

Class: 9
Subject: Physics
Topic: Motion in a Plane
No. of Questions: 20

Q1. From the top of a tower 45 m high, two stones are released. One vertically downwards and the other with a horizontal velocity of 30 m/s. How long will each stone take to strike the ground and how far from the tower will each stone strike the ground? ($g = 10 \text{ m/s}^2$)

Ans. Initial downward velocity of both the stones is 0. So both will reach ground in same time.

$$S = 45 \text{ m, } u = 0, \text{ } g = 10 \text{ m/s}^2$$

By second equation of motion

$$S = u t + (1/2) g t^2$$

$$45 = 0 + (1/2) \times 10 \times t^2 \Rightarrow 45 + 5t^2 \Rightarrow t^2 = 9 \Rightarrow t = 3\text{s}$$

So both stones will reach the ground in 3s.

The stone which is projected horizontally has a horizontal velocity of 30 m/s.

So its distance from the ground = velocity \times time = $30 \times 3 = 90 \text{ m}$

The other stones will fall just by the tower

Q2. Suppose a project is lunched with an initial velocity V_0 at an angle $[\theta]$ with respect to the x-axis. What is its Range R?

Ans. We start defining the coordinate system to be used in this position:

$$X(t = 0) = x_0 = 0$$

$$Y(t = 0) = y_0 = 0$$

The position of the projectile at any given time t can be obtained from the following expression:

$$X(t) = V_0 \cos(\theta) t$$

$$Y(t) = V_0 \sin(\theta) t - \frac{1}{2} g t^2$$

For the RANGE,

$$X = V_0 \cos(\theta) t$$

As we know time required to reach the ground again, i.e $t = \frac{2V_0 \sin(\theta)}{g}$

Substituting this value in the above equation, we will get

$$R = \frac{V_0^2 \sin 2(\theta)}{g}$$

Q3. At what angle of projection, Range is maximum?

Ans. At the angle of projection, $\theta = 45^\circ$, Range would be maximum.

Q4. An antitank gun is located on the edge of a plateau that is 60 m above the surrounding plain. The gun crew sights an enemy tank stationary on the plain at a horizontal distance of 2.2 km from the gun. At the same moment, the tank crew sees the gun and starts to move directly away from it with an acceleration of 0.90 m/s^2 . If the antitank gun fires a shell with a muzzle speed of 240 m/s at an elevation of 10° above the horizontal, How long should the gun crew wait before firing if they are to hit the tank.

Ans. Our starting point are the equation of motion of the shell.

$$X(t) = x_0 + V_{ox} t \dots\dots\dots(1)$$

$$Y(t) = Y_0 + V_{oy} t - \frac{1}{2} g t^2 \dots\dots\dots(2)$$

Our coordinate system is defined such that the shell is launched at $t = 0$ sec and its locations at that instant is specified by $x = 0$ m and $Y = h$. Therefore, $X_0 = 0$ m and $y_0 = h$. In order to determine the trajectory of the shell we first determine its time of flight between launched and impact. This time t_1 can be obtained from eq. (2) by requesting that on impact $y(t) = 0$ m. Thus the solution for t_1 are

$$Y(t_1) = Y_0 + V_{oy} t_1 - \frac{1}{2} g t_1^2 \dots\dots\dots(3)$$

$$T_1 = \frac{V_{oy} \pm \sqrt{V_{oy}^2 + 2hg}}{g} \dots\dots\dots(4)$$

Since the shell will hit the ground after being fired (at $t = 0$) we only need to consider the positive solution for t_1 . The range R of the shell can be obtained by substituting t_1 into eq(1):

$$R = x(t_1) = V_{ox} t_1 = V_{ox} \frac{V_{oy} \pm \sqrt{V_{oy}^2 + 2hg}}{g} \dots\dots\dots(5)$$

