CBSE
Class IX Mathematics
Term 1
Sample Paper – 1 Solution

Time: 3 hours **Total Marks: 90**

Section A

1. Correct answer: C
   π - 10

2. Correct answer: C
   $p(x) = x^3 + 10x^2 + px$
   $(x – 1)$ is the factor of $p(x)$
   So $p(1) = 0$
   $1 + 10 + p = 0$
   $P = -11$

3. Correct answer: B
   Two triangles will be congruent by SAS axiom if 2 sides and the included angle of one triangle are equal to the two sides and included angle of the other triangle. Thus, the triangles will be congruent when AC=DE.

4. Correct answer: A
   $$s = \frac{a+b+c}{2} = \frac{15+25+14}{2} = 27$$
   Using Heron's formula,
   $$\begin{aligned}\text{Area of a triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{27(27-15)(27-25)(27-14)} \\ &= \sqrt{27(12)(2)(13)} = 18\sqrt{26}\ \text{cm}^2\end{aligned}$$

Section B

5.

$$\left(\frac{81}{16}\right)^{-3/4}\times\left(\frac{25}{9}\right)^{-3/2}$$

$$=\left[\left(\frac{3}{2}\right)^{4}\right]^{-\frac{3}{4}}\times\left[\left(\frac{5}{3}\right)^{2}\right]^{-\frac{3}{2}}$$

$$=\left(\frac{3}{2}\right)^{-3}\times\left(\frac{5}{3}\right)^{-3}$$

$$=\left(\frac{2}{3}\right)^{3}\times\left(\frac{3}{5}\right)^{3}$$

$$=\frac{2^3}{3^3}\times\frac{3^3}{5^3}=\frac{2^3}{5^3}=\frac{8}{125}$$

6.

$$x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}$$

$$=\left(x^2+\frac{1}{x^2}+2\right)-2\left(x+\frac{1}{x}\right)$$

$$=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)$$

$$=\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}-2\right)$$

7.

(A) Point of the form (a, 0) lie on the x axis.
The point (-4, 0) will lie on the negative side of the x axis.

(B) (-, +) are the sign of the coordinate of points in the II quadrant.
$\therefore$ The point (-10, 2) lies in the II quadrant.

(C) Point of the form (0, a) lie on the y axis.
So, the point (0, 8) will lie on the positive side of y axis.

(D) (+, +) are the sign of the coordinates of points in the I quadrant.
$\therefore$ The point (10, 4) lies in the I quadrant.

8. Let x = 75, y = -25, z = -50

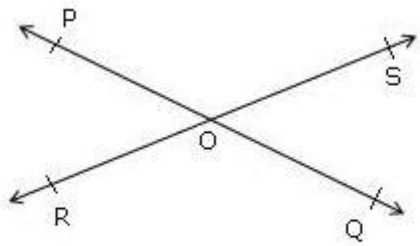
x + y + z = 75 - 25 - 50 = 0

We know, if x + y + z = 0 then $x^3 + y^3 + z^3 = 3xyz$

$75^3 - 25^3 - 50^3 = 3(75)(-25)(-50)$

= 281250

9.



$\angle POR + \angle ROQ = 180^{\circ}$ (Linear Pair)

But $\angle POR : \angle ROQ = 5 : 7$

$\therefore \angle POR = \frac{5}{12} \times 180^{\circ} = 75^{\circ}$

$\angle ROQ = \frac{7}{12} \times 180^{\circ} = 105^{\circ}$

$\angle POR = \angle SOQ = 75^{\circ}$ (Vertically opposite angles)

$\angle POS = \angle ROQ = 105^{\circ}$ (Vertically opposite angles)

10. AD is the bisector of $\angle A$

$\therefore \angle BAD = \angle CAD$

Exterior $\angle BDA > \angle CAD$

$\Rightarrow \angle BDA > \angle BAD$

$\Rightarrow AB > BD$ (side opposite the bigger angle is longer)

Section C

11.

$$\frac{(25)^{\frac{3}{2}} \times (343)^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$$

$$= \frac{(5^2)^{\frac{3}{2}} \times (7^3)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{9}{5}}}{2^5 \times 2^4 \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{9}{5}}}{2^9 \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{6}{5}}}{2^9}$$

12. Let $p(x) = x^3 + 13x^2 + 32x + 20$

$p(-1) = -1 + 13 - 32 + 20 = -33 + 33 = 0$

Therefore $(x + 1)$ is a factor of $p(x)$.

