PU-CET-2009

M

Code No.: 209101

Important : Please consult your Admit Card / Roll No. Slip before filling your Roll Number on the Test Booklet and Answer Sheet

Roll No. *In Figures* *In Words*

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O.M.R. Answer Sheet Serial No.

Signature of the Candidate : ______________________________

Paper : V

Subject : Mathematics

Time : 70 minutes **Number of Questions : 60** **Maximum Marks : 120**

DO NOT OPEN THE SEAL ON THE BOOKLET UNTIL ASKED TO DO SO

INSTRUCTIONS

1. Write your Roll No. on the Question Booklet and also on the OMR Answer Sheet in the space provided and nowhere else.
2. Enter the Subject and Code No. of Question Booklet on the OMR Answer Sheet. Darken the corresponding bubbles with **Black Ball Point / Black Gel pen.**
3. Do not make any identification mark on the Answer Sheet or Question Booklet.
4. To open the Question Booklet remove the paper seal (s) gently when asked to do so.
5. Please check that this Question Booklet contains **60** questions. In case of any discrepancy, inform the Assistant Superintendent within 10 minutes of the start of test.
6. Each question has four alternative answers (A, B, C, D) of which only one is correct. For each question, darken only one bubble (A or B or C or D), whichever you think is the correct answer, on the Answer Sheet with **Black Ball Point / Black Gel pen.**
7. If you do not want to answer a question, leave all the bubbles corresponding to that question blank in the Answer Sheet. No marks will be deducted in such cases.
8. Darken the bubbles in the OMR Answer Sheet according to the Serial No. of the questions given in the Question Booklet.
9. Negative marking will be adopted for evaluation i.e., 1/4th of the marks of the question will be deducted for each wrong answer. A wrong answer means incorrect answer or wrong filling of bubble.
10. For calculations, use of simple log tables is permitted. Borrowing of log tables and any other material is not allowed.
11. For rough work only the sheets marked "Rough Work" at the end of the Question Booklet be used.
12. The Answer Sheet is designed for **computer evaluation**. Therefore, if you do not follow the instructions given on the Answer Sheet, it may make evaluation by the computer difficult. **Any resultant loss to the candidate on the above account, i.e., not following the instructions completely, shall be of the candidate only.**
13. After the test, hand over the Question Booklet and the Answer Sheet to the Assistant Superintendent on duty.
14. In no case the Answer Sheet, the Question Booklet, or its part or any material copied/ noted from this Booklet is to be taken out of the examination hall. Any candidate found doing so would be expelled from the examination.
15. A candidate who creates disturbance of any kind or changes his/her seat or is found in possession of any paper possibly of any assistance or found giving or receiving assistance or found using any other unfair means during the examination will be expelled from the examination by the Centre Superintendent/ Observer whose decision shall be final.
16. **Telecommunication equipment such as pager, cellular phone, wireless, scanner, etc., is not permitted inside the examination hall. Use of calculators is not allowed.**

1. **If A, B and C are any three sets, then $A \times (B \cap C)$ is equal to :**

(A) $(A \times B) \cup (A \times C)$ (B) $(A \times B) \cap (A \times C)$

(C) $(A \cup B) \times (A \cup C)$ (D) $(A \cap B) \times (A \cap C)$

2. **Let X, Y, Z be three sets such that $X \cup Y = X \cup Z$ and $X \cap Y = X \cap Z$, then :**

(A) $X = Y$ (B) $X = Z$

(C) $Y = Z$ (D) $X = Y = Z$

3. **Let X and Y be two sets each with 10 elements. Then the number of all possible bijections from X to Y is :**

(A) 20 (B) 10 !

(C) 100 (D) 10^{10}

4. **A mapping $f : X \rightarrow Y$ is one-one if :**

(A) $x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$ (B) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

(C) $f(x_1) = f(x_2) \not\Rightarrow x_1 = x_2$ (D) $f(x_1) \neq f(x_2)$, for all $x_1, x_2 \in X$

5. **If $\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{2}{3} = \sin^{-1}x$, then the value of x is :**

(A) 0 (B) $\frac{\sqrt{5} - 4\sqrt{2}}{9}$

(C) $\frac{\sqrt{5} + 4\sqrt{2}}{9}$ (D) $\frac{\pi}{2}$

6. **If $3\sin\alpha = 5\sin\beta$, then $\tan\left(\frac{\alpha + \beta}{2}\right)\cot\left(\frac{\alpha - \beta}{2}\right)$ equals :**

(A) 1 (B) 2

(C) 3 (D) 4

7. **If $\tan(\theta + x)\tan(\theta - x) = 1$ for all x, then the value of θ must be :**

(A) 0^o (B) 30^o

(C) 45^o (D) 60^o

8. **The smallest positive integer n, for which $n! < \left(\frac{n+1}{2}\right)^n$ holds, is :**

(A) 1 (B) 2

(C) 3 (D) 4

9. **The constant term in the expansion of $\left(x + \frac{2}{x}\right)^6$ is :**

