

Class: 9
Subject: Mathematics
Topic: Triangle
No. of Questions: 20

Q1 The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol. Suppose the measures of four angles are $3x$, $5x$, $9x$, and $13x$.

$$\therefore 3x + 5x + 9x + 13x = 360^\circ \quad [\text{Angle sum property of a quadrilateral}]$$

$$\Rightarrow 30x = 360^\circ$$

$$\Rightarrow x = \frac{360^\circ}{30} = 12^\circ$$

$$\Rightarrow 3x = 3 \times 12^\circ = 36^\circ$$

$$5x = 5 \times 12^\circ = 60^\circ$$

$$9x = 9 \times 12^\circ = 108^\circ$$

$$13x = 13 \times 12^\circ = 156^\circ$$

\therefore the angles of the quadrilateral are 36° , 60° , 108° and 156°

Q2. If the diagonal of a parallelogram are equal, then show that it is a rectangle.

Sol. Given: ABCD is a parallelogram in which $AC = BD$.

To Prove: ABCD is a rectangle.

Proof: In $\triangle ABC$ and $\triangle BAD$

$$AB = AB \quad [\text{Common}]$$

$$BC = AD \quad [\text{Opposite sides of a parallelogram}]$$

$$AC = BD \quad [\text{Given}]$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{SSS congruence}]$$

$$\angle ABC = \angle BAD \quad \text{--- (i)} \quad [\text{CPCT}]$$

Since, ABCD is a parallelogram, thus,

$$\angle ABC + \angle BAD = 180^\circ \quad \text{--- (ii)} \quad [\text{Consecutive interior angles}]$$

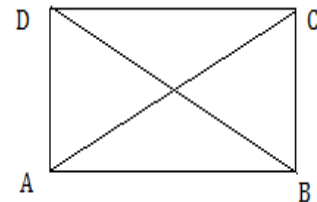
$$\angle ABC + \angle ABC = 180^\circ$$

$$\therefore 2\angle ABC = 180^\circ \quad [\text{from (i) and (ii)}]$$

$$\Rightarrow \angle ABC = \angle BAD = 90^\circ$$

This shows that ABCD is a parallelogram one of whose angle is 90° .

Hence, ABCD is a rectangle



Q3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Sol. Given: A quadrilateral ABCD, in which diagonals AC and BD bisect each other at right angles
 To prove: ABCD is a rhombus.

As shown in the figure

Proof: In $\triangle AOB$ and $\triangle BOC$

$$AO = OC \quad [\text{Diagonals AC and BD bisect each other}]$$

$$\angle AOB = \angle COB \quad [\text{Each} = 90^\circ]$$

$$BO = BO \quad [\text{Common}]$$

$$\therefore \triangle AOB \cong \triangle BOC \quad [\text{SAS Congruence}]$$

$$AB = BC \quad \text{---(i) [CPCT]}$$

Since, ABCD is a quadrilateral in which

$$AB = BC \quad \text{---[From (i)]}$$

Hence, ABCD is a rhombus. [\because if the diagonals of a quadrilateral bisect each other, then it is a parallelogram and equal]



Q4. Show that the diagonals of a square are equal and bisect each other at right angles.

Sol. Given: ABCD is a square in which AC and BD are diagonals.

To Prove: $AC = BD$ and AC bisects BD at right angles, i.e., $AC \perp BD$. $AO = OC$, $OB = OD$

As shown in the figure

Proof: In $\triangle ABC$ and $\triangle BAD$,

$$AB = AB \quad [\text{Common}]$$

$$BC = AD \quad [\text{Sides of a square}]$$

$$\angle ABC = \angle BAD = 90^\circ \quad [\text{Angles of a square}]$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{SAS congruence}]$$

$$\Rightarrow AC = BD \quad [\text{CPCT}]$$

Now, in $\triangle AOB$ and $\triangle COD$,

$$AB = DC \quad [\text{Sides of a square}]$$

$$\angle AOB = \angle COD \quad [\text{Vertically opposite angle}]$$

$$\angle OAB = \angle OCD \quad [\text{Alternate angles}]$$

$$\therefore \triangle AOB \cong \triangle COD \quad [\text{AAS congruence}]$$

$$\angle AO = \angle OC \quad [\text{CPCT}]$$

Similarly by taking $\triangle AOD$ and $\triangle BOC$, we can show that $OB = OD$.

