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Class: 9
Subject: Mathematics
Topic: Surface Area and Volume of solids
No. of Questions: 20

Q1. A Sphere is just enclosed inside a cube of volume 60 cm^2. Find the volume of the sphere.

Sol. The volume of a sphere is of radius r = $\frac{4}{3}\pi r^3$

It is fits exactly within a cube, this means the length/ height of the cube is the same as the diameter of the sphere i.e. 2r

The volume of a cube of side 2r = $(2r)^3 = 8r^3$

$8r^3 = 60\ cm^3$

This means r = 7.5

Putting this in the formula for the volume of the sphere, we get the volume = 10π cm^3

Q2. A sphere and a cone have the same radii. If the volume of the sphere is double of the volume of the cone. Find the ratio of the con's height and radius.

Sol. The volume of a cone is of radius r and height h = $1/3\ \pi r^2 h$

The volume of a sphere is of radius r = $4/3\ \pi r^3$

We know the volume of the sphere is double of the volume of the cone = $4/3\ \pi r^3 = 2 \times (1/3\pi\ r^2 h)$

= h/ r = 2 : 1

Q3. If the radius of a hemisphere is 4x, find its curved surface area.

Sol. The surface area of a sphere of radius x is given by $4\pi x^2$

The curved surface area of a hemisphere is half of that i.e. $2\pi x^2$

Here the radius is specified as 4x. Substituting this into the formula, we get the answer is $(2\pi)x(4x)^2$

This gives us the answer $32\pi x^2$

Q4. A cone made completely of metal (i.e. it is not hollow) has a base radius of 7 cm, and height of 28 cm. If we melt it and recast it into a sphere, what will be the radius of sphere?

Sol. The volume of a cone is of radius r and height h = $\frac{1}{3}\pi r^2 h$

The volume of a sphere of radius x = $\frac{4}{3}\pi x^3$

We know that the cone of base radius 7 cm and height 28 was melted down

The volume of metal resulting from this = $\frac{1}{3}\pi 7^2 28$

When we recast it into a sphere, we get a sphere of this volume

This is to say $\frac{4}{3}\pi x^3 = \frac{1}{3}\pi 7^2 28$

Solving for x, we get x = 7 cm

Q5. Find the surface area of the biggest sphere which can fit inside a cube of side 6a.

Sol. The biggest sphere that can fit inside a cube of side 6a will have a diameter of 6a (anything larger will not fit in, as opposite sides are separated by a distance of 6a)

This means that the radius of this sphere is $\frac{1}{2}$ 6a

The surface area of a sphere of radius x is $4\pi x^2$

Therefore the surface area of this sphere is $4\pi\left(\frac{1}{2}\, 6a\right)^2$

The answer is $36\pi a^2$

Q6. If radius of a hemisphere is 4b, find its volume.

(a) $2/3\ \pi b^3$

(b) $16/3\ b^3$

(c) $128/3\ \pi b^3$

(d) $18\ \pi b^3$

Sol. (c)

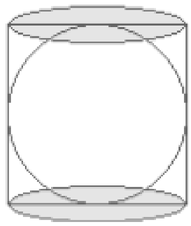
The volume of a hemisphere of radius x is given by $(4/3)\pi x^3$

The volume of hemisphere is half that i.e. $(2/3)\ \pi x^3$

Here the radius is specified as 4b. Substituting this into the formula, we get the answer is $(2/3\pi) \times (4b)^3$

This gives us the answer $128/3\ \pi b^3$

Q7. A sphere is just enclosed inside a right circular cylinder. If surface area of sphere is 180 cm^2, find total surface area of cylinder.



(a) 540 cm^2
(b) 360 cm^2
(c) 270 cm^2
(d) 135 cm^2

Sol. (c)

There are three equations we need to know this type of question – the total area of a cylinder, the curved area of a cylinder, and the surface area of sphere

The curved surface area of a cylinder of radius ‘r’ and height ‘h’ is $2\pi rh$. Here we know the sphere will fit in exactly in the cylinder, so $h = 2r$, and the formula now becomes $4\pi r^2$

The total surface area of the same cylinder will be the sum of the curved area and the surface area of the two circles at top and bottom. So $4\pi r^2 + 2\pi r^2 = 6\pi r^2$

And of course, the sphere will have the radius r too, so it’s surface area is $4\pi r^2$

From these equations, we see that for this case, the surface area of the sphere is the same as the curved surface are of the cylinder, and 2/3 of the total surface area of the cylinder

Here we know that surface area of sphere is 180 cm^2, and need to find total surface area of cylinder

Substituting from the equations above, we get total surface area of cylinder = 270 cm^3

Q8. If radiuses of two hemispheres are in ratio 5:2, find the ratio of their volumes.