The problem provides the following information concerning the firing of the projectile:

$$H = 60 \text{ m}$$

$$V_0 = 240 \text{ m / s}$$

$$[\theta] = 10^\circ$$

Using this information we can calculate V_{0x} and V_{0y} :

$$V_{0x} = V_0 \cos(\theta) = 236 \text{ m/s}$$

$$V_{0y} = V_0 \sin(\theta) = 42 \text{ m/s}$$

Substituting these values in eq.(4) and eq. (5) we obtain:

$$T_1 = 9.8 \text{ s}$$

$$R = 2320 \text{ m}$$

The distance between the impact point and the original position of the tank is 120 m. The tank starts from rest ($v = 0 \text{ m/s}$) and an acceleration of 0.9 m/s^2 . The time t_2 it takes for the tank to travel the impact point can be found by solving the following equation:

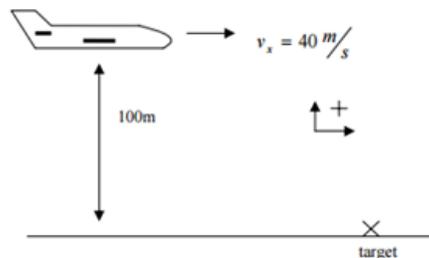
$$X(t_2) = \frac{1}{2} a t_2^2 = d = 120 \text{ m} \dots\dots (6)$$

This shows that

$$T_2 = \sqrt{\frac{2d}{a}} = 163 \text{ s} \dots\dots(7)$$

If she shell is fired at the same time that the tank starts to move, the tank will not reach the impact until $(16.3 - 9.8)\text{s} = 6.5 \text{ s}$ after the shell has landed. This means that the gun crew has to wait 6.5 s before firing the antitank gun if they are to hit the tank.

Q5. Consider a supply airplane attempting to airdrop a box onto a target marked on the ground as shown below.



At what range from the target should the package be dropped?

Ans. Determine time of flight from a height of 100m:

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$$

$$-100m = 0 - (4.9m \cdot s^{-2})t^2$$

$$t = 4.5s$$

For a velocity of 40 m/s in the + x direction, what is the range?

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x - x_0 = (+40m \cdot s^{-1})(4.5s) + 0$$

$$x - x_0 = 180m$$

So the package should be released when the plane is 180 meters from the target as long as the plane is in straight and level flight with a constant velocity of 40 m/s.

Q6. A BASE jumper ascends El Capitan (1200 m) in Yosemite Valley CA and in order to BASE jump rather than walk 8 arduous miles down the Yosemite falls trail. If she leaps horizontally with a velocity of 5 m/s, and due to a technical malfunction is unable to open her parachute until 7 seconds have elapsed, does she live or die?

Ans. Given $y - y_0 = -1200$ m

$$a = -9.8m/s^2$$

$$t = 7s$$

- Try solving for the vertical distance an object in free fall covers in 7 seconds. Note: There is no initial velocity in the y direction.

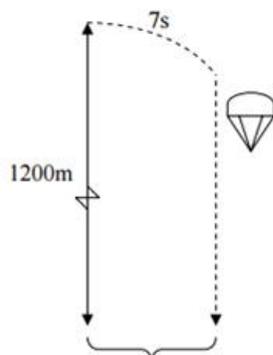
$$y - y_0 = v_{0y}t + \frac{1}{2}at^2$$

$$y - y_0 = (-4.9m/s^2)(7s)^2$$

$$y - y_0 = -240m$$

Since $240m < 1200m$ she lives.

Where does she land? (Assume she falls vertically once the parachute opens)



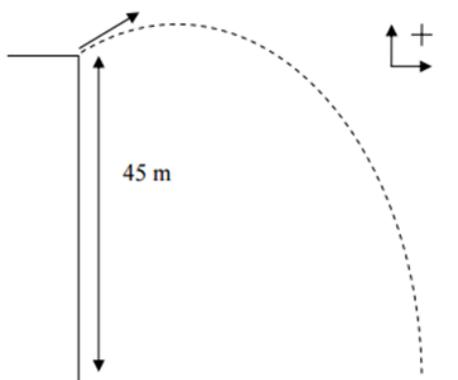
Since $v_x = v_{0x}$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$= (5\text{ m/s})(7\text{ s})$$

$$= 35\text{ m}$$

- Q7. A stone is thrown from the top of a building upward at an angle of 30° to the horizontal with an initial speed of 20 m/s . If the height of the building is 45 m , find: time of flight; range; and the velocity of the stone just before it hits the ground.