$$\text{In } \triangle ABC, \angle BAC + \angle BCA = 90^\circ \quad [\because \angle B = 90^\circ]$$

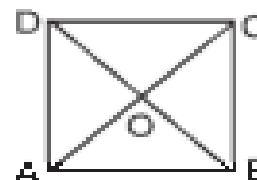
$$\Rightarrow 2\angle BAC = 90^\circ \quad [\angle BAC = \angle BCA \text{ as } BC = AD]$$

$$\Rightarrow \angle BCA = 45^\circ \text{ or } \angle BCO = 45^\circ$$

$$\text{Similarly } \angle CBO = 45^\circ$$

In $\triangle BCO$.

$$\angle BCO + \angle CBO + \angle BOC = 180^\circ$$



$\Rightarrow 90^\circ + \angle BOC = 180^\circ$
 $\Rightarrow \angle BOC = 90^\circ$
 $\Rightarrow BO \perp OC \Rightarrow BO \perp AC$
 Hence, $AC = BD$, $AC \perp BD$, $AO = OC$ and $OB = OD$.

Q5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Sol. Given: A quadrilateral ABCD, in which diagonals AC and BD are equal and bisect each other at right angles.

To Prove: ABCD is a square.

As shows in the figure

Proof: Since ABCD is a quadrilateral whose diagonals bisect each other, so it is a parallelogram. Also, its diagonals bisect each other at right angles, therefore, ABCD is a rhombus.

$\Rightarrow AB = BC = CD = DA$ [Sides of a rhombus]

In $\triangle ABC$ and $\triangle BAD$, we have

$AB = AB$ [Common]

$BC = AD$ [Sides of a rhombus]

$AC = BD$ [Given]

$\therefore \triangle ABC \cong \triangle BAD$ [SSS congruence]

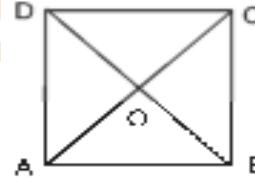
$\therefore \angle ABC = \angle BAD$ [CPCT]

But, $\angle ABC + \angle BAD = 180^\circ$ [Consecutive interior angles]

$\angle ABC = \angle BAD = 90^\circ$

$\angle A = \angle B = \angle C = \angle D = 90^\circ$ [Opposite angles of a \parallel^m]

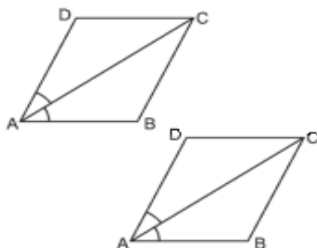
\Rightarrow ABCD is a rhombus whose angles are of 90° each.



Q6. Diagonals AC of a parallelogram ABCD bisect $\angle A$ (see Fig.) Show that

(a) It bisects $\angle C$ also.

(b) ABCD is a rhombus.



Sol. Given: A parallelogram ABCD, in which diagonal AC bisects $\angle A$, i.e., $\angle DAC = \angle BAC$

To prove: (a) Diagonal AC bisects $\angle C$ i.e., $\angle DCA = \angle BCA$

(b) ABCD is a rhombus.

Proof: (a) $\angle DAC = \angle BCA$ [Alternative angles]
 $\angle BAC = \angle DCA$ [Alternative angles]
But, $\angle DAC = \angle BAC$ [Given]
 $\therefore \angle BCA = \angle DCA$

Hence, AC bisects $\angle DCB$

Or, AC bisect $\angle C$

(b) In $\triangle ABC$ and $\triangle CDA$

$AC = AC$ [Common]

$\angle BAC = \angle DAC$ [Given]

And $\angle BCA = \angle DCA$ [Proved above]

$\therefore \triangle ABC \cong \triangle ADC$ [ASA congruence]

$\therefore BC = DC$ [CPCT]

But $AB = DC$ [Given]

$\therefore AB = BC = DC = AD$

Hence, ABCD is a rhombus [\because opposite angles are equal]

Q7. ABCD is a rhombus show that diagonals AC bisect $\angle A$ as well as $\angle C$ and diagonal BD bisect $\angle B$ as well as $\angle D$

Sol. Given: ABCD is a rhombus, i.e., $AB = BC = CD = DA$.