(a) 25 : 4
(b) 125 : 8
(c) 4 : 25
(d) 8 : 125

Sol. (b)

The volume of a hemispheres of radius x is given by $(4/3)\pi x^3$

The volume of a hemisphere is half that i.e. $(2/3)\ \pi x^3$

We see that the volume is proportional to the 3^{rd} power of the radius

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To see this more clearly, assume the radii of these two hemispheres are 5x and 2x (not that this allows us to get the ratio of 5 : 2, which is the only thing we know about these radii)
The volume of the first one then is 2/3 $5x^3$, and the volume of the second one is $2/3\pi 2x^3$
The ratio of the volumes is therefore $2/3\pi(5x)^3$. $2/3\pi(2x)^3$
This simplifies to $(5x)^3 : (2x)^3$, and further to 5^3 . 2^3
Therefore the answer is 125: 8

Q9. Find the volume of the biggest hemisphere, which can fit in a cube of side 8a.
(a) $2/3\ \pi\ a^3$
(b) $16/3\ \pi\ a^3$
(c) $128/3\ \pi\ a^3$
(d) $18\ \pi\ a^3$

Sol. (c)
The biggest hemisphere that can fit inside a cube of side 8a will have a diameter of 8a (anything larger will not fit in, as opposite sides are separated by a distance of 8a).
This means that the radius of this sphere is (1/2)8a
The volume of a hemisphere of radius x is $(2/3)\pi x^3$
Therefore the volume of this hemisphere is $(2/3)\pi\ ((1/2)8a)^3$
Solving for this gives us $128/3\ \pi a^3$

Q10. If a cylinder and hemisphere stands on equal bases, and have the same height, Find the ratio of their volumes.
(a) 2 : 3
(b) 3 : 1
(c) 3 : 2
(d) 2: 1

Sol. (c)
The volume of a cylinder of radius ‘r’ and height ‘h’ is $\pi r^2 h$.
The volume of a hemisphere of radius ‘r’ is $\frac{2}{3}\pi r^3$.
From these equations we can cancel out the equal terms (remember the heights are also equal) to find the ratio as 3 : 2

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Q11. An sphere is expanded to a bigger sphere such that its volume increases by a factor of 64, Find the change in its radius.

(a) 64 times
(b) 16 times
(c) 4 times
(d) None of these

Sol. (c)

The volume of a sphere of radius x = $\frac{4}{3}\pi x^3$

The surface area of a sphere of radius x = $4\pi x^2$

This means that the surface area will increase as a square of the increase in radius And the volume will increase as a cube of the increase in radius

Here we know that the volume increased by a factor of 64

This means that the radius grew by the cube root of this value i.e. $64^{\frac{1}{3}}$

Solving this, we get 4 times

Q12. Find the volume of the biggest cone that can fit inside a cube of side 5 cm.

(a) $\frac{125\pi}{3}$ cm^3
(b) $\frac{125\pi}{12}$ cm^3
(c) $\frac{25\pi}{12}$ cm^3
(d) $\frac{125\pi}{6}$ cm^3

Sol. (b)

The volume of a cone is of radius r and height h = $\frac{1}{3}\pi r^2 h$

Since we have to fit it inside a cube of side 5 cm, we see that the diameter of the cone will be 5 cm, and the height will be 5 cm (a cone larger than this in the diameter or the height will not fit inside the cube)

So the radius of the cone is $\frac{5}{2}$ = 2.5

Putting these values into the equations of the volume, we get the volume of the cone =

$\frac{1}{3} \times \pi \times \frac{5^2}{2} \times 5$

Solving we get the volume of the cone = $\frac{125\pi}{12}\ cm^3$

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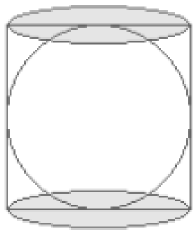
Q13. A sphere is just enclosed inside a right circular cylinder. If volume of the gap between cylinder and sphere is 10 cm^3, find volume of the sphere



