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Class: 9
Subject: Mathematics
Topic: Triangles
No. of Questions: 20

Q1. If two sides of a triangle are unequal, then the angle opposite to the longer side is larger (or greater).

Sol. **Given:** A triangle ABC in which AB > AC

To Prove: $\angle C > \angle B$

Construction: Take a point D on AB such that AC = AD join CD.

Proof: In ΔACD , AC = AD

Therefore, $\angle ACD = \angle ADC$ ....(i)

But $\angle ADC$ is an exterior angle of ΔBDC

$\therefore \angle ADC > \angle B$ ...(ii)

From (i) and (ii) , we have

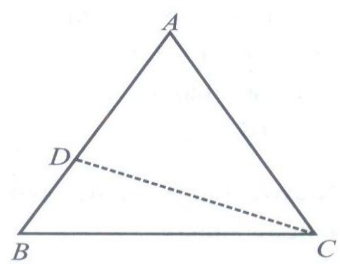
$\angle ACD > \angle B$ ...(iii)

By figure, $\angle ACB > \angle ACD$ ...(iv)

$\angle ACB > \angle ACD > \angle B$

$\Rightarrow \angle ACB > \angle B$

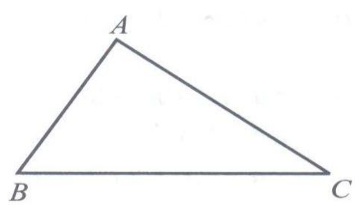
$\Rightarrow \angle C > \angle B$



Q2. In a triangle, the side opposite to the longer (greater) angle is longer.

Sol. **Given:** A triangle ABC in which $\angle B > \angle C$

To Prove: AC > AB



Proof: We have the following three possibilities for sides AB and AC of ΔABC.

(i) AC = AB

(ii) AC < AB

(iii) AC > AB

Case (i) : If AC = AB:

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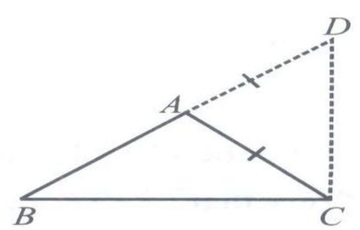
If AC = AB, then opposite angles of equal sides are equal. Hence, $\angle B = \angle C$.
But it is given that $\angle B > \angle C$
Hence $AC \neq AB$
Case (ii): if $AC < AB$:
We know that the angle opposite to longer side is larger.
$\therefore AC < AB \Rightarrow \angle C > \angle B$,
Which is also contrary to given $\angle B > \angle C$
Hence, $AC \neq AB$
Case (iii): if $AC > AB$:
We are left only this possibility which must be true.
Hence, AC> AB.

Q3. The sum of any two sides of a triangle is greater than its third side.

Sol. **Given:** A triangle ABC.
To Prove:
(i) $AB + AC > AC$
$BC + AC > AB$
$AC + AB > BC$
Construction: Produce BA to D, such that $AD = AC$ and Join DC.



Proof: In ΔADC, by construction $AD = AC$, then opposite angles are equal,
$\therefore \angle ACD = \angle ADC$ ...(i)
Now, $\angle BCD > \angle ACD$ ...(ii)
From (i) and (ii), we have
$\angle BCD > \angle ACD = \angle ADC$
Therefore, $BD > BC$ [Side opposite to larger angle in triangle is longer]
$\Rightarrow BA + AD > BC$ [$\because BD = BA + AD$]
$\Rightarrow BA + AC > BC$ [By construction $AD = AC$]
$AB + AC > BC$
Similarly, we can show that
$AB + BC > AC$
$BC + AC > AB$

