

Class: 9
Subject: Mathematics
Topic: Herons Formula
No. of Questions: 20

Q1. Madhav makes the kite using two piece of paper. 1st piece of paper is cut in the shape of square where one diagonal is of the length 33cm. At one of the vertex of this square a second piece of paper is attached which is of the shape of an equilateral triangle of length 6 cm to give the shape of a kite. Find the area of this kite.

Sol. 560.09 cm^2

Q2. The area of an equilateral triangle with side of Q cm is:

Sol. As per Heron's formula, the area of a triangle with sides a, b and c, and perimeter

$$2S = \sqrt{S(S-a)(S-b)(S-c)}$$

$$\text{Here, } a = b = c = Q \text{ and } S = \frac{3}{2} \times a = \frac{3Q}{2}$$

$$\text{Therefore Area} = \sqrt{\frac{3Q}{2} \times \left(\frac{3Q}{2} - Q\right) \times \left(\frac{3Q}{2} - Q\right) \times \left(\frac{3Q}{2} - Q\right)}$$

$$\text{Area} = \sqrt{\frac{3Q}{2} \times \frac{Q}{2} \times \frac{Q}{2} \times \frac{Q}{2}}$$

$$\text{Area} = \frac{(Q^2 \sqrt{3})}{4} \text{ cm}^2$$

Q3. The perimeter of a rhombus is 164 cm and one of its diagonals is 80 cm. What is the length of other diagonal?

Sol. On way to solve this is as follows

We know that the

(a) The sides of a rhombus are equal. Therefore one side $= \frac{1}{4} \times 164 = 41$

(b) A diagonals of a rhombus divides the rhombus into 2 equal triangles

(c) The area of a rhombus is $\frac{1}{4} (\text{Diagonal 1} \times \text{Diagonal 2})$

Taking one of the two triangles formed by the diagonal with length 80 cm

$$\text{Area (suing Heron's formula)} = \sqrt{S(S-41)(S-41)(S-80)}$$

$$\text{Where } S = \frac{2 \times 41 + 80}{2} = \frac{162}{2} = 81$$

$$\text{Area} = \sqrt{81(81-41)(81-41)(81-80)} = 720 \text{ (the details of this computation are left for the students)}$$

From c) above, Area = $720 = \frac{1}{4} (\text{Diagonal1} \times \text{Diagonal2}) = \frac{1}{4} (80 \times \text{Diagonal2})$

$$\text{Diagonal 2} = 2 \times \frac{720}{80} = 18 \text{ cm}$$

Q4. The area of a triangle with sides 26 cm, 25 cm and 3 cm is:

Sol. The area of a triangle with sides a, b, c is given by Heron's formula as

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

Where S is half of the perimeter, i.e. $S = \frac{a+b+c}{2}$

$$\text{Here } S = \frac{26+25+3}{2} = \frac{54}{2} = 27$$

$$\text{Area} = \sqrt{27(27-26)(27-25)(27-3)} = \sqrt{1296} = 36\text{cm}^2$$

Q5. A Carpenter has cut a board in the shape of Trapezium. If the parallel sides of the trapezium are 23 cm and 12 cm and non-parallel sides are 14 cm and 14 cm, find the area of the board.

- (a) 154.49 cm^2
- (b) 22.53 cm^2
- (c) 225.3 cm^2
- (d) 450.6 cm^2

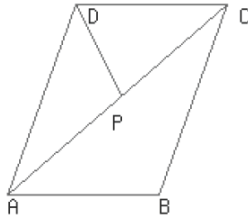
Sol. (c) 225.3 cm^2

Q6. A parallelogram has a diagonal of 8 cm. The perpendicular distance of this diagonal from an opposite vertex is 5 cm. Find the area of the parallelogram.

- (a) 40 cm^2
- (b) 10 cm^2
- (c) 13 cm^2
- (d) 20 cm^2

Sol. (a) 40 cm^2

Consider a parallelogram ABCD as shown in the figure below



P is the point where the perpendicular from point D meets diagonal AC
 From the diagram, we see that ACD is a triangle. The area of ACD is half the area of parallelogram ABCD

The area of ACD is $\frac{1}{2}$ x base x height

Here base = length of diagonal = 8 cm

Height = length of DP = 5 cm

Area of ACD = $\frac{1}{2}$ X 5X 8= 20

Area of parallelogram ABCD = 2 X area of ACD = 2 x 20 = 40 cm²

Q7. The area of an equilateral triangle with a side of 12 cm is:

- (a) $36\sqrt{3}$ cm²
- (b) 1728 cm²
- (c) 36 cm²
- (d) 144 cm²

Sol. (a) $36\sqrt{3}$ cm²

As per heron's formula, the area of a triangle with sides a, b and c, and perimeter

$$2S = \sqrt{S(S-a)(S-b)(S-c)}$$

Here, a=b=c= 12 and $S = \frac{3}{2} \times a = 18$

$$\text{Therefore Area} = \sqrt{18 \times (18 - 12) \times (18 - 12) \times (18 - 12)}$$

$$\text{Area} = \sqrt{(18 \times 6 \times 6 \times 6)}$$

$$\text{Area} = 36 \sqrt{3} \text{ cm}^2$$

Q8. The base of an isosceles triangle is J cm and its perimeter is T cm. find the area of the triangle.

- (a) $(J(4T^2 - J^2)) / 4$ cm²
- (b) $(J\sqrt{T^2 - J^2}) / 4$ cm²
- (c) $(\frac{J}{4})\sqrt{(T^2 - 2TJ)}$ cm²
- (d) $(J^2\sqrt{4T^2 - J^2}) / 4$ cm²

Sol. (c) $\left(\frac{J}{4}\right)\sqrt{(T^2 - 2TJ)} \text{ cm}^2$

As per Heron's formula, the area of a triangle with sides a, b and c, and perimeter 2S =

$$\sqrt{S(S-a)(S-b)(S-c)}$$

Here $S = \frac{T}{2}$

Here we have an isosceles triangle, so two sides are equal

Let's assume $a=b$, and $c=J$ is the base

Also, perimeter $T = a + b + c = 2a + J$

$$a = \frac{T-J}{2}$$

$$\text{Area} = \sqrt{S(S-a)(S-a)(S-J)} = (S-a)\sqrt{S(S-J)} = \left(S - \frac{T-J}{2}\right)\sqrt{S(S-J)}$$

Substituting $S = \frac{T}{2}$ in the equation we get

$$\text{Area} = \left(\frac{J}{2}\right)\sqrt{\frac{T}{2}\left(\frac{T}{2} - J\right)} = \left(\frac{J}{4}\right)\sqrt{(T^2 - 2TJ)}$$

- Q9. A field in the shape of trapezium has its parallel sides as 25 m and 13 m while the non-parallel sides are 15 m and 16 m. Find the amount of money farmer has to pay if the cost of sowing the seeds per m^2 area is Rs. 125.
- (a) Rs. 10681.25
(b) Rs. 50735.625
(c) Rs. 270.59
(d) Rs. 33823.75

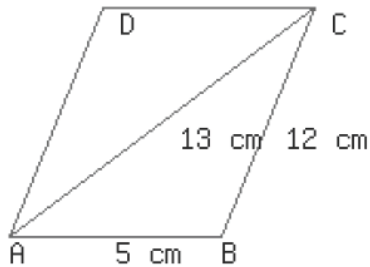
Sol. (d) Rs. 33823.75

- Q10. An umbrella is made by stitching 12 triangular pieces of cloth each piece measuring 25 cm, 47 cm and 47 cm. How much cloth is required for this umbrella.
- (a) 6796.08 cm^2
(b) 10194.12 cm^2
(c) 566.34 cm^2
(d) 3398.04 cm^2

Sol. (a) 6796.08 cm^2

- Q11. The two adjacent sides of a parallelogram are 5 cm and 12 cm. if one of its diagonals is 13 cm, find the area of the triangle.
- (a) 60 cm^2
(b) 780 cm^2
(c) 30 cm^2

Sol. (A) Take a look at the representative image below



We can see that ABC is a triangle

Also We know the length of all the sides. We can therefore use Heron's Formula

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}, \text{ where } a, b, c \text{ are the sides, and } S = \frac{a+b+c}{2} = \frac{5+12+13}{2} = 15$$

$$\text{Area} = \sqrt{15(15-5)(15-12)(15-13)} = 60 \text{ cm}^2$$

- Q12. In heron's formula, S is equal to:
- (a) $A + b + c$
(b) $\frac{a \times b \times c}{2}$
(c) Half of perimeter of the triangle
(d) $\frac{a+b+c}{abc}$

Sol. (C) Half of perimeter of the triangle

- Q13. From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. If the lengths of the perpendiculars are a, b and c, find the area of the triangle.
- (a) $(abc)^2 / \sqrt{3}$
(b) $abc / \sqrt{3}$
(c) $(a+b+c)^2 / \sqrt{3}$
(d) $(a+b+c) \sqrt{3}$

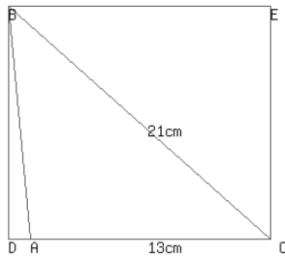
Sol. (c) $(a + b + c)^2 / \sqrt{3}$

Q14. Find the area of a quadrilateral ABCD where AB = 9 cm, BC = 10 cm, CD = 16 cm, DB = 19 cm and AC = 11 cm.