- Ans. Find the components of the initial velocity

$$v_{0x} = v_0 \cos \theta_0 = (20\text{ m} \cdot \text{s}^{-1})(\cos 30^\circ) = +17.3\text{ m} \cdot \text{s}^{-1}$$

$$v_{0y} = v_0 \sin \theta_0 = (20\text{ m} \cdot \text{s}^{-1})(\sin 30^\circ) = +10.0\text{ m} \cdot \text{s}^{-1}$$

- Find the time of flight. Note that this is the same as it would be for a stone tossed straight up at an initial velocity of 10 m/s. Why?

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$$

$$-45\text{m} = (+10\text{m} \cdot \text{s}^{-1})(t) - (4.9\text{m} \cdot \text{s}^{-2})(t^2)$$

$$t = 4.22\text{s}$$

- Find the range. In order to do this we use the time of flight (4.22s) and ask ourselves how far in the x direction can the stone move at a constant velocity of 17.3 m/s during this time.

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$= (17.3\text{m} \cdot \text{s}^{-1})(4.22\text{s}) + 0$$

$$= 73\text{m}$$

- What is the velocity of the stone just before striking the ground? Remember that the x component of this vector has a constant value that we have already computed (+17.3 m/s). All we have to do is compute the y component which we do with our kinematic equations in y .

$$v_{fy} = v_{0y} + gt$$

$$= (+10\text{m} \cdot \text{s}^{-1}) - (9.8\text{m} \cdot \text{s}^{-2})(4.22\text{s})$$

$$v_{fy} = -31.4\text{m} \cdot \text{s}^{-1}$$

$$v_{fx} = v_{0x} = +17.3\text{m/s}$$

Now resolve the x and y components into the resultant velocity vector

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(17.3)^2 + (-31.4)^2} \text{ m} \cdot \text{s}$$
$$= 35.9 \text{ m} \cdot \text{s}^{-1}$$

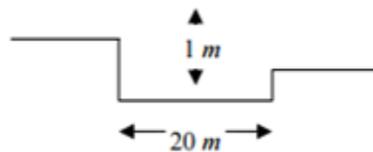
$$\tan \theta = \frac{y}{x} = \frac{-31.4 \text{ m} \cdot \text{s}^{-1}}{+17.3 \text{ m} \cdot \text{s}^{-1}}$$

$$\theta = -61^\circ$$

So the velocity just before the stone hits the ground is:

$$\vec{v} = 35.9 \text{ m} \cdot \text{s}^{-1} @ -61^\circ$$

- Q8. A mountain biker approaches a ditch from the left at a speed of 16 m/s. The ditch is 20 m wide and the bank on the opposite side is 1 meter lower. Does the mountain biker make it across?



- Ans. Find the time of flight for a vertical distance of 1 meter, i.e. the time it takes to drop a distance of 1 meter which places the airborne biker below the opposite lip of the ditch.

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$$

$$-1 = 0 - (4.9 \text{ m/s}^2)t^2$$

$$t = .452 \text{ s}$$

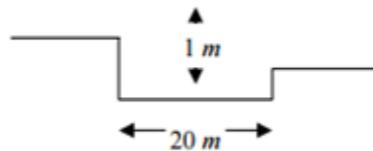
Now find the range for this time of flight.

$$\begin{aligned}\text{Now: } x - x_0 &= v_{0x}t \\ &= (16 \text{ m/s})(.452 \text{ s}) \\ &= 7.2 \text{ m}\end{aligned}$$

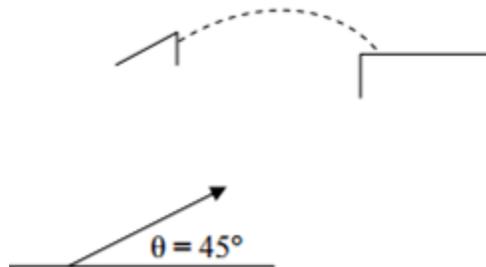
So the mountain biker doesn't clear the ditch.