On dividing $p(x)$ by $(x + 1)$ we get

$p(x) \div (x + 1) = x^2 + 12x + 20$

Thus,

$x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20)$

$= (x + 1) (x^2 + 10x + 2x + 20)$

$= (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1) (x + 2) (x + 10)$

Hence, $x^3 + 13x^2 + 32x + 20 = (x + 1) (x + 2) (x + 10)$.

13.

LM = MN &. Given

$\Rightarrow \angle MLN = \angle MNL$ (angles opposite equal sides are equal)

$\Rightarrow \angle MLQ = \angle MNP$

LP = QN (Given)

$\Rightarrow$ LP + PQ = PQ + QN (adding PQ on both sides)

$\Rightarrow$ LQ = PN

In ΔLMQ and Δ NMP

LM = MN

$\angle MLQ = \angle MNP$

LQ = PN

$\Delta LMQ \cong \Delta NMP$ (SAS congruence rule)

14.

When $p(x) = ax^3 + 3x^2 - 3$ is divided by $(x - 4)$, the remainder is given by

$R_1 = a(4)^3 + 3(4)^2 - 3 = 64a + 45$

When $q(x) = 2x^3 - 5x + a$ is divided by $(x - 4)$, the remainder is given by

$R_2 = 2(4)^3 - 5(4) + a = 108 + a$

Given: $R_1 + R_2 = 0$

$\Rightarrow 65a + 153 = 0$

$\Rightarrow a = \dfrac{-153}{65}$

By hit and trial we find x = 3 is factor of given polynomial, as

$2(3)^3 - 9 - 39 - 6 = 54 - 54 = 0$

By dividing $2x^3 - x^2 - 13x - 6$ by $x - 3$ we get

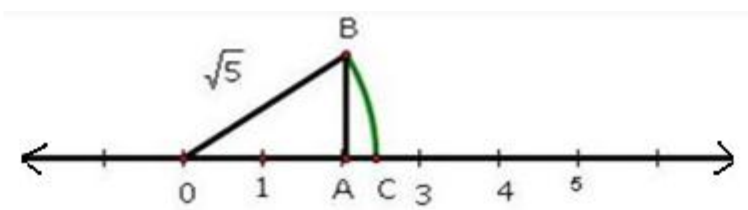
$2x^2 + 5x + 2$ as quotient.
Factorising this further
$2x^2 + 5x + 2 = 2x^2 + 4x + x + 2$ [1+ 4 = 5]
$= 2x(x + 2) + 1(x + 2)$
$= (2x + 1)(x + 2)$
So, $2x^3 - x^2 - 13x - 6 = (2x + 1)(x + 2)(x - 3)$

15.

In ΔDCB, $\angle DBC = \angle DCB$ (given)
DC = DB [Side opp. To equal $\angle$'s are equal].....(i)
In ΔABD and ΔACD
AB = AC (given)
BD = CD [from (i)]
AD = AD common
$\Delta ABD \cong \Delta ACD$ [SSS Rule]
$\angle BAD = \angle CAD$ (CPCT)
Hence, AD is bisector of $\angle BAC$

16.

Using Pythagoras theorem: $5 = 2^2 + 1^2$
Taking positive square root, we get, $\sqrt{5} = \sqrt{(2)^2 + (1)^2}$
(i) We mark a point 'A' representing 2 units on number line.
(ii) Construct AB of unit length perpendicular to OA. Then, join OB.
(iii) Taking O as centre and OB as radius, draw an arc intersecting number line at point C.
(iv) Point C represents $\sqrt{5}$ on number line.



17. In ΔAOD and ΔAOB

AD = AB (given)

AO =AO (common side)

OD = OB (given)

$\therefore \Delta AOD \cong \Delta AOB$ (SSS congruence rule)

$\therefore\ \angle AOD = \angle AOB$ (c.p.c.t)

Similarly, $\Delta DOC \cong \Delta BOC$ (SSS congruence rule)

$\therefore \angle DOC = \angle BOC$ (c.p.c.t)

$\angle AOD + \angle AOB + \angle DOC + \angle BOC = 360^o$ (angles at a point)

$2\angle AOD + 2\angle DOC = 360^o$

$\angle AOD + \angle DOC = 180^o$

Hence, AO and OC are in one and the same straight line.

18.