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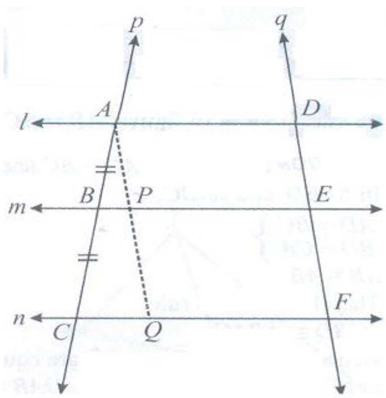
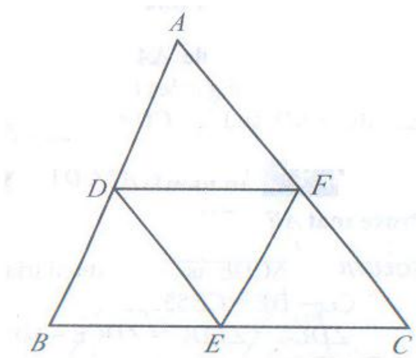
Q4. If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

Sol. **Given:** Three parallel l, m and n i.e., l ∥ m ∥ n.
A transversal p meets these parallel lines at point A, B and C respectively such that AB = BC.
Another transversal q also meets these parallel lines at l, m and n at points D, E and F respectively.
To Prove: DE = EF
Construction: Through point A, draw a line parallel to DEF; which meets BE at point P and CF at point Q.



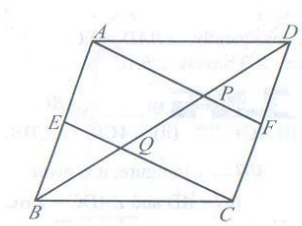
Proof: In ΔACQ, B is mid-point of AC and BP is parallel to CQ and we know that the line through the mid-point of one side of a triangle and parallel to another side bisects its third side.
∴ AP = PQ ...(i)
∵ AP ∥ DE and AD ∥ PE [By construction]
⇒ APED is a parallelogram
∴ AP = DE ...(ii)
And PQ ∥ EF and PE ∥ QF [By construction]
⇒ PQFE is a parallelogram
⇒ PQ = EF ...(iii)
From (i), (ii) and (iii) we get, DE = EF

Q5. In ΔABC, D, E and F are respectively the mid-points of sides AB, BC and CA. Show that ΔABC is divided into four congruent triangles by joining D, E and F.



Sol. As D and E mid-points of sides AB and BC respectively of the triangle ABC.
∴ DE ∥ AC
Similarly, DF ∥ BC and EF ∥ AB
Therefore, ADEF, BDFE and DFCE are all parallelograms.
Now, DE is a diagonal of the parallelogram BDFE.
Therefore, ΔBDE ≅ ΔFED
Similarly, ΔDAF ≅ ΔFED
And ΔEFC ≅ ΔFED
So, all the four triangles are congruent.

Q6. ABCD is a parallelogram in which E and F are the mid-points of the sides AB and CD respectively. Prove that the segments CE and AF trisect the diagonal BD.



Sol. AF and CE intersect BD at P and Q respectively.
Clearly, AE ∥ CF and AE = CF.
∴ AECF is a parallelogram. So, FA ∥ CE.
In ΔCDQ, F is the mid-point of CD and FP ∥ CQ [∵ FA ∥ CE]
So, P is the mid-points of QD.
Consequently, QP = PD ...(i)
In ΔBPA, E is the mid-point of AB and EQ ∥ AP
So , Q is the mid-point of BP.

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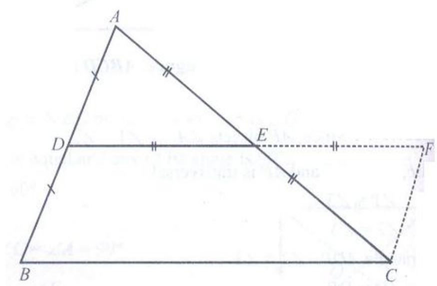
Consequently, BQ = QP ...(ii)
From (i) and (ii),
Hence, BQ = QP = PD.

Q7. The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal of half of it.

Sol. **Given:** A ΔABC in which D and E are the mid-points of AB and AC respectively. DE is joined.
To Prove: DE ∥ BC and DE = $\frac{1}{2}$ BC
Construction: Produced DE to F such that DE = EF. Join CF.