- Q9. The mountain biker in the previous example is being chased by a bear and really needs to get across the ditch. Unfortunately he can't pedal any faster. Any suggestion?

(A mountain biker approaches a ditch from the left at a speed of 16 m/s. The ditch is 20 m wide and the bank on the opposite side is 1 meter lower)



- Ans. One could increase the time of flight by launching at an angle by use of a “kicker” or incline. Let's assume that the top of the incline is till 1 meter above the other side of the ditch. It may easily be determined by experimentation that the maximum horizontal range is achieved through a launched angle of 45° for most circumstances.



The components of the initial velocity are:

$$v_{0x} = v_0 \cos 45^\circ = 11.3 \text{ m} \cdot \text{s}^{-1}$$

$$v_{0y} = v_0 \sin 45^\circ = 11.3 \text{ m} \cdot \text{s}^{-1}$$

Time of flight at 45° is:

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$$

$$-1\text{m} = (+11.3 \text{ m} \cdot \text{s}^{-1})t - (4.9 \text{ m} \cdot \text{s}^{-2})t^2$$

The y time of flight equation is a quadratic in t.

$$\frac{-b \pm (b^2 - 4ac)^{1/2}}{2a}$$

$a = -4.9$
 $b = +11.3$
 $c = 1$

$$t = \frac{-11.3 \pm \sqrt{(11.3)^2 - 4(-4.9)(1)}}{(2)(-4.9)}$$

$$t = \frac{-11.3 \pm (12.1)}{-9.8}$$

$$t = 2.39\text{s}$$

Since $V_x = 11.3 \text{ m/s}$, in 2.39s the biker travels is 27 m, so he makes it across.

Q10. A kicker is capable of booting a football at an angle of $\theta = 37^\circ$ with an initial velocity of 20 m/s.

Find:

- Maximum height
- Time of flight
- Range
- Velocity at max. height
- Maximum field goal range if bar is 3 m tall

Ans. First let's find the x and y component of initial velocity

$$v_{0x} = v_0 \cos \theta = v_0 \cos 37^\circ = 16 \frac{m}{s}$$

$$v_{0y} = v_0 \sin \theta = v_0 \sin 37^\circ = 12 \frac{m}{s}$$

Does it make sense that the x component is larger? Why? Next, noting that maximum height occurs when $v_y = 0$

$$v_{fy} = v_{0y} + a_y t$$

$$\therefore \frac{v_{0y}}{g} = t = \frac{12 \text{ m/s}}{9.8 \text{ m/s}^2} = 1.22 \text{ s}$$

So maximum height occurs 1.22 seconds into the airborne trajectory of the football. To compute the height we simply use this information to compute the vertical displacement (Without any further calculation can you tell what the total time of flight will be given the symmetry of this problem?).

$$\begin{aligned} y - y_0 &= v_{0y} t - \frac{1}{2} g t^2 \\ &= (12 \text{ m/s})(1.22 \text{ s}) - (4.9 \text{ m/s}^2)(1.22 \text{ s})^2 \\ y - y_0 &= +7.35 \text{ m} \end{aligned}$$

Maximum height is 7.35 m above the ground.

The total time of flight may be calculated (if you have not already figured it out using the symmetry of the problem) with the knowledge that when $y - y_0 = u t - \frac{1}{2} g t^2$, the ball is on the ground. This occurs twice, when $t = t_0$ and when $t = t_f$. Hence:

$$y - y_0 = (12 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2$$

Which is a quadratic in t .

$$t_0 = 0$$

$$t_f = 2.45 \text{ s}$$

The total time of flight may be calculated (if you have not already figured it out using the symmetry of the problem) with the knowledge that when $y - y_0 = 0$ the ball is on the ground. This occurs twice, when $t = t_0$ and when $t = t_f$. Hence:

$$y - y_0 = (12 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2$$

which is a quadratic in t :

$$t_0 = 0$$

$$t_f = 2.45 \text{ s}$$

Now to find the range we use the time of flight and the constant value of the velocity in the x direction: Since $v_x = v_{0x}$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$= (16 \text{ m/s})(2.45 \text{ s})$$

$$= 39.2 \text{ m}$$

Or use the range equation:

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

$$= \frac{(20 \text{ m/s})^2 \sin 74^\circ}{9.8 \text{ m/s}^2}$$

$$R = 39.2 \text{ m}$$

To determine the velocity at the apex we exploit the facts that v_x is constant and v_y is zero. Hence:

$$v_x = v_{0x} = 16 \text{ m/s}$$

$$v_y = 0$$

$$\therefore v = 16 \text{ m/s}$$

To determine the acceleration at apex we exploit the facts that acceleration in the y direction is constant and acceleration in the x direction is zero. Hence:

$$a_y = -g = -9.8 \text{ m/s}^2$$

$$a_x = 0$$

$$\therefore a = -9.8 \text{ m/s}^2$$

To determine the maximum field goal range for a 3 m high crossbar, we must cast the physics problem in mathematical terms. We want to find $x-x_0$ at the same time $y-y_0 = +3\text{m}$. We can do this by solving for the time when this relationship is true.

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$$

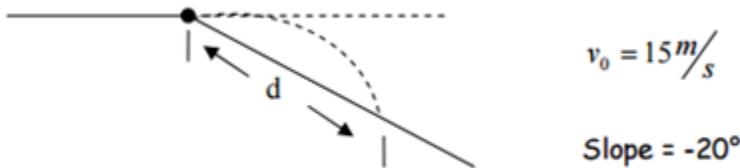
$$+3\text{m} = (12 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2$$

Quadratic:	a = -4.9	}	t = 0.29s ↑ up
	b = 12		
	c = -3		

So the football is at a height of 3 metres at $t = 0.29$ seconds on its way up and $t = 2.16$ seconds on its way down (which is what we want for maximum range). The next step is to find out how far the ball has traveled from horizontally from the origin in this time Hence:

$$\begin{aligned} x - x_0 &= v_{0x}t \\ &= (16 \frac{m}{s})(2.16s) \\ &= 34.6 \text{ meters} \end{aligned}$$

- Q11 A skier cranking down the ridge at pebble Creek hits the cat track with a velocity of 15 m/s. If the skier leaves the car track horizontally and if the ground falls away with a slope of -20° how long is the skier in the air? What is the skier's velocity upon landing? What does this suggest to you about the desirability of steep landings after if you are going to jump off something on skis or on a dirt or mountain bike?



Ans. Since the ground falls away. We have to divide the displacement vector, d to the landing into components:

$$\begin{aligned} \therefore x - x_0 &= v_0 t + \frac{1}{2} a t^2 \\ d \cos \theta &= x - x_0 \quad + \quad d \sin \theta = y - y_0 \\ \Rightarrow 1. \quad d \cos \theta &= v_{0x} t + \frac{1}{2} a_x t^2 & 2. \quad d \sin \theta &= v_{0y} t - \frac{1}{2} g t^2 \\ \underbrace{d \cos \theta = 15 \frac{m}{s} t} & & \underbrace{d \sin \theta = -\frac{1}{2} g t^2} & \\ & \text{2 equations, 2 unknowns, } d \text{ and } t & & \end{aligned}$$

One method of solution is to divide equation 2 by equation 1, eliminate the variable d while solving for t :

$$\frac{d \sin \theta}{d \cos \theta} = \frac{-\frac{1}{2}gt^2}{15m/s \cdot t} \Rightarrow \frac{-\frac{1}{2}(9.8m/s^2)t}{15m/s} = \tan 20^\circ$$

$$t = 1.11s$$

Next we solve for the velocity components on landing:

$$v_{fx} = v_{0x} = 15m/s \quad v_{fy} = -(9.8m/s^2)(1.11s) \approx -11m/s \quad \Rightarrow \tan^{-1}\left(\frac{-11m/s}{+15m/s}\right) = -36^\circ$$

$$v = \sqrt{(15m/s)^2 + (11m/s)^2} = 18.5m/s$$

So $\vec{v} = 18.5m/s$ at an angle of 36° below horizontal

The steeper the landing, in general, the closer it comes to aligning with the velocity vector. This insures soft (albeit high speed) landings because there is little immediate change in acceleration due to a direction change on landing.