Given: $8x^3 + 27y^3 = 730$

$2x^2y + 3xy^2 = 15$

Now, $(2x + 3y)^3 = (2x)^3 + (3y)^3 + 3\ (2x)\ (3y)\ (2x + 3y)$

$= 8x^3 + 27y^3 + 18xy\ (2x + 3y)$

So, $(2x + 3y)^3 = 8x^3 + 27y^3 + 18\ (2x^2y + 3xy^2)$

$(2x + 3y)^3 = 730 + 18\ (15)$ [Using the given conditions]

$= 730 + 270$

$(2x + 3y)^3 = 1000$

Therefore, $2x + 3y = 10$

19.

In Δ PQR, PQ > PR.

$\Rightarrow \angle PRQ > \angle PQR$ (Angle opposite to longer side is greater)

$\Rightarrow \frac{1}{2}\angle PRQ > \frac{1}{2}\angle PQR$

$\Rightarrow \angle SRQ > \angle SQR$ (RS and QS are the bisectors of $\angle PRQ$ and $\angle PQR$)

In Δ SQR, $\angle SRQ > \angle SQR$ (Proved above)

$\Rightarrow$ SQ > SR (side opposite to greater angle is longer)

20.

Let p(x) = $x^3 - 23x^2 + 142x - 120$

Then p(1) = $(1)^3 - 23(1)^2 + 142(1) - 120 = 1 - 23 + 142 - 120 = 0$

Thus (x - 1) is a factor of p(x).

Now by long division

$$\begin{array}{r|l} & x^2 - 22x + 120 \\ \hline x-1 & x^3 - 23x^2 + 142x - 120 \\ & x^3 - x^2 \\ \hline & -22x^2 + 142x - 120 \\ & -22x^2 + 22x \\ \hline & 120x - 120 \\ & 120x - 120 \\ \hline & 0 \end{array}$$

Thus, $x^3 - 23x^2 + 142x - 120 = (x - 1)(x^2 - 22x + 120)$

$=(x - 1)(x^2 - 12x - 10x + 120)$

$=(x - 1)(x(x - 12) - 10(x - 12))$

$=(x - 1)(x - 12)(x - 10)$

Section D

21.

$$\frac{1}{3-\sqrt{8}} = \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} = \frac{3+\sqrt{8}}{9-8} = 3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}} = \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} = \frac{\sqrt{8}+\sqrt{7}}{8-7} = \sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$$

$$\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}+\sqrt{5}}{6-5} = \sqrt{6}-\sqrt{5}$$

$$\frac{1}{\sqrt{5}-2} = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$$

$$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

$$= 3+\sqrt{8} - \left(\sqrt{8}+\sqrt{7}\right) + \left(\sqrt{7}+\sqrt{6}\right) - \left(\sqrt{6}-\sqrt{5}\right) + \left(\sqrt{5}+2\right)$$

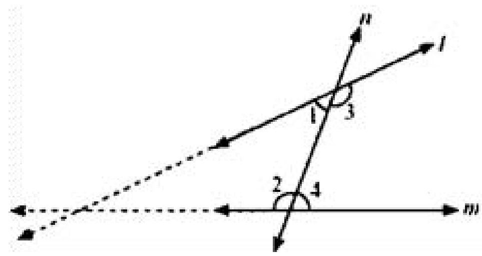
$$= 5$$

22. Euclid's 5th postulate states that:

If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

This implies that if n intersects lines l and m and if

$\angle 1 + \angle 2 < 180^0$, then $\angle 3 + \angle 4 > 180^0$. In that case, producing line l and further will meet in the side of $\angle 1$ and $\angle 2$ which is less than 180^o



If $\angle 1 + \angle 2 < 180^0$, then $\angle 3 + \angle 4 > 180^0$

In that case, the lines l and m neither meet at the side of $\angle 1$ and $\angle 2$ nor at the side of $\angle 3$ and $\angle 4$ implying that the lines l and m will never intersect each other. Therefore, the lines are parallel.

23.