(a) 25 cm^3
(b) 10 cm^3
(c) 40 cm^3
(d) 20 cm^3

Sol. (d)

There are three equations we need to know this type of question- the total volume of a cylinder, the volume a sphere, and the remaining volume after the sphere fits in the cylinder

The volume of a cylinder of radius 'r' and height 'h' is $\pi r^2 h$. Here we know the sphere will fit in exactly in the cylinder, so h = 2r, and the formula now becomes $2\pi r^3$

The sphere will have the radius r too (see the figure here), so it's volume is $\frac{4}{3}\pi r^3$

The volume of the gap between the cylinder and the sphere is all the volume inside the cylinder not taken up by the sphere.

This is the difference between the volume of the cylinder and the volume of the sphere.

i.e. volume of the gap = $2\pi r^3 - \frac{4}{3}\pi r^3$

Simplifying, volume of the gap = $\frac{2}{3}\pi r^3$

So we have 3 equations

Volume (cylinder) = $2\pi r^3$

Volume (sphere) = $\frac{4}{3}\pi r^3$

Volume (gap) = $\frac{2}{3}\pi r^3$

Here we know that volume of the gap between cylinder and sphere is 10 cm^3, and need to find volume of the sphere

Substituting from the equation above, we get volume of the sphere = 20 cm^3

Q14. A sphere and a right circular have the same radius. If the volume of the sphere is triple the volume of the cylinder, then what is the ratio of cylinder's height and radius?

(a) 4 : 9
(b) 4 : 3
(c) 3 : 4
(d) 9 : 4

Sol. The volume of a sphere is of radius r = $\frac{4}{3}\ \pi r^3$

The volume of a cylinder of radius 'r' and height 'h' is $\pi r^2 h$.

Here, we are told the volume of the sphere is triple times the volume of the cylinder

So $\frac{4}{3}\ \pi r^3 = 3 \times (\pi r^2 h)$

Cancelling out the terms, we get 9 × h = 4 ×r

Therefore, the ratio of the cylinder's height to radius is 9 : 4

Q15. The radius of a cylinder is halved and the height is tripled. What is the area of the curved surface now compared to the previous surface area?

(a) Two times
(b) Three times
(c) 1.5 times
(d) Same

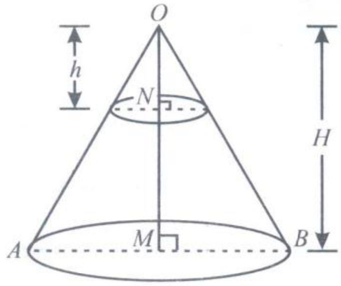
Sol. The curved surface area of a cylinder is 2πrh

Here we halved the radius and tripled the height

Putting this into the formula, we see that the curved surface area becomes 1.5 times

Q16. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume be 1/27 of the volume of the give cone, at which height above the base is the section cut?

Sol. Let OAB be the given cone of height, H = 30 cm, and base radius R cm. Let this cone be cut by the plane CND to obtain the cone OCD with height h cm and base radius r cm



Then, ΔOND ~ ΔOMB

So, $\frac{ND}{NB} = \frac{ON}{OM} \Rightarrow \frac{r}{R} = \frac{h}{30}$ ...(i)

Volume of OCD $= \frac{1}{3} \times Volume\ of\ cone\ OAB$ (Given)

$\Rightarrow \frac{1}{3}\ \pi r^2 h = \frac{1}{27} \times \frac{1}{3} \pi R^2 \times 30 \Rightarrow \left(\frac{r}{R}\right)^2 = \frac{10}{9h} \Rightarrow \left(\frac{h}{30}\right)^2 = \frac{10}{9h}$ (From (i))

$\Rightarrow 9h^3 = 9000 \Rightarrow 1000 \Rightarrow h = 10$

∴ Height of the cone OCD = 10 cm.

Hence, the section is cut at the height of (30-10)cm, i.e. 20 cm from the base.

Q17. The surface area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate (i) the radius of the solid sphere, (ii) the number of cones recast (Take π = 3.14)

Sol. (i) Let the radius of the sphere be r cm.