(A) 156 (B) 165

(C) 162 (D) 160

10. A value of $\sqrt{i} + \sqrt{-i}$ is :

(A) 0 (B) $\sqrt{2}$

(C) $-i$ (D) i

11. The quadratic equation $8\sec^2\theta - 6\sec\theta + 1 = 0$ has :

(A) infinitely many roots (B) exactly two roots

(C) exactly four roots (D) no root

12. There are five roads leading to a town from a village. The number of different ways in which a villager can go to the town and return back is :

(A) 25 (B) 20

(C) 10 (D) 5

13. A box contains two white balls, three black balls and four red balls. The number of ways in which three balls can be drawn from the box so that at least one of the balls is black is :

(A) 74 (B) 84

(C) 64 (D) 20

14. If the sum of first n positive integers is $\frac{1}{5}$ times the sum of their squares, then n equals :

(A) 5 (B) 6

(C) 7 (D) 8

15. Sum to infinity of the series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots$ is :

(A) $\frac{16}{35}$ (B) $\frac{11}{8}$

(C) $\frac{35}{16}$ (D) $\frac{8}{11}$

16. The equation of the straight line which passes through the point (1, –2) and cuts off equal intercepts from the axes will be :

(A) $x + y = 1$ (B) $x - y = 1$

(C) $x + y + 1 = 0$ (D) $x - y - 2 = 0$

17. If the sum of the distances of a point from two perpendicular lines in the plane is 1, then its locus is :

(A) a straight line (B) a circle

(C) a parabola (D) an ellipse

18. The latus rectum of the hyperbola $9x^2 - 16y^2 - 18x - 32y - 151 = 0$ is :

(A) $\frac{9}{4}$ (B) 9

(C) $\frac{3}{2}$ (D) $\frac{9}{2}$

19. **If one end of the diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ is (–3, 2), then the other end is :**

(A) (5, 3) (B) (6, 2)

(C) (1, –8) (D) (11, 2)

20. **The angle between the two planes $3x - 4y + 5z = 0$ and $2x - y - 2z = 5$ is :**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

21. **The distance of the point (2, 3, 4) from the plane $3x - 6y + 2z + 11 = 0$ is :**

(A) 9 (B) 10

(C) 2 (D) 1

22. **If $y = \log\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by :**

(A) 1 (B) 0

(C) $\frac{1}{2}$ (D) ∞

23. **The derivative of $\sin^{-1}x$ with respect to $\cos^{-1}\sqrt{1-x^2}$ is :**

(A) $\frac{1}{\sqrt{1-x^2}}$ (B) 1

(C) $\cos^{-1}x$ (D) $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$

24. **$\frac{d}{dx}\left[\tan^{-1}(\sec x + \tan x)\right]$ is equal to :**

(A) 0 (B) $\sec x - \tan x$

(C) $\frac{1}{2}$ (D) 2

25. **If f(x) is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = 2$ then $f'(-3)$ equals :**

(A) 0 (B) 1

(C) 2 (D) 4

26. **The variance of the data 2, 4, 6, 8, 10 is :**

(A) 6 (B) 7

(C) 8 (D) 40

27. **A group of 10 items has mean 6. If the mean of 4 of these items is 7.5, then the mean of the remaining items is :**

(A) 6.5 (B) 5.5

(C) 4.5 (D) 5.0

28. Which one of the following measures of marks is the most suitable one of central location for computing intelligence of students ?

(A) Mode (B) Arithmetic Mean

(C) Geometric Mean (D) Median

29. There are four letters and four addressed envelopes. The probability that all letters are not dispatched in the right envelope is :

(A) $\frac{20}{24}$ (B) $\frac{21}{24}$

(C) $\frac{22}{24}$ (D) $\frac{23}{24}$

30. A and B are two independent events. The probability that both A and B occurs is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. Then the probability of the two events are respectively :

(A) $\frac{1}{2}$ and $\frac{1}{3}$ (B) $\frac{1}{5}$ and $\frac{1}{6}$

(C) $\frac{1}{2}$ and $\frac{1}{6}$ (D) $\frac{2}{3}$ and $\frac{1}{4}$

31. If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1} x + 2\sin^{-1} x)$ is :

(A) $-\sqrt{\frac{24}{25}}$ (B) $\sqrt{\frac{24}{25}}$

(C) $-\frac{1}{5}$ (D) $\frac{1}{5}$

32. The equation $2\cos^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ is valid for all x satisfying :

(A) $-1 \le x \le 1$ (B) $0 \le x \le 1$

(C) $0 \le x \le \frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{2}} \le x \le 1$

33. If g = {(1, 1), (2, 3), (3, 5), (4, 7)} is a function described by $g(x) = \alpha x + \beta$, then what values should be assigned to α and β ?