Proof: In Δs AED and CEF, We have
ED = EF [By construction]
EA = EC [∵ E is the mid-point of AC]
And ∠AED = ∠CEF [vert. opp. ∠s]
∴ ΔAED ≅ ΔCEF [S.A.S.]
So, AD = CF and AD = DB together imply that DB = CF.
Also, ∠ADE = ∠EFC ⇒ AD ∥ CF [∵ ∠ADE & ∠EFC are alt. ∠s]
⇒ DB ∥ CF
Thus, DB ∥ CF and DB = CF
∴ BCFD is a parallelogram
Hence, DF ∥ BC and DF = BC.
But, D, E, F are collinear and DE = EF.
∴ DE ∥ BC and DE = $\frac{1}{2}$ BC.

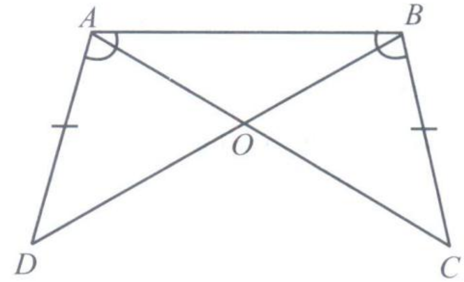
Q8. In the adjoining figure, ABC and BAD are two triangle on the same base AB such that BC = AD ∠ABC = ∠BAD.

Prove that:

(i) AC = BD

(ii) ∠ACB = BDA

(iii) CO = DO



Sol. In ∆s ABC and BAD, we have:

AB = BA (Common)

BC = AD (Given)

∠ABC = ∠BAD (Given)

∴ ∆ABC ≅ ∆BAD (SAS rule)

∴ AC = BD and ∠ACB =∠BDA and therefore,

∠OCB = ∠ODA

Again, in ∆s BOC and AOD, we have:

BC = AD (Given)

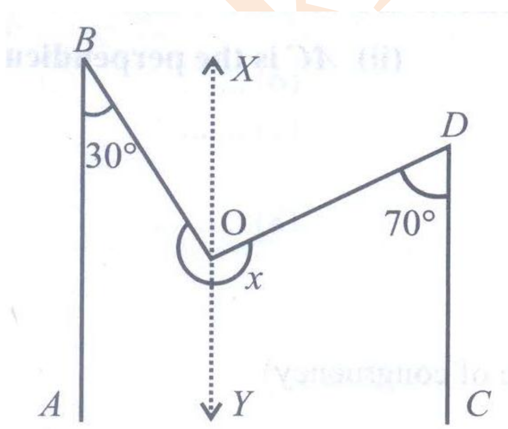
∠OCB = ∠ODA (Proved)

And ∠BOC = ∠AOD (Vertically opposite angles)

∴ ∆BOC ≅ ∆AOD (AAS rule)

Hence, CO = DO.

Q9. In the given figure, AB ∥ CD and ∠ABO = 30°, ∠ODC = 70°, find x.



Sol. Draw a line XY passing through O parallel to AB and CD.

Now ∠B and ∠BOY are interior angles on the same side of transversal OA between parallel line AB and XY.

⇒ ∠B + ∠BOY = 180°

30° + ∠BOY = 180°

∠BOY = 150°

Similarly ∠D and ∠DOY are interior angles on the same side of transversal OD between parallel line XY and CD.

