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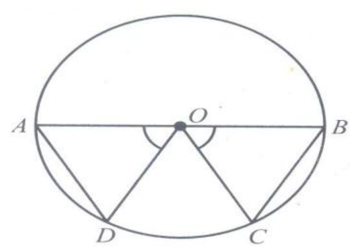
Class: 9
Subject: Mathematics
Topic: Circles
No. of Questions: 20

Q1. Equal chord of a circle subtend equal angles at the centre.

Sol. **Given:** AD and CB are two equal chords of a circle with centre O. AD and CB subtends an angles ∠AOD and ∠COB at the center O respectively.

To Prove: ∠AOD = ∠COB



Proof: In ΔAOD and Δ COB,

AD = CB [Given]

OA = OC [Radii of the same circle]

OB = OD

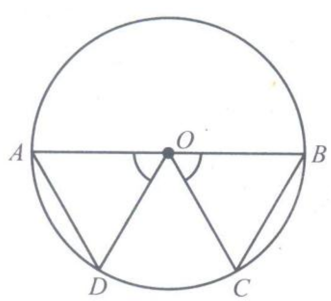
∴ ΔAOD ≅ ΔCOB [By SSS congruent Rule i.e. side side side]

∴ ∠AOD = ∠COB [Corresponding parts of congruent triangles are equal]

Q2. If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Sol. **Given:** AD and CB are two chords of a circle with centre O. Chords AD and CB subtends equal angle ∠AOD and ∠COB at the centre respectively.

To Prove: AD = CB



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Proof: In ΔAOD and ΔCOB,

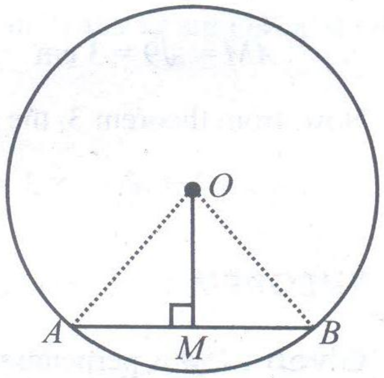
	OA = OC	[Radii of the same circle]
	OB = OD	[Radii of the same circle]
	∠AOD = ∠COB	[Given]
∴	ΔAOD ≅ ΔCOB	[By SSS congruent Rule i.e. side side side]
∴	AD = CB	[Corresponding part of congruent triangle are equal]

Q3. The perpendicular from the centre of a circle to a chord bisects the chord.

Sol. **Given:** AB is chord of a circle with centre O. OM be the perpendicular from O to chord AB.

To Prove: OM bisect AB i.e. AM = MB

Construction: Join OA and OB



Proof: In ΔAOMand ΔBOM

OA = OB	[Radii of the same circle]
OM = OM	[Common]
∠OMA = ∠OMB = 90°	
∴ ΔOAM ≅ ΔOBM	[By RHS congruent rule]
∴ AM = BM	[Corresponding parts of two congruent triangles are equal]

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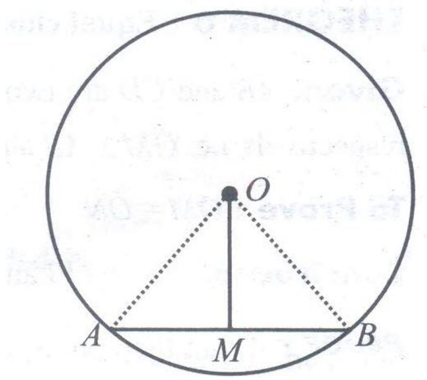
Q4. The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

Sol. **Given:** AB is a chord of a circle whose centre is O. M is the mid-point of the chord AB, i.e. AM = MB.

To Prove: OM is perpendicular to AB i.e. OM ⊥ AB

Construction: Join OA and OB.