To Prove: $\angle DAC = \angle BAC$,

$\angle BCA = \angle DCA$

$\angle ADB = \angle CDB, \angle ABD = \angle CBD$

As shows in the figure

Proof: In $\triangle ABC$ and $\triangle CDA$, we have

$AB = AD$ [Sides of a rhombus]

$AC = AC$ [Common]

$BC = CD$ [Sides of a rhombus]

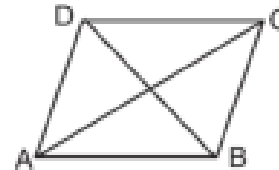
$\triangle ABC \cong \triangle ADC$ [SSS congruence]

So, $\angle DAC = \angle BAC$

$\angle BCA = \angle DCA$ [CPCT]

Similarly, $\angle ADB = \angle CDB$ and $\angle ABD = \angle CBD$.

Hence, diagonals AC bisect $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.



Q8. ABCD is a rectangle in which diagonals AC bisect $\angle A$ as well as $\angle C$. Show that: (i) ABCD is a square (ii) diagonals BD bisect $\angle B$ as well as $\angle D$.

Sol. Given: ABCD is a rectangle in which diagonal AC bisect $\angle A$ as well as $\angle C$

To Prove: (i) ABCD is a square. (ii) Diagonals BD bisect $\angle B$ as well as $\angle D$

As shows in the figure

Proof: (i) In $\triangle ABC$ and $\triangle ADC$, we have

$$\angle BAC = \angle DAC \quad [\text{Given}]$$

$$\angle BCA = \angle DCA \quad [\text{Given}]$$

$$AC = AC \quad [\text{Common}]$$

$$\therefore \triangle ABC \cong \triangle ADC \quad [\text{ASA congruence}]$$

$$\therefore AB = AD \text{ and } CB = CD \quad [\text{CPCT}]$$

But, in a rectangular opposite sides are equal,

i.e., $AB = DC$ and $BC = AD$

$$\therefore AB = BC = CD = DA$$

Hence, ABCD is a square

(ii) In $\triangle ABD$ and $\triangle ACB$, we have

$$AD = CD \quad [\text{Sides of square}]$$

$$AB = CB \quad [\text{Sides of a square}]$$

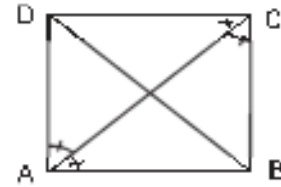
$$BD = BD \quad [\text{Common}]$$

$$\therefore \triangle ABD \cong \triangle ACB \quad [\text{SSS congruence}]$$

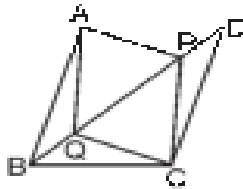
$$\text{So, } \angle ABD = \angle CBD \quad [\text{CPCT}]$$

$$\angle ADB = \angle CDB \quad [\text{CPCT}]$$

Hence, diagonals BD bisect $\angle B$ as well as $\angle D$



- Q9. In parallelogram ABCD, two points P and Q are taken on diagonals BD such that $DP = BQ$ (See Fig.) Show that:



- (i) $\triangle APD \cong \triangle CQB$
- (ii) $AP = CQ$
- (iii) $\triangle AQB \cong \triangle CPD$
- (iv) $AQ = CP$
- (v) APCQ is a parallelogram

- Sol. Given: ABCD is a parallelogram and P and Q are points on diagonals BD such that $DP = BQ$.

To prove:

- (i) $\triangle APD \cong \triangle CQB$
- (ii) $AP = CQ$
- (iii) $\triangle AQB \cong \triangle CPD$

- (iv) $AQ = CP$
(v) APCQ is a parallelogram

Proof:

(i) In $\triangle APD$ and $\triangle CQB$, we have

$$\begin{aligned} AD &= BC && \text{[Opposite sides of a || gm]} \\ DP &= BQ && \text{[Given]} \\ \angle ADP &= \angle CBQ && \text{[Alternative angles]} \\ \therefore \triangle APD &\cong \triangle CQB && \text{[SAS congruence]} \end{aligned}$$

(ii) $\therefore AP = CQ$ [CPCT]

(iii) In $\triangle AQB$ and $\triangle CDP$, we have

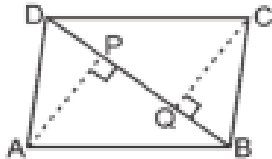
$$\begin{aligned} AB &= CD && \text{[Opposite sides of a || gm]} \\ DP &= BQ && \text{[Given]} \\ \angle ABQ &= \angle CDP && \text{[Alternative angles]} \\ \therefore \triangle AQB &\cong \triangle CDP && \text{[SAS congruence]} \end{aligned}$$

(iv) $\therefore AQ = CP$ [CPCT]

(v) Since in APCQ, opposite sides are equal, therefore it is a parallelogram.