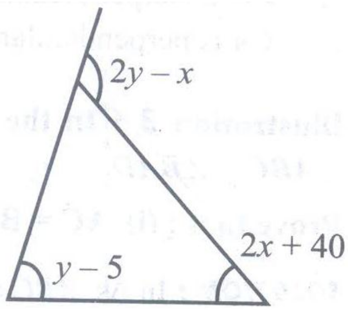
⇒ ∠D + ∠DOY = 180°

70° + ∠DOY = 180°

∠DOY = 180° - 70° = 110°

Now, x = ∠BOY + ∠DOY = 150° + 110° = 260°

Q10. In the given figure find y, if x = 5°



Sol. We know that in a triangle exterior angle = sum of two interior opposite angles

2y - x = y – 5 + 2x + 40

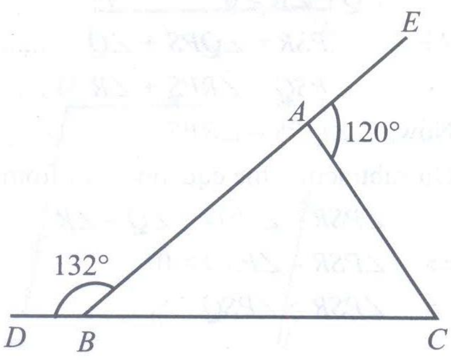
2y – y = 2x + 35 + x

Y = 3x + 35

On substituting x = 5°, we get

Y = 3 ×5 + 35 = 15 + 35 = 50°

Q11. In figure, ∠DBA = 132° and ∠EAC = 120°. Show that AB > AC



Sol. As DBC is a straight line,

132° + ∠ABC = 180°

⇒ ∠ABC = 180° - 132° = 48°

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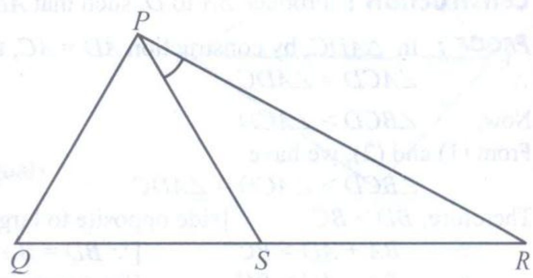
For ΔABC, ∠EAC is an exterior angle
120° = ∠ABC + ∠BCA (Ext. ∠ = sum of two opp. Int. ∠s)
⇒ 120° = 48° + ∠BCA
⇒ ∠BCA = 120° - 48° = 72°
Thus, we find that ∠BCA > ∠ABC
⇒ AB > AC (side opposite to greater angle is greater)

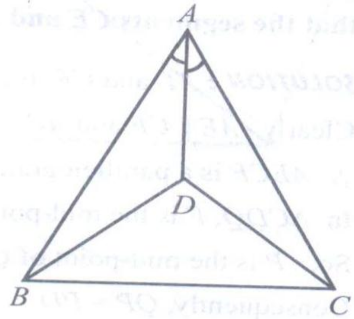
Q12. In the given figure, PR > PQ and PS bisect ∠QPR.
Prove that: ∠PSR > ∠PSQ.



Sol. In ΔPQR
PR > PQ (Given)
⇒ ∠Q > ∠R (Angle opposite to larger side is greater)
⇒ ∠Q - ∠R > 0
Also, ∠PSR = ∠QPS + ∠Q ... (i)
∠PSQ = ∠RPS + ∠R ... (ii) (Exterior angle is equal to sum of interior opposite angles)
Now, ∠QPS = ∠RPS (PS bisects ∠QPR)
On subtracting the equation (ii) from (i), we get
∠PSR - ∠PSQ = ∠Q - ∠R
⇒ ∠PSR - ∠PSQ > 0
⇒ ∠PSR > ∠PSQ

Q13. In figure, AB = AC. D is a point in the interior of ΔABC such that ∠DBC = ∠DCB. Prove that AD bisects ∠BAC.

Sol. In ΔBDC, ∠DBC = ∠DCB then the opposite sides are equal.
i.e. CD = BD ...(i)
Now, in ΔABD and ΔACD
BD = CD [by (i)]
AD = AD [common side]



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AB = AC [Given]

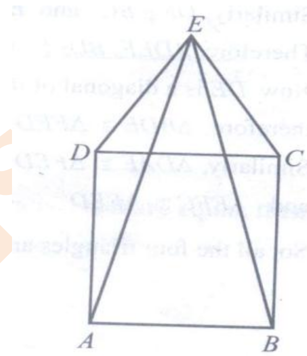
Therefore by SSS rule,

ΔABD ≅ ΔACD

Consequently, ∠BAD = ∠CAD

⇒ AD bisects ∠BAC

Q14. In figure, ABCD is a square and ΔCDE is an equilateral triangle. Prove that AE = BE.