Proof: In ΔAOM ≅ ΔBOM

OA = OB	[Radii of the same circle]
OM = OM	[Common]
AM = MB	[Given]
∴ ΔAOM ≅ ΔBOM	[By SSS congruent rule]
∴ ∠AMO = ∠BMO … (i)	[Corresponding part of two congruent triangle are equal]
Now, ∠AMO + ∠BMO = 180°	[Linear pair angles]
But ∠AMO+ ∠BMO	[From (i)]

∴ ∠AMO + ∠AMO = 180°

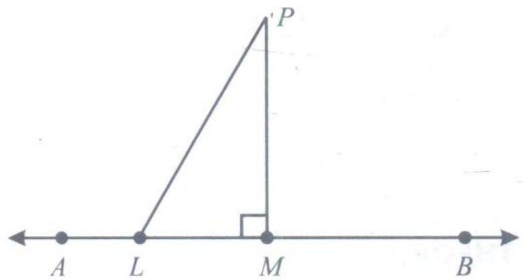
2∠AMO = 180°

∠AMO = $\frac{180°}{2}$ = 90°

So, OM ⊥ AB

Q5. Perpendicular distance of a line from a point is the shortest distance of the line from the point.

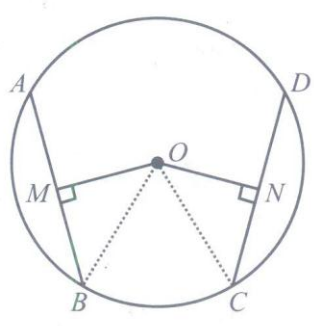
Sol. **Given**: PM is perpendicular to a line AB from point P.
L is any point other than point M on the line AB.
To Prove: PM < PL



Proof: PL is the hypotenuse of the triangle right angled at M.
We know that hypotenuse of a triangle is always greater any one of the other two sides.
∴ PL > PM
Or PM < PL

Q6. Equal chord of a circle of a circle (or of congruent circles) are equidistance from the centre.

Sol. **Given:** AB and CD are two equal chords of a circle having centre O. OM and ON are the distance of AB and CD from centre O respectively i.e. OM ⊥ AB and ON ⊥ CD.
To Prove: OM = ON
Construction: Join OB and OC.



Proof: From theorem-3, perpendicular from centre to chord bisect the chord, so OM bisect AB and ON bisects CD.

$\therefore AM = MB$ ...(i)

$\text{And } CN = ND$ ...(ii)

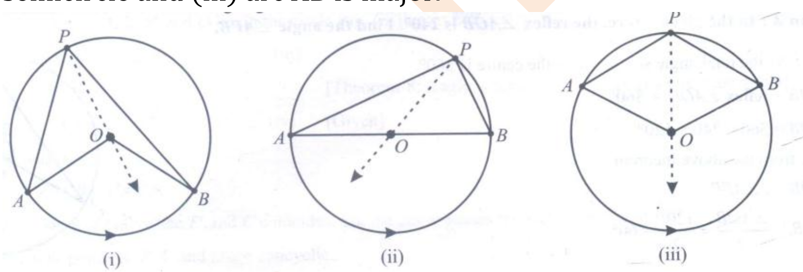
$\text{But } AB = CD$ [Given]

$AM + MB = CN + ND$

MB + MB = CN + CN [from (i) and (ii)]
∴ 2 MB = 2CN ...(iii)
Now, In ΔOMB and ΔONC
MB = CN [from (iii)]
∴ OB = OC [Radii of the same circle]
∴ ∠OMB = ∠ONC = 90° [OM ⊥ AB, ON ⊥ CD]
ΔOMB ≅ ΔONC [By RHS congruent rule]
∴ OM = ON [Corresponding parts of congruent triangles are equal]

Q7. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Sol. **Given:** AB is an arc of a circle subtending angles AOB at the centre O and APB at a point P on the remaining part of the circle.
To Prove: ∠AOB = 2 ∠APB (if arc AB is minor)
180° = 2 ∠APB (if arc AB is semicircle)
Reflex ∠AOB = 2 ∠APB (if arc AB is major arc)
Construction: Join P to O and extends it to point C.
We consider the three cases as shown in the above figure (i) arc AB is minor (ii) arc AB is a semicircle and (iii) arc AB is major.