Consider $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

We know that,

If x + y + x = 0 then $x^3 + y^3 + z^3 = 3xyz$

Now, $a^2 - b^2 + b^2 - c^2 - a^2 = 0$

And, $a - b + b - c + c - a = 0$

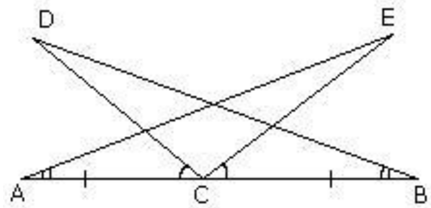
$$\therefore \frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$$

$$= \frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)}$$

$$= \frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$$

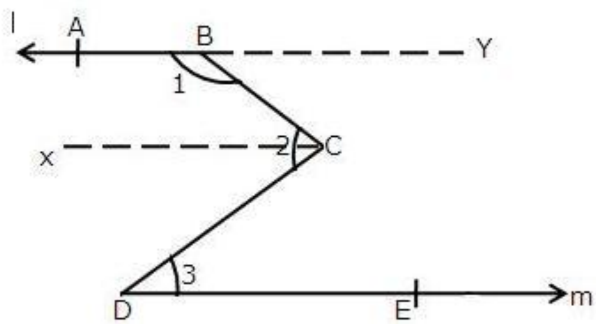
$$= (a+b)(b+c)(c+a)$$

24.



Given that ∠DCA = ∠ECB
⇒∠DCA + ∠ECD = ∠ECB + ∠ECD
⇒ ∠ECA = ∠DCB .....(i)
Now in ΔDBC and ΔEAC
∠ DCB = ∠ ECA [from (i)]
BC = AC (Given)
∠ DBC = ∠ EAC (Given)
Δ DBC ≅ Δ EAC (ASA Congruence)
Therefore, BD =AE (CPCT)

25.



Given: l||m
To prove: $\angle 1 + \angle 2 - \angle 3 = 180^o$
Construction: Draw XC||AB and extend AB to Y.
Proof:
$\angle BCX = 180^o - \angle 1$ (sum of int. $\angle_s$ on same side of transversal is 180^o)
$\angle XCD = \angle 3$ (Alternate int. $\angle_s$)
Now, $\angle 2 = \angle BCX + \angle XCD$
Or, $\angle 2 = (180^o - \angle 1) + \angle 3$
$\therefore \angle 1 + \angle 2 - \angle 3 = 180^o$

26.

On rationalising, $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$

$= \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$= \frac{19-9\sqrt{5}}{9-20}$

$= \frac{19-9\sqrt{5}}{-11}$

$= \frac{-19}{11} + \frac{9\sqrt{5}}{11}$

and, $a\sqrt{5} - b = \frac{9}{11}\sqrt{5} - \frac{19}{11}$

$\Rightarrow$ a = $\frac{9}{11}$ and b = $\frac{19}{11}$

27.

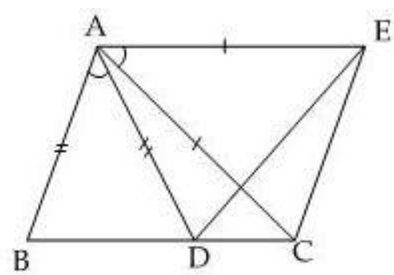
Given: AC = AE, AB = AD and ∠BAD = ∠EAC

To prove: BC = DE

Proof: ∠BAD = ∠EAC (given)