Q12. A boat lies at anchor 100 meters off shore. I launch a water balloon from the beach at this boat with an initial velocity of 50 m/s. At what angle must I launch the balloon in order to strike the boat?

Ans. Given: $V_0 = 50$ m/s

$$x-x_0 = 100 \text{ m}$$

Find: θ

The challenge here is to solve for the launch angle – information that has been supplied with problem, we've looked at so far. Since we don't have an equation that can be solved directly for θ (without also having to solve for other variables, time, velocity, etc.) We will have to derive what we need from our list of kinematic equations.

Using kinematic equations and substituting for known quantities:

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \rightarrow 0 = (v_0 \sin \theta)t - \frac{1}{2}gt^2 \Rightarrow \frac{2v_0 \sin \theta}{g} = t$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \rightarrow x - x_0 = (v_0 \cos \theta)t \Rightarrow \frac{x - x_0}{v_0 \cos \theta} = t$$

If these equations are set equal to each other t is eliminated and we are left to solve for θ in terms of initial velocity (V_0) and displacement ($x-x_0$) both which are given

$$\frac{2v_0 \sin \theta}{g} = \frac{x - x_0}{v_0 \cos \theta} \therefore 2 \sin \theta \cos \theta = \frac{(x - x_0)g}{v_0^2}$$

$$\sin 2\theta = \frac{(100m)(9.8m \cdot s^{-2})}{(50m \cdot s^{-1})^2}$$

$2\theta = \sin^{-1} 0.392 \therefore \theta \approx 11.54^\circ$ So I'd need to launch the balloon upwards at an angle of about 11.5 degrees.

- Q13. An Object is projected from the origin. The initial velocity components are $V_{ix} = 7.07$ m/s, and $V_{iy} = 7.07$ m/s. Determine the x and y position of the object at 0.2 second intervals for 1.4 seconds. Also plot the results.

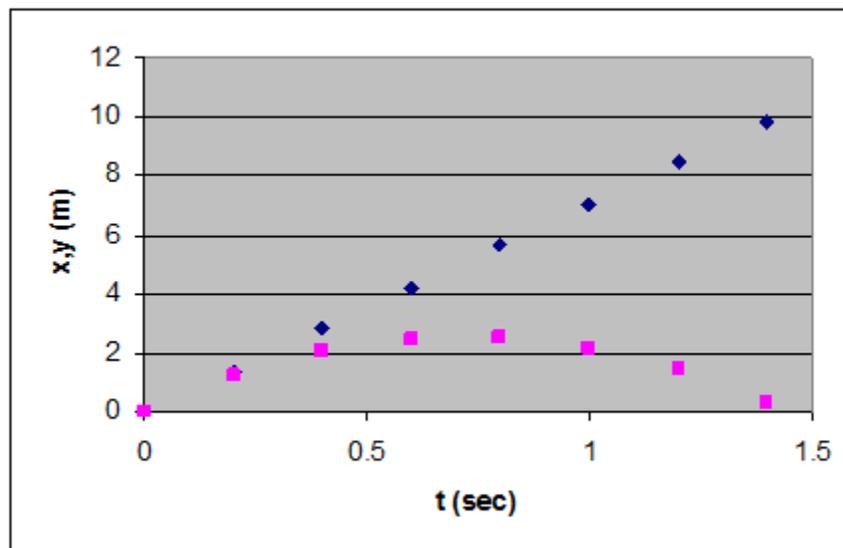
$$\Delta y = y_f - y_i = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$\Delta x = x_f - x_i = v_{ix}\Delta t$$

Ans. Since the object starts from the origin, Δy and Δx will represent the location of the objects at time Δt .

t (sec)	x (meters)	y (meters)
0	0	0
0.2	1.41	1.22
0.4	2.83	2.04
0.6	4.24	2.48
0.8	5.66	2.52
1.0	7.07	2.17
1.2	8.48	1.43
1.4	9.89	0.29

This is a plot of the x position (black points) and y position (red points) of the object as a function of time.