Proof: In all the three cases ∠AOC is the exterior angle of ΔAOP.
∴ ∠AOC = ∠APO + ∠OAP ...(i) [Exterior angles of a triangle is equal to the sum of the two interior opposite angles]
Now, In ΔOAP
OP = OA [Radii]
∴ ∠APO = ∠OAP [Angles opp. Equal sides in a triangle are equal]
∴ ∠AOC = ∠APO + ∠APO
∠AOC = 2 ∠APO ...(ii)
Similarly, ∠BOC = 2 ∠OPB ...(iii)

Case- I : Adding (ii) & (iii), we get
∠AOC + ∠BOC = 2[∠APO + ∠OPB]

∠AOB = 2 ∠APB

Case-II : Angle subtended by semicircle at the centre of the circle is 180° .

Adding (ii) and (iii), we get

∠AOC + ∠BOC = 2 ∠APO + 2 ∠OPB

⇒ 180° = 2(∠APO + ∠OPB)

⇒ 180° = 2 ∠APB

Case- III : Angle subtended by the major arc AB at the centre of the circle is the reflex angle ∠AOB.

Adding (ii) and (iii), we get

∠AOC + ∠BOC = 2 ∠APO + 2∠OPB

⇒ Reflex ∠AOB = 2(∠APO + ∠OPB)

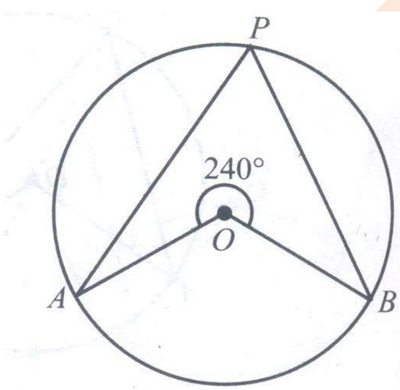
⇒ Reflex ∠AOB = 2 ∠APB

Note: In case (ii) ∠ AOB = 2 ∠APB, but ∠AOB = 180° [AB is the diameter]

∴ ∠APB = $\frac{180°}{2}$ = 90°

This proves the property that the angle is the semicircle is a right angle.

Q8. In the given figure, the reflex, the reflex ∠AOB = 240°. Find the angle ∠APB.



Sol. As the total angle subtended at the center is 360°

∴ ∠AOB + reflex ∠AOB = 360°

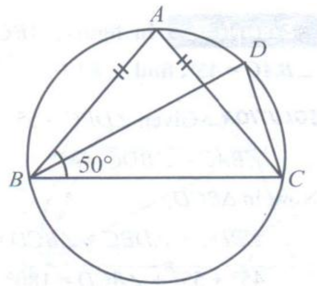
∴ ∠AOB = 360- 240 = 120°

Now, from the above theorem

∠AOB = 2 ∠APB

∠APB = $\frac{\angle AOB}{2} = \frac{120°}{2} = 60°$

Q9. In the given figure, ABC is an isosceles triangle in which AB = AC and $\angle ABC = 50°$, find $\angle BDC$.



Sol. $AB = AC, \therefore \angle ABC = \angle ACB = 50°$

$\therefore$ In ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180°$

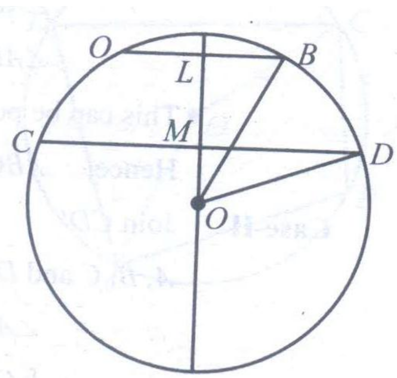
$50° + 50° + \angle BAC = 180°$

$\angle BAC = 180° - 100° = 80°$

Now, $\angle BAC = \angle BDC$ [Angle in the same segment of a circle are equal]

$\therefore \angle BDC = 80°$

Q10. Two chords AB and CD of lengths 6 cm, 12 cm respectively of a circle are parallel. If the ⊥ distance between AB and CD is 3 cm. Find the radius of the circle