- (a) 130.34 cm^2
- (b) 1303.4 cm^2
- (c) 87.91 cm^2
- (d) 42.43 cm^2

Sol. (A) 130.34 cm^2

Q15. If in the figure below BC = 21 cm, CA = 13cm and BD = 16.99cm, find the area of the triangle ABC.



- (a) 165.63 cm^2
- (b) 25.5 cm^2
- (c) 110.42 cm^2
- (d) 220.84 cm^2

Sol. (c) 110.42 cm^2

Q16. The semi perimeter of a triangle having the length of its sides as 20 cm, 15 cm and 9 cm is:

- (a) 44 cm
- (b) 21 cm
- (c) 22 cm
- (d) None

Sol. (c)

Q16. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find area of the triangle.

Sol. An isosceles triangle has two sides equal $a = b = 12$ cm.

Given perimeter = 30 cm

\Rightarrow Third side of the Δ is $= c = 30 - (12+12) = 6$ cm

Semi-perimeter $s = 30/2 = 15$ cm

Area of $\Delta = [(15 \times (15-12) \times (15-12) \times (15-6))]^{1/2}$

$\Rightarrow = [15 \times 3 \times 3 \times 9]^{1/2}$

$\Rightarrow = 9 \sqrt{(15)} \text{ cm}^2$

Q17. Find the area of an equilateral triangle with side 10 cm.

Sol. Area of equilateral $\Delta = (\sqrt{3}/4)a^2 = (\sqrt{3}/4) \times 10^2 = (\sqrt{3}/4) \times 100 = 25\sqrt{3} \text{ cm}^2$

Using Heron's formula, $s = 30/2 = 15$

Area = $[(15 \times (15-10) \times (15-10) \times (15-10))]^{1/2} = [15 \times 5 \times 5 \times 5]^{1/2} = 25\sqrt{3} \text{ cm}^2$

Q18. The sides of a Δ are 7 cm, 24 cm and 25 cm. Its area is:

(a) 168 cm^2

(b) 84 cm^2

(c) 87.5 cm^2

(d) 300 cm^2

Sol. (b)

$S = (7+24+25)/2 = 28$

Area = $[(28) \times (28-7) \times (28-24) \times (28-25)]^{1/2} = [(28) \times (21) \times (4) \times (3)]^{1/2}$

$= [(7 \times 4) \times (7 \times 3) \times (4) \times (3)]^{1/2} = 7 \times 4 \times 3 = 84 \text{ cm}^2$

Q19. A square and an equilateral triangle have equal perimeters. If the diagonal of the square is $12\sqrt{2}$ cm, then area of the of triangle is:

(a) $24\sqrt{2} \text{ cm}^2$

(b) $24\sqrt{3} \text{ cm}^2$

(c) $48\sqrt{3} \text{ cm}^2$

(d) $64\sqrt{3} \text{ cm}^2$

Sol. (b)

Let side of square = a

Using Pythagoras theorem, $2a^2 = 144 \times 2 \Rightarrow a = 12 \text{ cm}$

Perimeter of equilateral $\Delta = 4 \times 12 = 48$

Side of equilateral Δ (b) = $48/3 = 16 \text{ cm}$

Area of equilateral $\Delta = (\sqrt{3}/4)b^2 = (\sqrt{3}/4)(16)^2 = 64\sqrt{3} \text{ cm}^2$

Q20. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 15 cm, 14 cm and 13 cm, and the parallelogram stands on the base 15 cm, find the height of the parallelogram.

Sol. Given, Area of Triangle = Area of the Parallelogram and both have same base = 15 cm.

Sides of the triangle are: a = 15 cm, b = 14 cm, c = 13 cm

Semi-perimeter $s = (15+14+13)/2 = 42/2 = 21 \text{ cm}$

Area of $\Delta = [(21 \times (21-15) \times (21-14) \times (21-13))]^{1/2}$

$= (21 \times 6 \times 7 \times 8)^{1/2}$

$= (3 \times 7 \times 2 \times 3 \times 7 \times 2 \times 4)^{1/2} = 3 \times 7 \times 2 \times 2 = 84 \text{ cm}^2$

\therefore Area of parallelogram = base \times height = 84

\Rightarrow Height = $84/15 = 5.6 \text{ cm}$