Q10. ABCD is a parallelogram and AP and CQ are perpendicular from vertices A and C on diagonals BD (see Fig.) Show that

- (i) $\triangle APB \cong \triangle CQD$
(ii) $AP = CQ$



Sol. Given: ABCD is a parallelogram and AP and CQ are perpendicular from vertices A and C on BD.

To Prove:

- (i) $\triangle APB \cong \triangle CQD$
(ii) $AP = CQ$

Proof:

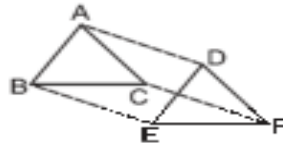
(i) In $\triangle APB$ and $\triangle CQD$, we have

$$\begin{aligned} \angle ABP &= \angle CDQ && \text{[Alternate angle]} \\ AB &= CD && \text{[Opposite sides of a parallelogram]} \\ \angle APB &= \angle CQD && \text{[Each = } 90^\circ \text{]} \end{aligned}$$

$$\therefore \triangle APB \cong \triangle CQD \quad \text{[ASA congruence]}$$

(ii) So, $AP = CQ$ [CPCT]

Q11. In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig.) Show that



- (i) Quadrilateral ABED is a parallelogram
- (ii) Quadrilateral BEFC is a parallelogram
- (iii) $AD \parallel CF$ and $AD = CF$
- (iv) Quadrilateral ACFD is a parallelogram
- (v) $AC = DF$
- (vi) $\triangle ABC \cong \triangle DEF$

Sol. Given: In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F.

To Prove:

- (i) Quadrilateral ABED is a parallelogram
- (ii) Quadrilateral BEFC is a parallelogram
- (iii) $AD \parallel CF$ and $AD = CF$
- (iv) Quadrilateral ACFD is a parallelogram
- (v) $AC = DF$
- (vi) $\triangle ABC \cong \triangle DEF$

Proof: (i) In quadrilateral ABED, we have

$$AB = DE \text{ and } AB \parallel DE, \quad [\text{Given}]$$

\Rightarrow ABED is a parallelogram. [One pair of opposite sides is parallel and equal]

(ii) In quadrilateral BEFC, we have

$$BC = EF \text{ and } BC \parallel EF \quad [\text{Given}]$$

\Rightarrow BEFC is a parallelogram. [One pair of opposite sides is parallel and equal]

(iii) $BE = CF$ and $BE \parallel BECF$ [BEFC is Parallelogram]

$$AD = BE \text{ and } AD \parallel BE \quad [\text{ABED is a parallelogram}]$$

\Rightarrow $AD = CF$ and $AD \parallel CF$

(iv) ACFD is a parallelogram [One pair of opposite sides is parallel and equal]

(v) $AC = DF$ [Opposite sides of parallelogram ACFD]

(vi) In $\triangle ABC$ and $\triangle DEF$, we have

$$AB = DE \quad [\text{Given}]$$

$$BC = EF \quad [\text{Given}]$$

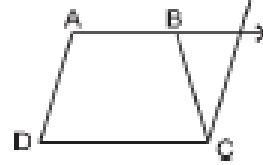
$$AC = DF \quad [\text{Prove above}]$$

$\therefore \triangle ABC \cong \triangle DEF$ [SSS axiom]

Q12. ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$ (see Fig.).

Show that

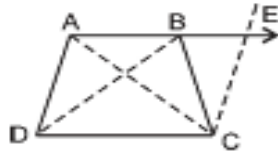
- (i) $\angle A = \angle B$
- (ii) $\angle C = \angle D$
- (iii) $\triangle ABC \cong \triangle BAD$
- (iv) Diagonal $AC = BD$



Sol. Given: In trapezium ABCD, $AB \parallel CD$ and $AD = BC$.