Sol. Here AB = 6 cm ⇒ AL = LB = 3B

CD = 12 cm ⇒ CM = MD = 6 cm

Also LM = 3 cm. Let OM x

In right triangle OLB,

$OL^2 + LB^2 = OB^2$ [By Pythagoras theorem]

⇒ $(3+x)^2 + 3^2 = OB^2$ ...(i)

Now in right OMD,

$OM^2 + MD^2 = OD^2$

⇒ $x^2 + 6^2 = OB^2$ ...(ii) (since OD = OB = radius)

From (i) & (ii), $(3 + x)^2 + 3^2 = x^2 + 6^2$

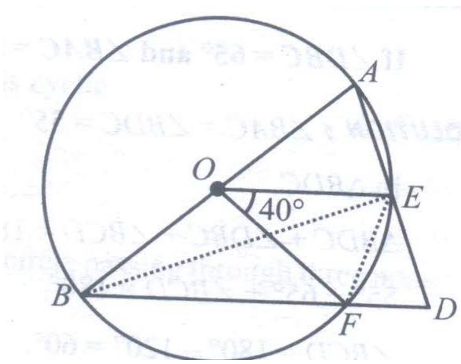
⇒ 9 +6x = 36 – 9 or x = 3.

From (i), $OB^2 = (3+3)^2 + 3^2 = 36 + 9$

OB = 3 cm.

Hence radius = 3 cm

Q11. In the given figure, O is the centre of a circle and AB is a diameter. If $\angle EOF = 40°$, Find $\angle EDF$.



Sol. Join BE and EF.

Since AB is the diameter of the circle

$\angle AEB = 90°$ [Angle in the semicircles is 90°]

$\angle EBF = \frac{1}{2}\ \angle EOF = \frac{40°}{2} = 20°$

[Angle made by a chord at any point on the circumference is half the angle made by the chord at the center of the circle].

Also $\angle AEB = 90°$

$\therefore \angle BED = 180°$ - $90° = 90°$

In ΔBFD,

$\angle BED + \angle EBD + \angle EDB = 180°$

$90° + \angle EBF + \angle EDB = 180°$ [$\angle EBD = \angle EBF$]

$90° + 20° + \angle EDB = 180°$

$\angle EDB = 180↑8 - 110° = 70°$

i.e. $\angle EDF = 90°$

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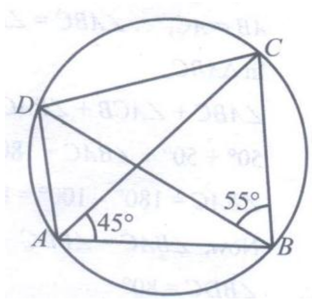
Q12. In figure, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If $\angle DBC = 55°$ and $\angle BAC = 45°$, find $\angle BCD$.



Sol. Given $\angle DBC = 55°$ and $\angle BAC = 45°$

$\angle BAC = \angle BDC = 45°$ ...(i) [Angle in the same segment of a circle are equal]

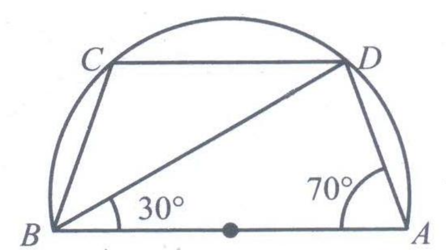
Now, in ΔBCD,

$\angle BDC + \angle DBC + \angle BCD = 180°$

$45° + 55° + \angle BCD = 180°$ [from (i)]

$\angle BCD = 180° - 100° = 80°$

Q13. In the given figure AB is the diameter, $\angle BAD = 70°$ and $\angle DBC = 30°$. Find $\angle BDC$.



Sol. ABCD is a cyclic quadrilateral

$\therefore \angle BAD + \angle BCD = 180°$ [Sum of opposite angles of a cyclic quadrilateral is 180°]

$\angle BCD = 180° - \angle BAD$

$= 180° - 70° = 110°$ [$\angle BAD = 70°$, Given] ..(i)

Now, In ΔBCD.