Then, its surface area = $(4\pi r^2)\ cm^2$

∴ $4\pi r^2 = 1256 \Rightarrow 4 \times 3.14 \times r^2 = 1256$

$\Rightarrow r^2 = \frac{1256}{4\times3.14} = \frac{1256}{12.56} = 100 \Rightarrow r = 10$

Hence, the radius of the sphere = 10 cm.

(ii) Volume of the sphere = $\frac{4}{3}\ \pi r^3 = \left[\frac{4}{3}\ \pi \times (10)^3\right] cm^3 = \left(\frac{4000}{3}\pi\right)\ cm^3$

Volume of a cone = $\frac{1}{3}\ \pi^2 h = \left[\frac{1}{3}\ \pi \times (2.5)^2 \times 8\right] cm^3 = \left(\frac{50}{3}\ \pi\right)\ cm^3$

∴ Number of cones recast = $\frac{Volume\ of\ the\ sphere}{Volume\ of\ 1\ sphere} = \left(\frac{4000}{3}\pi \times \frac{3}{50\pi}\right) = 80$

Q18. An ice-cream cone is the union of a right circular cone and a hemisphere that has the same circular base the cone. Find the volume office cream if the height of the cone is 9 cm and radius of its base is 2.5 cm.

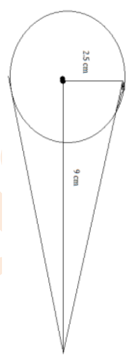
Sol. Given height of the cone = 9 cm.

And radius of the base = 2.5 cm = $\frac{5}{2}$ cm.

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9 = \frac{11 \times 25 \times 3}{7 \times 2}\ cm^3 = \frac{825}{14}\ cm^3$

Volume of the hemisphere = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{11 \times 125}{42} = \frac{1375}{42}\ cm^3$

Volume of the ice cream cone = $\frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42} = \frac{3850}{42}\ cm^3 = 91.66\ cm^3$



Q19. Water in a canal, 30 dm wide and 12 dm deep, is flowing with a velocity of 20 km per hour. How much area will it irrigate in 30 min, if 9 cm of standing water is desired?

Sol. Water in the canal forms a cuboid of breadth = 12 dm = 12/10 m = 1.2 m, height = 30 dm = 30/10 = 3 m and, Length = Distance covered by water in 30 minutes

= Velocity of water × time

$= 20000 \times \frac{30}{60}$ m = 10000 m

∴ Volume of water flown in 30 min = lbh = (1.2 ×3 ×10000) m^3

Suppose area irrigated be Am^2. Then,

$\Rightarrow A \times \frac{9}{100} = 36000 \Rightarrow A = 400000 \text{ m}^2$

Hence, area irrigated = 400000m^2

Q20. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled into it. The diameter pencil is 7 mm, the diameter of the graphite is 1 mm and the length of the pencil is 14 cm. Find the: (i) Volume of the graphite (ii) Volume of the wood (iii) The weight of the whole pencil, if the specific gravity of the wood is 0.7 gm/cm^3 and that of the graphite is 2.1 gm/cm^3.

Sol. (i) We have,

Diameter of the graphite cylinder = 1 mm = $\frac{1}{10}$ cm

$\therefore$ Radius of the graphite cylinder $= \frac{1}{20}$ cm

Length of the graphite cylinder = 14 cm

V_1 = Volume of the graphite cylinder $= \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 14\ cm^3 = 0.11\ cm^3$

(ii) We have,

Diameter of pencil = 7mm $= \frac{7}{10}\ cm$

Radius $= \frac{7}{20}$

∴ V_2 = Volume of pencil $= \frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 14\ cm^3 = 5.39\ cm^3$

Volume of wood $= V_2 - V_1 = (5.39 - 0.11) cm^3 = 5.28\ cm^3$

(iii) We have,

Specific gravity of wood = 0.7 gm/cm^3

and, specific gravity of wood graphite = 2.1 gm/cm^3

∴ Weight of the pencil = Volume of wood × Specific gravity of wood + Volume of graphite × Specific gravity of wood graphite

$= (5.28 \times 0.7 + 0.11 \times 2.1)$gm = 3.927 gm.