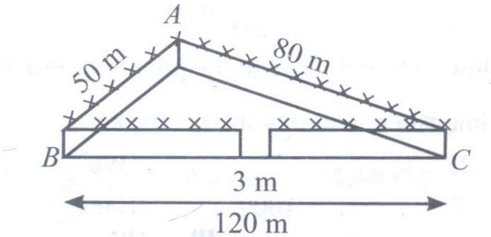
2s = 50m +80m + 120m = 250 m.

S = 125 m

i.e., s-a = (125-120) m = 5 m,

S-b = (125-80) m = 45 m,

S-c = (125-50) m = 75 m.



Therefore, area of the park = $\sqrt{S\,(S-a)(S-b)(S-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75}\ \ m^2 = 375\sqrt{15}\,m^2$

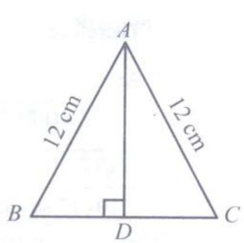
Also, Perimeter of the park = AB+BC+CA = 250 m

Therefore, length of the wire needed for fencing = 250 m – 3m (to be left for gate) = 247 m

And so the cost of fencing = Rs. 20 ×247 = Rs. 4940

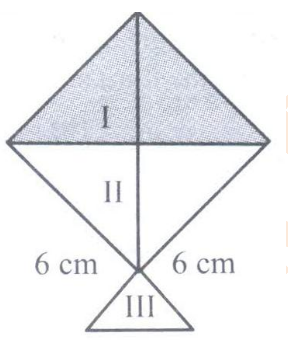
Q18. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Sol.



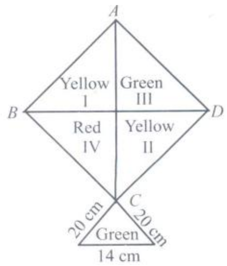
Let a, b, c be the sides of isosceles Δ.
Given, a = 12 cm, b = 12 cm
Perimeter = 30 cm
⇒ a+b+c = 30
⇒ 12 +12 + c = 30
⇒ c = 30-24
⇒ c = 6 cm
Now, S = $\frac{a+b+c}{2} = \frac{12+12+6}{2}$
⇒ S = $\frac{30}{2}$ cm $= 15$ cm
∴ Area of the triangle = $\sqrt{S(S-a)(S-b)(S-c)}$
$= \sqrt{15(15-12)(15-12)(15-6)}$
$= \sqrt{15\,(3)(3)(9)} = 9\sqrt{15}\ \text{cm}^2$

Q19. A kit in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 c m and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?



Sol. Area of paper of shade I = $\frac{1}{2}\left[\frac{1}{2}\, d_1 d_2\right]$ where d_1 and d_2 denotes the diagonals
$= \frac{1}{2}\left(\frac{1}{2} \times 32 \times 32\right) = 256\ \text{cm}^2$
Area of paper II = 256 cm^2
For paper of shade III, sides are given as a = 8 cm, b = 6 cm c = 6 cm
∴ $\frac{1}{2}\left(\frac{1}{2} \times 32 \times 32\right) = 256\text{cm}^2$
∴ Area of paper of shade III
$= \sqrt{S(S-a)(S-b)(S-c)}$
$= \sqrt{10(10-8)(10-6)(10-6)}$
$= \sqrt{10(2)(4)(4)} = 8\sqrt{5} = 17.92\ \text{cm}^2$

Q20. How much paper of each shade is needed to make a kite given in Fig. in which ABCD is a square with diagonal 44 cm?



Sol. Yellow: 484 m^2; Red: 242 m^2
Green: 373.04 m^2