Sol. ΔCDE is a equilateral triangle.

CD = DE = CE ...(i)

∠DEC = ∠EDC = ∠DCE = 60° ...(ii)

And ABCD is a square

∠ADC = ∠BCD = 90°

On adding ∠EDC to both sides

∠ADC + ∠EDC = ∠BCD + ∠EDC

⇒ ∠EDA = ∠ECB ... (iii) [∵ ∠EDC = ∠DCE]

Now in ΔADE and ΔBCE

AD = BC [sides of the square]

∠EDA = ∠ECB [by (iii)]

DE = EC [by (i)]

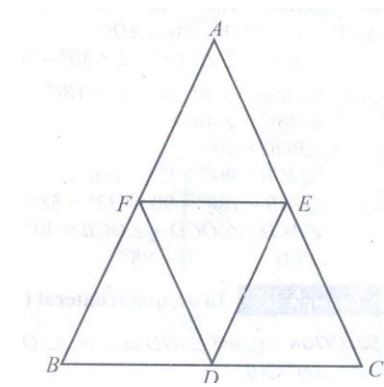
Therefore by SAS rule

ΔADE ≅ ΔBCE

Consequently corresponding sides are equal.

i.e., AE = BE.

Q15. In an equilateral triangle ABC the mid-point of the side BC, CA and AB are D, E and F, respectively. Prove that ΔDEF is an equilateral triangle.



Sol. In ΔABC, D, E and F are midpoint of BC, CA and AB respectively

Thus DE = $\frac{1}{2}$ AB ... (i)

EF = $\frac{1}{2}$ BC ... (ii)

FD = $\frac{1}{2}$ AC ... (iii)

But ΔABC is an equilateral triangle

Hence, AB = BC = AC

From (i), (ii) and (iii), we get

DE = EF = FD

Therefore ΔDEF is an equilateral triangle.

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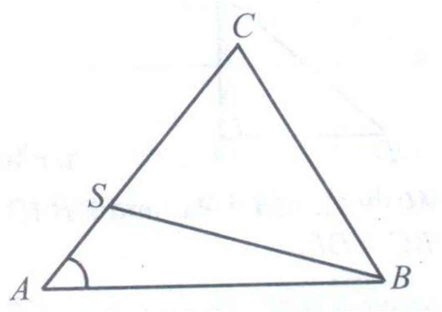
Q16. In figure, AB = AC and S is any point on side AC. Prove that CS < BS.



Sol. In ΔABC,

AB = AC (Given)

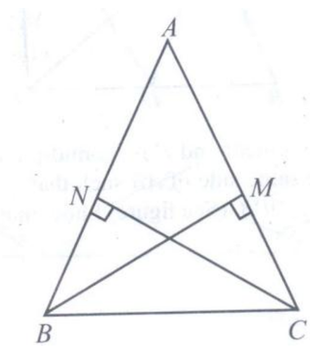
∴ ∠ACB = ∠ABC ..(i)

[Angles opposite to equal to greater angle is longer]

∴ ∠ACB > ∠ CBS

∴ BS > CS [Side opposite to greater angle is longer]

Q17. In the adjoining figure, AB = AC. BM ⊥ AC and CN ⊥ AB. Prove that BM = CN



Sol. In ΔABC,

AB = AC

∴ ∠ABC = ∠ACB [angles opposite equal sides are equal]

In Δs BCM and BCN, ∠N = ∠M [each = 90°]

∠ABC = ∠ACB [from above]

BC = BC [common]

∴ ΔBCM ≅ ΔCBM [A. A. S. axiom of congruency]