To Prove:

- (i) $\angle A = \angle B$
- (ii) $\angle C = \angle D$
- (iii) $\triangle ABC \cong \triangle BAD$
- (iv) Diagonal $AC = BD$



Construction: Join AC and BD. Extend AB and draw a line through C parallel to AD meeting AB produced at E.

Proof:

- (i) Since $AB \parallel DC$
 $\Rightarrow AE \parallel DC$... (i)
 And $AD \parallel CE$... (ii) [Construction]
 $\Rightarrow ADCE$ is parallelogram [Opposite pairs of sides are parallel]
 $\angle A + \angle E = 180^\circ$... (iii) [Consecutive interior angles]
 $\angle B + \angle CBE = 180^\circ$... (iv) [Linear pair]
 $AD = CE$... (v) [Opposite sides of a \parallel^m]
 $AD = BC$... (vi) [Given]
 $\Rightarrow BC = CE$ [From (v) and (vi)]
 $\Rightarrow \angle E = \angle CBE$ (vii) [Angles opposite to equal sides]
 $\therefore \angle B + \angle E = 180^\circ$... (viii) [From (iv) and (vii)]

Now from (iii) and (viii) we have

$$\begin{aligned} \angle A + \angle E &= \angle B + \angle E \\ \Rightarrow \angle A &= \angle B \end{aligned}$$

- (ii) $\angle A + \angle D = 180^\circ$ [Consecutive interior angles]
 $\angle B + \angle C = 180^\circ$ [Consecutive interior angles]
 $\Rightarrow \angle A + \angle D = \angle B + \angle C$ [$\because \angle A = \angle B$]
 $\Rightarrow \angle D = \angle C$
 Or $\angle C = \angle D$
- (iii) In $\triangle ABC$ and $\triangle BAD$, We have
 $AD = BC$ [Given]

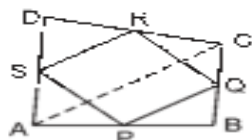
$$\angle A = \angle B \quad [\text{Proved}]$$

$$AB = CD \quad [\text{Common}]$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{ASA congruence}]$$

$$(iv) \quad \text{Diagonal } AC = \text{Diagonal } BD \quad [\text{CPCT}]$$

Q13. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides, AB, BC, CD and DA respectively. (See Fig.). AC is a diagonal. Show that:



$$(i) \quad SR \parallel AC \text{ and } SR = \frac{1}{2} AC$$

$$(ii) \quad PQ = SR$$

$$(iii) \quad PQRS \text{ is a parallelogram.}$$

Sol. Given: ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA. AC is diagonal.

To Prove:

$$(i) \quad SR \parallel AC \text{ and } SR = \frac{1}{2} AC$$

$$(ii) \quad PQ = SR$$

$$(iii) \quad PQRS \text{ is a parallelogram.}$$

Proof: (i) In $\triangle ABC$, P is the mid-point of AB and Q is the mid-point of BC.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots (1) \quad [\text{Mid-point theorem}]$$

In $\triangle ADC$, R is the mid-point of CD and S is the mid-point of AD

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots (2) \quad [\text{Mid-point theorem}]$$

(ii) From (1) and (2), we get

$$PQ \parallel SR \text{ and } PQ = SR$$

(iii) Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is equal and parallel.

$$\therefore PQRS \text{ is a parallelogram.}$$

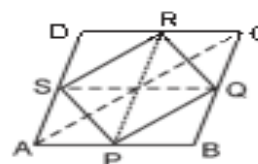
Q14. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Sol. Given: ABCD is a rhombus in which P, Q, R and S are mid points of sides AB, BC, CD and DA respectively:

To Prove: PQRS is a rectangle.

Construction: Join AC, PR and SQ.

Proof: In $\triangle ABC$



P is mid-point of AB [Given]
Q is mid-point of BC [Given]
 $\Rightarrow PQ \parallel AC$ and $PQ = \frac{1}{2} AC$... (i) [Mid-point theorem]
Similarly, in $\triangle DAC$,
 $SR \parallel AC$ and $SR = \frac{1}{2} AC$... (ii)
From (i) and (ii), we have $PQ \parallel SR$ and $PQ = SR$
 $\Rightarrow PQRS$ is a parallelogram [One pair of opposite sides is parallel and equal]
Since $ABQS$ is a parallelogram
 $\Rightarrow AB = SQ$ [Opposite sides of a \parallel gm]
Similarly, since $PBCR$ is a parallelogram.
 $\Rightarrow BC = PR$
Thus, $SQ = PR$ [AB = BC]
Since SQ and PR are diagonal of parallelogram $PQRS$, which are equal.
 $\Rightarrow PQRS$ is a rectangle.