⇒∠BAD + ∠DAC = ∠EAC + ∠DAC

⇒∠BAC = ∠DAE



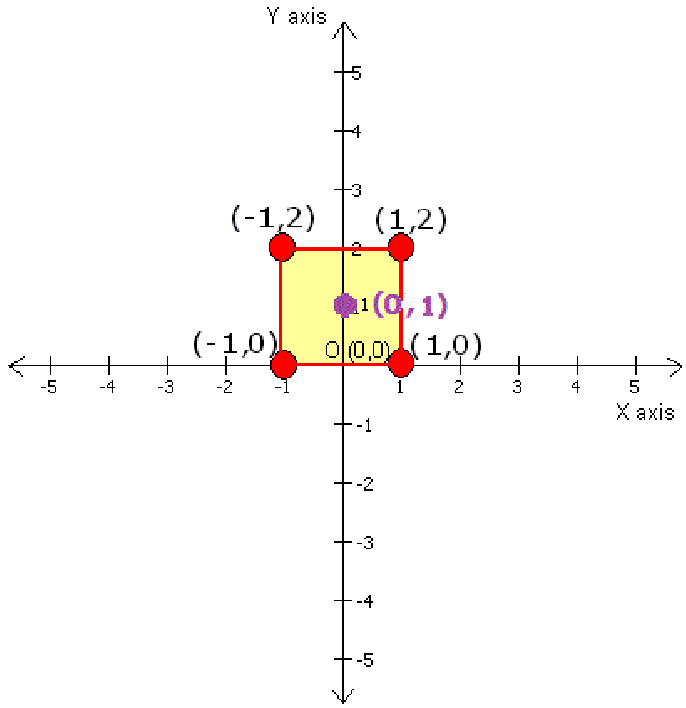
Now Δ ABC and Δ ADE

AB = AD
∠BAC = ∠DAE
AC = AE

By SAS rule Δ ABC ≅ ΔADE

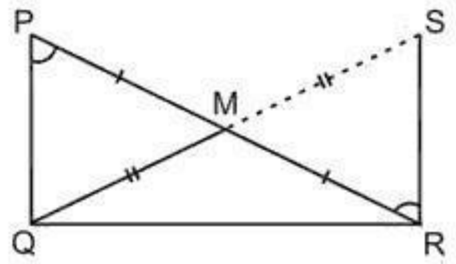
⇒ BC = DE

28. The points (-1,0) (1,0) (1,2) (-1,2) can be plotted on a graph as follows:



Shape of the lawn is square. Tree is to be planted at the centre of the lawn, so it must be planted at the point of intersection of its diagonals, i.e. at (0,1). Value indicated: Protection of environment

29.



Produce QM to S such that QM = MS. Join SR

In ΔPMQ and ΔRMS

PM = MR (M is the mid point)

QM = MS (By construction)

$\angle PMQ = \angle RMS$...vertically opposite angles

$\therefore \Delta PMQ \cong \Delta RMS$ (SAS congruence criterion)

$\therefore PQ = SR$ and $\angle QPM = \angle SRM$ (c.p.c.t)

$\angle QPM = \angle SRM$ (alternate angles) $\therefore RS \parallel PQ$

$\angle PQR + \angle QRS = 180^\circ$ (Co-interior angles)

$\Rightarrow 90^\circ + \angle QRS = 180^\circ$

$\Rightarrow \angle QRS = 90^\circ$

In ΔPQR and ΔQRS

QR = RQ (common)

PQ = SR

$\angle PQR = \angle QRS$ (90° each)

$\therefore \Delta PQR \cong \Delta SRQ$ (SAS congruence criterion)

$\therefore PR = QS \Rightarrow \frac{1}{2}SQ = \frac{1}{2}PR$

$\Rightarrow \frac{1}{2}SQ = QM = \frac{1}{2}PR$

Hence, proved

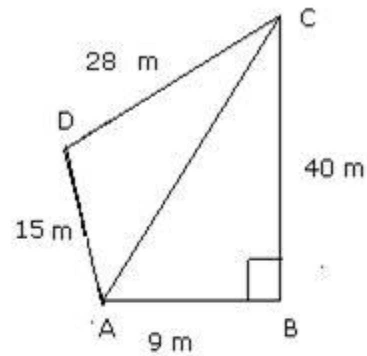
30. Let ABCD be the garden.

Δ ABC,

$AC^2 = 9^2 + 40^2 = 1681$

AC = 41

Area of the garden = Area of Δ ABC + area of ΔACD



Area of Δ ABC $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 9 \times 40 = 180 cm^2$

Area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$\left[\because s = \frac{15+28+41}{2} = 42m\right]$

$\therefore$ Area of $\Delta ACD = \sqrt{42 \times 27 \times 14 \times 1} = 7 \times 3 \times 3 \times 2 = 126 m^2$

$\therefore$ Area of the garden = 180 + 126 = 306 m^2.

31. Let $f(x) = x^3 + 2x^2 - 5ax - 8$ and

$g(x) = x^3 + ax^2 - 12x - 6$

When divided by (x-2) and (x-3), f(x) and g(x) leave remainder p and q respectively

$F(x) = x^3 + 2x^2 - 5ax - 8$

$\therefore f(2) = 2^3 + 2 \times 2^2 - 5a \times 2 - 8$

$= 8 + 8 - 10a - 8$

$p = 8 - 10a$ _ _ _ _ _ (1)

$g(x) = x^3 + ax^2 - 12x - 6$

$g(3) = 3^3 + a \times 3^2 - 12 \times 3 - 6$

$= 27 + 9a - 36 - 6$

$\therefore q = -15 + 9a$ _ _ _ _ (2)

If q – p = 10

$\Rightarrow -15 + 9a - 8 + 10a = 10$

$\Rightarrow 19a - 23 = 10$

$\Rightarrow 19a = 33$

$\therefore a = \frac{33}{19}$