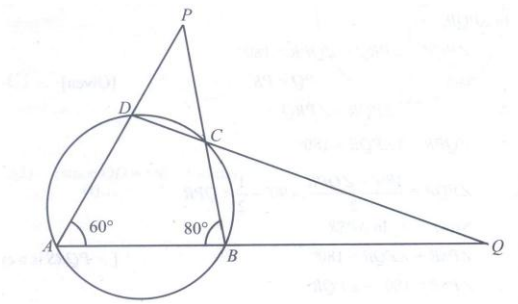
$\angle DBC + \angle BCD + \angle BDC = 180°$

$30° + 110° + \angle BDC = 180°$ [$\angle DBC = 30°$, $\angle BCD = 110°$, Given]

$\therefore \angle BDC = 180° - 140° = 40°$

$\angle BDC = 40°$

Q14. In the given figure, $\angle A = 60°$ and $\angle ABC = 80°$, find $\angle DPC$ and $\angle BQC$.



Sol. In a cyclic quadrilateral, exterior angle is equal to opposite interior angle. So, in cyclic quadrilateral ABCD, we have

$\angle PDC = \angle ABC$ and $\angle DCP = \angle A$

$\Rightarrow \angle PDC = 80°$ and $\angle DCP = 60°$ [$\angle ABC = 80°$ and $\angle A = 60°$]

In ΔDPC, we have

$\angle DPC = 180° - (\angle PDC + \angle DCP)$

$\Rightarrow \angle DPC = 180° - (80° + 60°) = 40°$

Similarly, we have

$\angle QBC = \angle ADC$ and $\angle BCQ = \angle A$

$\Rightarrow \angle QBC = 180° - \angle ABC$ and $\angle BCQ = 60°$

[$\angle ADC + \angle ABC = 180°$ and $\angle A = 60°$]

$\Rightarrow \angle QBC = 180° - 80° = 100°$ and $\angle BCQ = 60°$

Now, in ΔBQC, we have

$\angle BQC = 180° - (\angle QBC + \angle BCQ) = 180° - (100° + 60°) = 20°$

Q15. A chord of a circle is equal to its radius. Find the angle subtended by this chord at a point in major segment.

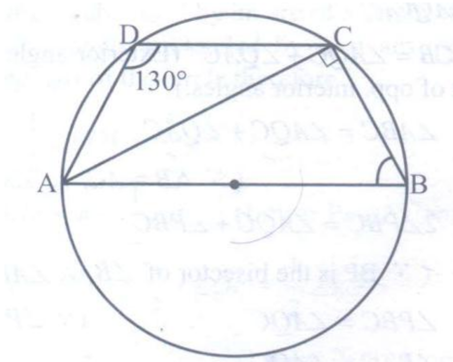
Sol. AB = OA = OB (Given)

$\therefore \angle AOB = 60°$ $\angle AOB = 2\angle ACB$

$\therefore \angle ACB = 30°$

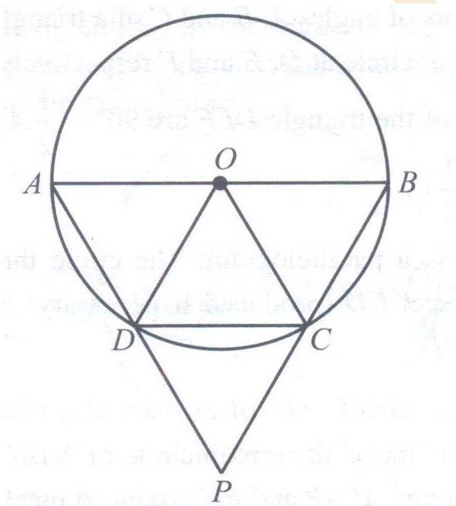
Q16. A quadrilateral ABCD is inscribed in a circle such that AB is a diameter and ∠ADC = 130°. Find ∠BAC

Sol.