Q15. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that quadrilateral PQRS is a rhombus.

Sol. Given: A rectangle ABCD in which P, Q, R, S are the mid-points of AB, BC, CD and DA respectively, PQ, QR, RS and SP are joined.

To prove: PQRS is a rhombus.

Construction: Join AC

Proof: In $\triangle ABC$, P and Q are mid-points of the sides AB and BC.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots (i) \quad [\text{Mid-point theorem}]$$

Similarly, in $\triangle ADC$,

$$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots (ii)$$

From (i) and (ii), we get

$$PQ \parallel SR \text{ and } PQ = SR \quad \dots (iii)$$

Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is parallel and equal [From (iii)]

$\therefore PQRS$ is a parallelogram.

Now $AD = BC$... (iv) [Opposite sides of a rectangle ABCD]

$$\therefore \frac{1}{2} AD = \frac{1}{2} BC$$

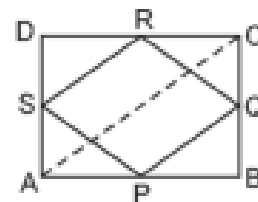
$$\Rightarrow AS = BQ$$

In $\triangle APS$ and $\triangle BPQ$

$$AP = BP \quad [\because P \text{ is the mid-point of } AB]$$

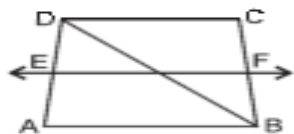
$$AS = BQ \quad [\text{Prove above}]$$

$$\angle PAS = \angle PBQ \quad [\text{Each} = 90^\circ]$$

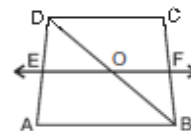


$\triangle APS \cong \triangle BPQ$ [SAS axiom]
 $\therefore PS = PQ$... (v)
 From (iii) and (v), we have
 PQRS is a rhombus.

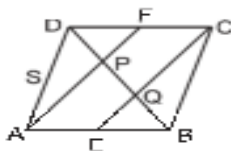
- Q16. ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (See Fig.). Show that F is the mid-point of BC.



Sol. Given: A trapezium ABCD with $AD \parallel DC$, E is the mid-point of AD and $EF \parallel AB$.
 To Prove: $AB \parallel DC$ and $EF \parallel AB$
 Proof: $AB \parallel DC$ and $EF \parallel AB$
 $\Rightarrow AB, EF$ and DC are parallel.
 Intercepts made by parallel lines AB, EF and DC on transversal AD are equal.
 \therefore Intercepts made by those parallel lines on transversal BC are also equal.
 i.e., $BF = FC$
 $\Rightarrow F$ is the mid-point of BC .



- Q17. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig.). Show that the line segments. AF and EC trisect the diagonals BD.



Sol. Given: A parallelogram ABCD, in which E and F are mid-points of sides AB and DC respectively.
 To prove: $DP = PQ = QB$
 Proof: Since E and F are mid-point of AB and DC respectively
 $\Rightarrow AE = \frac{1}{2} AB$ and $CF = \frac{1}{2} DC$... (i)
 But, $AB = DC$ and $AB \parallel DC$... (ii) [Opposite sides of parallelogram]
 $\therefore AE = CF$ and $AE \parallel CF$.
 $\Rightarrow AECF$ is a parallelogram. [One pair of opposite sides is parallel and equal]

In $\triangle BAP$,
E is the mid-point of AB
 $EQ \parallel AP$
 $\Rightarrow Q$ is mid-point of PB [Converse of mid-point theorem]
 $\Rightarrow PQ = QB$... (iii)
Similarly, In $\triangle DQC$,
P is the mid-point of DQ
 $DP = PQ$... (iv)
From (iii) and (iv), we have
 $DP = PQ = QB$
Or line segments AF and EC trisect the diagonal BD.

Q18. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Sol. Given: ABCD is a quadrilateral in which EG and FH are the line segments joining the mid-points of opposite sides.

To Prove: EG and FH bisect each other.