⇒ BM = CN

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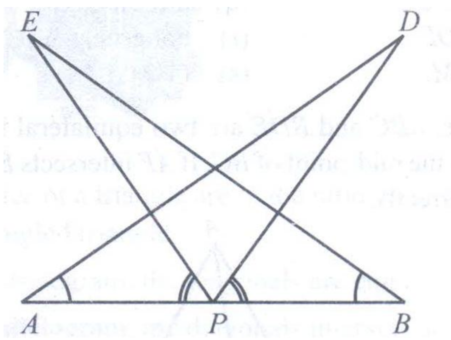
Q18. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB. (See figure)
Show that
(i) ΔDAP ≅ ΔEBP
(ii) AD = BE

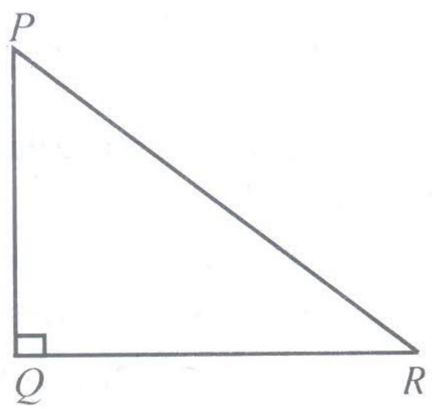


Sol. (i) Since, P is the mid-point of the line segment AB
∴ from Δ DAP and ΔEBP,
AP = BP
Also, given ∠DAP = ∠EBP and ∠EPA = ∠DPB
Adding ∠EPD to both sides
⇒ ∠EPA + ∠EPD = ∠EPD + ∠DPB
⇒ ∠APD = ∠BPE
Thus, by ASA rule
∴ ΔDAP ≅ ∠EBP

(ii)Since, ΔDAP ≅ ΔEBP [from above]
∴ AD = BE [CPCT]

Q19. Show that in a right angles triangle, the hypotenuse is the longest side.

Sol. Let PQR be a right angled triangle in which ∠Q = 90°
Then, ∠P + ∠R = 90° [By angle sum property]
∴ ∠Q = ∠P + ∠R
⇒ ∠Q > ∠P



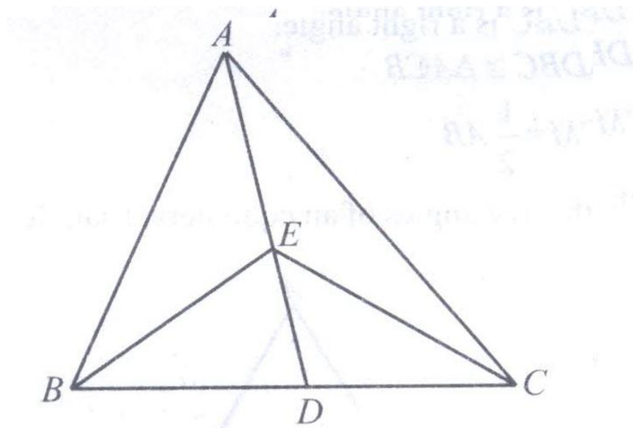
And ∠ Q > ∠R

∴ PR > QR [∵ Side opposite to greater is longer]

Similarly PR > PQ

∴ PR is the longest side, i.e, hypotenuse is the longest side.

20. In figure, E is any point on medium AD of a ΔABC. Show that ar(ΔABE) = ar(ΔACE).



Sol. Since, AD is a median in ΔABC which divides it into two triangles of equal areas.

∴ ar(ΔABD) = ar(ΔACD) ...(i)

Similarly, ar(ΔEBD) = ar (ΔECD) ...(ii)

(∵ ED is a medium in ΔEBC)

Subtracting (ii) from (i), we get

ar(ΔABD) − ar(ΔEBD) = ar(ΔACD) - ar (ΔECD)

⇒ ar (ΔABE) = ar(ΔACE).