∠B = 180° - 130° = 50° (∴ ABCD is a cyclic quadrilateral)
∠ACB = 90° (Angle in the semi-circle)
∠BAC = 180° - (90° +50°) = 40°

Q17. AB is a diameter of the circle with centre O and chord CD is equal to radius OC. AD and BC produced, which meet at P. Prove that ∠CPD = 60°



Sol. Join AD, In ΔOCD,
OC = OD ...(i) [Radii of the same circle]
OC = CD ...(ii) [Given]
From (i) and (ii),
OC = OD = CD
ΔOCD is an equilateral triangle
∵ ∠COD = 60°

$\because \angle CAD = \frac{1}{2}\angle COD = \frac{1}{2}(60°) = 30°$

($\because$ Angle subtended by an arc of a circle at the centre is twice the angle subtended by it any point of the remaining part of the circle.)

$\Rightarrow \angle PAD = 30°$ ...(iii)

$\angle ADB = 90°$ ...(iv) [angle in a semi-circle]

$\angle ADB + \angle ADP = 180°$ [Linear Pair Axiom]

$\Rightarrow 90° + \angle ADP = 180°$ [From (iv)]

$\Rightarrow \angle ADP = 90°$ ... (v)

In ΔADP,

$\angle APD + \angle PAD + \angle ADP = 180°$

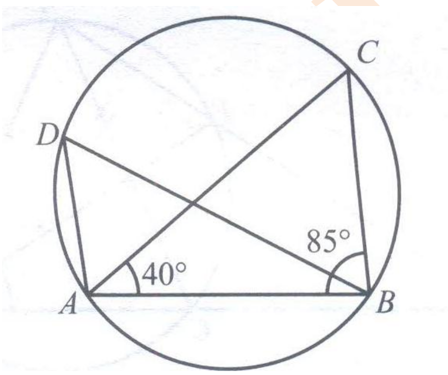
$\Rightarrow \angle APD + 30° + 90° = 180°$ [From (iii) and (v)]

$\Rightarrow \angle APD + 120° = 180°$

$\Rightarrow \angle APD = 180° - 120° = 60°$

$\Rightarrow \angle CPD = 60°$

Q18. In the given figure, points A, B, C and D lie on a circle. If $\angle CAB = 40°$ and $\angle ABC = 85°$ then find $\angle ADB$.



Sol. In ΔABC

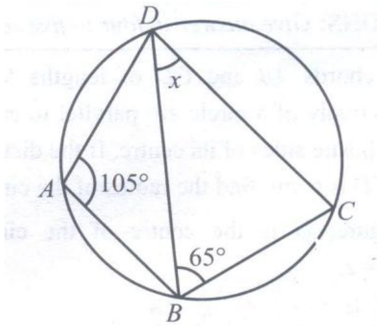
$\angle ACB + 40° + 85° = 180°$

$\Rightarrow \angle ACB = 180° - 40° - 85° = 55°$

Now, $\angle ADB = \angle ACB = 55°$

($\because$ angle in the same segment of a circle)

Q19. In the given figure, ABCD is a cyclic quadrilateral. If $\angle BAD = 105°$ and $\angle CBD = 65°$ then find the value of x.



Sol. $\angle BCD + \angle BAD = 180°$ (sum of the opp. Angles of a cyclic quad.)

$\Rightarrow \angle BCD + 105° = 180° \Rightarrow \angle BCD = 75°$

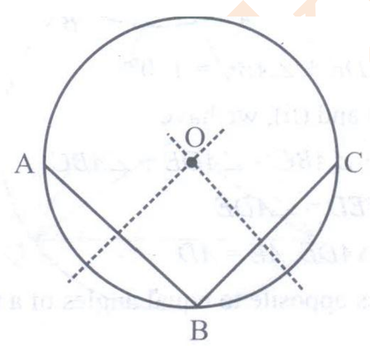
In ΔBCD, $x + 65° + 75° = 180° \Rightarrow x = 40°$

Q20. Suppose you are given a circle. Give a construction to find its centre.

Sol. Steps of construction:

(i) Take any three points A, B and C on the circle.

(ii) Join AB and BC.



(iii) Draw the perpendicular bisect of AB and BC.

Let these intersect at O. Then, O is the center of the circle.