Construction: Join EF, FG, GH, HE and AC.

Proof: In $\triangle ABC$, E and F are mid-points of AB and BC respectively.

$$\therefore EF = \frac{1}{2} AC \text{ and } EF \parallel AC \quad \dots (i)$$

In $\triangle ADC$, H and G are mid-points of AD and CD respectively.

$$\therefore HG = \frac{1}{2} AC \text{ and } HG \parallel AC \quad \dots (ii)$$

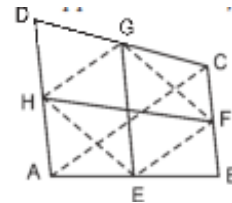
From (i) and (ii), we get

$$EF = HG \text{ and } EF \parallel HG$$

\therefore EFGH is a parallelogram [∵ a quadrilateral is a parallelogram if its one pair of opposite sides is equal and parallel]

Now, EG and FH are diagonals of the parallelogram EFGH.

$$\therefore EG \text{ and } FH \text{ bisect each other} \quad [\text{Diagonals of a parallelogram bisect each other}]$$



Q19. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC.

(ii) $MD \perp BC$

(iii) $CM = MA = \frac{1}{2} AB$

Sol. Given: A triangle ABC, in which $\angle C = 90^\circ$ and M is the mid-point of B and BC \parallel DM.

To Prove:

- (i) D is the mid-point of AC.
- (ii) $MD \perp BC$
- (iii) $CM = MA = \frac{1}{2} AB$

Construction: Join CM.

Proof: (i) In $\triangle ABC$,

M is the mid-point of AB

[Given]

$BC \parallel DM$

[Given]

D is the mid-points of AC

[Converse of mid-point theorem]

(ii) $\angle ADM = \angle ACB$

[\because Corresponding angles]

But $\angle ACB = 90^\circ$

[Given]

$\therefore \angle ADM = 90^\circ$

But $\angle ADM + \angle CDM = 180^\circ$ [Linear pair]

$\therefore \angle CDM = 90^\circ$

Hence, $MD \perp AC$

(iii) $AD = DC$... (i)

[\because D is the mid-point of AC]

Now, in $\triangle ADM$ and $\triangle CMD$, we have

$\angle ADM = \angle CDM$

[Each = 90°]

$AD = DC$

[From (i)]

$DM = DM$

[Common]

$\therefore \triangle ADM \cong \triangle CMD$

[SAS congruence]

$\Rightarrow CM = MA$

... (ii) [CPCT]

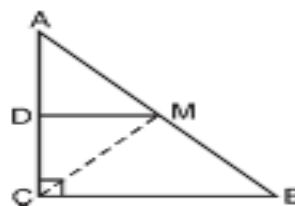
Since M is mid-point of AB,

$\therefore MA = \frac{1}{2} AB$

... (iii)

Hence, $CM = MA = \frac{1}{2} AB$

[From (ii) and (iii)]

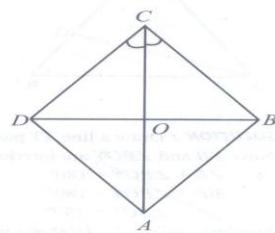


Q20. In a quadrilateral ABCD, AC bisect $\angle C$ and $BC = CD$.

Prove that:

(i) $AB = AD$

(ii) AC is the perpendicular bisect of BD.



Sol. (i) In triangle ADC and ABC,

$CD = CB$

[Given]

$CA = CA$

[Common]

$$\angle DCA = \angle BCA \quad [\text{Given}]$$

$$\therefore \triangle DCA \cong \triangle BCA \quad [\text{By SAS rule of congruency}]$$

$$\therefore AD = AB$$

(ii) In triangle DCO and BCO

$$CD = CB \quad [\text{Proved in (i) above}]$$

$$CO = CO \quad [\text{common}]$$

$$\angle DCO = \angle BCO \quad [\text{Given}]$$

$$\therefore \triangle DCO \cong \triangle BCO$$

Since $\angle COD$ and $\angle COB$ are linear pair angles, therefore $\angle DOC + \angle BOC = 180^\circ$

$$\Rightarrow \angle COD + \angle COB = 180^\circ$$

$$\Rightarrow \angle COD = 90^\circ$$

\therefore CO is perpendicular to DB

\therefore CA is perpendicular to DB

askITians