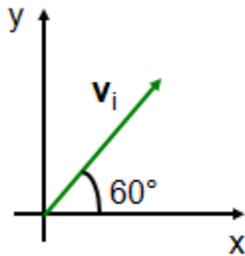


Q14. Example (text problem 3.50): An arrow is shot into the air with $\theta = 60^\circ$ and $V_i = 20.0$ m/s.

(a) What are V_x and V_y of the arrow when $t = 3$ sec?

(b) What are the x and y components of the displacement of the arrow during the 3.0 sec interval?

Ans. (a) The components of the initial velocity are:

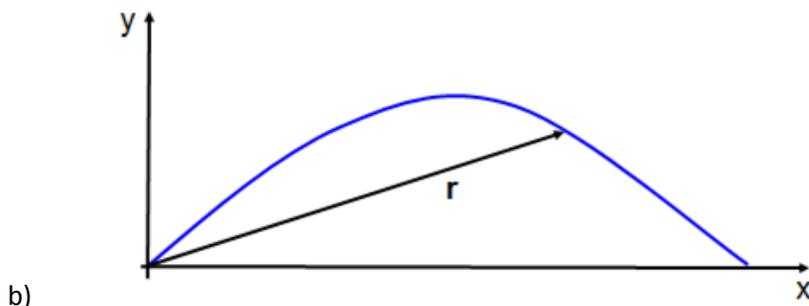


$$v_{ix} = v_i \cos \theta = 10.0 \text{ m/s}$$

$$v_{iy} = v_i \sin \theta = 17.3 \text{ m/s}$$

At $t = 3$ sec: $v_{fx} = v_{ix} + a_x \Delta t = v_{ix} = 10.0 \text{ m/s} \leftarrow \text{CONSTANT}$

$$v_{fy} = v_{iy} + a_y \Delta t = v_{iy} - g \Delta t = -12.1 \text{ m/s}$$



$$\Delta r_x = \Delta x = x_f - x_i = v_{ix} \Delta t + \frac{1}{2} a_x \Delta t^2 = v_{ix} \Delta t + 0 = 30.0 \text{ m}$$

$$\Delta r_y = \Delta y = y_f - y_i = v_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2 = v_{iy} \Delta t - \frac{1}{2} g \Delta t^2 = 7.80 \text{ m}$$

Q15. Example: How far does the arrow in the previous example land from where it is released?

Ans. The arrow lands when $\Delta y = 0$.

$$\Delta y = v_{iy} \Delta t - \frac{1}{2} g \Delta t^2 = 0$$

$$\Delta y = (v_{iy} - \frac{1}{2} g \Delta t) \Delta t = 0$$

Solving for Δt : $\Delta t = \frac{2v_{iy}}{g} = 3.53 \text{ sec}$

The distance traveled is: $\Delta x = v_{ix} \Delta t = 35.3 \text{ m}$

Q16. A plane can travel with a speed of 80 mi/hr. with respect to the air. Determine the resultant velocity of the plane (magnitude only) if it encounters a

- (a) 10 mi/hr. headwind.
- (b) 10 mi/hr. tailwind.
- (c) 10 mi/hr. crosswind.
- (d) 60 mi/hr. crosswind.

Ans. (a) A headwind would decrease the resultant velocity of the plane to 70 mi/hr.

(b) A tailwind would increase the resultant velocity of the plane to 90 mi/hr.

(c) A 10 mi/hr. crosswind would increase the resultant velocity of the plane to 80.6 mi/hr. This can be determined using the Pythagorean: $\text{SQRT} [(80 \text{ mi/hr.})^2 + (10 \text{ mi/hr.})^2]$

(d) A 60 mi/hr. crossed would increase the resultant velocity of the plane to 100 mi/hr. This can be determined using the Pythagorean Theorem: $\sqrt{[(80\text{mi/hr.})^2 + (60\text{mi/hr.})^2]}$

Q17 A motorboat traveling 5 m/s, East encounters a current traveling 2.5 m/s, North.

- (a) What is the resultant velocity of the motor boat?
- (b) If the width of the river is 80 meters wide, then how much time does it take the boat to travel shore to shore?
- (c) What distance downstream does the boat reach the opposite shore?

Ans. (a) The resultant velocity can be found using the Pythagorean. The resultant is the hypotenuse of a right triangle with sides of 5 m/s and 2.5 m/s. It is $\sqrt{[(5\text{m/s})^2 + (2.5\text{ m/s})^2]} = 5.59\text{ m/s}$. Its direction can be determined using a trigonometric function.

$$\text{Direction} = \tan^{-1}[(2.5\text{ m/s}) / (5\text{ m/s})] = 26.6\text{ degrees.}$$

(b) The time to cross the river is $t = d / v = (80\text{ m}) / (5\text{ m/s}) = 16.0\text{s}$

(c) The distance traveled downstream is $d = v \cdot t = (2.5\text{ m/s}) \cdot (16.0) = 40\text{ m}$

Q18. A soccer ball is kicked horizontally off a 22.0-meter high hill and lands a distance of 35.0 meters from the edge of the hill. Determine the initial horizontal velocity of the soccer ball.

Ans.	Horizontal Info:	Vertical Info:
	$X = 35.0\text{ m}$	$Y = -22.0\text{ m}$
	$V_{ix} = ?$	$V_{iy} = 0\text{ m/s}$
	$a_x = 0\text{ m/s/s}$	$a_y = -9.8\text{ m/s/s}$

Use $y = V_{iy} \cdot t + 0.5 \cdot a_y \cdot t^2$ to solve for time: the time of flight is 2.12 seconds.

Now use $x = V_{ix} \cdot t + 0.5 \cdot a_x \cdot t^2$ to solve for V_{ix}

Note that a_x is 0 m/s/s so the last term on the right side of the equation cancels. By substituting 35.0 m for x and 2.12 s for t, the V_{ix} can be found to be 16.5 m/s.

Q19. A motor boat traveling 5 m/s. East encounters a current traveling 2.5 m/s, south.

- (a) What is the resultant velocity of the motor boat?
- (b) If the width of river is 80 meters wide, then how much time does it take the boat to travel shore to shore?
- (c) What distance downstream does the boat reach the opposite shore?

Ans. (a) The resultant velocity can be found using the Pythagorean Theorem. The resultant is the hypotenuse of a triangle with sides of 5 m/s and 2.5 m/s. It is

$$\sqrt{[(5\text{ m/s})^2 + (2.5\text{ m/s})^2]} = 5.59\text{ m/s}$$

Its direction can be determined using a trigonometric function.

$$\text{Direction} = 360^\circ - \tan^{-1}[(2.5\text{ m/s})] = 333.4\text{ degrees}$$

NOTE: The direction of the resultant velocity (like any vector) is expected as the counterclockwise angle of rotation from due east.

(b) The time to cross the river is $t = d/v = (80) / (5\text{ m/s}) = 16.0\text{ s}$

(c) The distance traveled downstream is $d = v \cdot t = (2.5\text{ m/s}) \cdot (16.0\text{ s}) = 40\text{ m}$

Q20. A motor boat traveling 6 m/s, East encounters a current traveling 3.8 m/s, South.

- (a) What is the resultant velocity of the motor boat?
- (b) If the width of the river is 120 meters wide, then how much time does it take the boat to travel shore to shore?
- (c) What distance downstream does the boat reach the opposite shore?

Ans. (a) The resultant velocity can be found using the Pythagorean Theorem. The resultant is the hypotenuse of a right triangle with sides of 6 m/s and 3.8 m/s. It is

$$\text{SQRT} [(6\text{ m/s})^2 + (3.8\text{ m/s})^2] = 7.10\text{ m/s}$$

Its direction can be determined using a trigonometric function.

$$\text{Direction} = 360\text{ degrees} - \tan^{-1}[(3.8\text{ m/s}) / (6\text{ m/s})] = 327.6\text{ degrees}$$

NOTE: the direction of the resultant velocity (like any vector) is expressed as the counterclockwise direction of rotation function from due East.

(b) The time to cross the river is $t = d/v = (120\text{ m}) / (6\text{ m/s}) = 20.0\text{ s}$

(c) The distance traveled downstream is $d = v \cdot t = (3.8 \text{ m/s}) \cdot (20.0 \text{ m/s}) = 76 \text{ m}$.