(A) $\alpha = 1, \beta = 1$ (B) $\alpha = 2, \beta = -1$

(C) $\alpha = 1, \beta = -2$ (D) $\alpha = -2, \beta = -1$

34. A function f from the set of natural numbers to integers defined by $f(n) = \begin{cases} \frac{n-1}{2} & n \text{ odd} \\ -\frac{n}{2} & n \text{ even} \end{cases}$ is :

(A) neither one-one nor onto

(B) one-one but not onto

(C) onto but not one-one

(D) both one-one and onto

35. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$. Then the value of α for which $A^2 = B$ is :

(A) 1

(B) –1

(C) i

(D) –i

36. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$. Then α is equal to :

(A) ± 3

(B) ± 2

(C) ± 5

(D) 0

37. The value of x for which the matrix $A = \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$ is the inverse of $B = \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$ is :

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{5}$

38. If $a + b + c > 0$ and $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$. Then :

(A) $\Delta > 0$

(B) $\Delta \leq 0$

(C) $\Delta < 0$

(D) $\Delta = 0$

39. If $f(x) = \begin{vmatrix} 1 & 3\cos x & 1 \\ \sin x & 1 & 3\cos x \\ 1 & \sin x & 1 \end{vmatrix}$. Then the maximum value of f(x) is :

(A) 10

(B) 5

(C) –10

(D) –5

40. If $f(x)=\begin{cases}\dfrac{[x]^2+\sin[x]}{[x]} & , [x]\neq 0\\ 0 & , [x]=0\end{cases}$, where [x] denotes the greatest integer less than or equal to x.

Then $\lim_{x\to 0} f(x)$ is :

(A) 1 (B) –1

(C) does not exist (D) 0

41. If the function $f(x)=\begin{cases}\dfrac{x^2-(A+2)x+A}{x-2} & , x\neq 2\\ 2 & , x=2\end{cases}$, is continuous at x = 2, then :

(A) A = 0 (B) A = 1

(C) A = –1 (D) A = 4

42. If $\lim_{x\to 0}\dfrac{x(1+a\cos x)-b\sin x}{x^3}=1$. Then :

(A) $a=\frac{5}{2},\ b=\frac{3}{2}$ (B) $a=-\frac{5}{2},\ b=-\frac{3}{2}$

(C) $a=\frac{5}{2},\ b=-\frac{3}{2}$ (D) $a=-\frac{5}{2},\ b=\frac{3}{2}$

43. If $\cos^{-1}\left(\dfrac{x^2-y^2}{x^2+y^2}\right)=\log a$, then $\dfrac{dy}{dx}$ is equal to :

(A) $\frac{y}{x}$ (B) $\frac{x}{y}$

(C) $\frac{x^2}{y^2}$ (D) $\frac{y^2}{x^2}$

44. The length of the longest interval in which the function $3\sin x-4\sin^3 x$ is increasing is :

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{2}$ (D) π

45. A curve passes through the point (2, 0) and the slope of the tangent at any point (x, y) is x^2-2x for all values of x. Then $3y_{max}$ is equal to :

(A) $\frac{4}{3}$ (B) 4

(C) $\frac{3}{4}$ (D) 12

46. If $I = \int \frac{dx}{\sqrt{(1-x)(x-2)}}$. Then I equals :

(A) $\sin^{-1}(2x+5)+c$ (B) $\sin^{-1}(2x-3)+c$

(C) $\sin^{-1}(3-2x)+c$ (D) $\sin^{-1}(5-2x)+c$

47. If $I = \int \sqrt{\frac{5-x}{2+x}}\,dx$. Then I equals :

(A) $\sqrt{x+2}\,\sqrt{5-x} + 3\sin^{-1}\sqrt{\frac{x+2}{3}} + c$ (B) $\sqrt{x+2}\,\sqrt{5-x} + 7\sin^{-1}\sqrt{\frac{x+2}{7}} + c$

(C) $\sqrt{x+2}\,\sqrt{5-x} + 5\sin^{-1}\sqrt{\frac{x+2}{5}} + c$ (D) $\sqrt{x+2}\,\sqrt{5+x} + 5\sin^{-1}\sqrt{\frac{x+2}{5}} + c$

48. If $I = \int_3^5 \frac{\sqrt{x}}{\sqrt{8-x}+\sqrt{x}}\,dx$. Then I equals :

(A) 1 (B) 2

(C) 3 (D) 3.5

49. If $I = \int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}}$. Then I equals :

(A) $\frac{1}{2}\log\frac{5}{3}$ (B) $2\log\frac{1}{3}$

(C) $\frac{1}{2}\log\frac{1}{5}$ (D) $2\log\frac{5}{3}$

50. The area bounded by the curves $x = y^2$ and $x = 3 - 2y^2$ is :

(A) 2 (B) $\frac{4}{3}$

(C) 4 (D) $\frac{3}{4}$

51. The differential equation of all the circles passing through the origin and having centres on x – axis is :

(A) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (B) $x^2 = y^2 + xy\frac{dy}{dx}$

(C) $y^2 = x^2 - 2xy\frac{dy}{dx}$ (D) $x^2 = y^2 + 3xy\frac{dy}{dx}$

52. The solution of $\frac{dy}{dx} = \log(x+1)$ is :

(A) $(x+1)\log(x+1)+x+c$ (B) $x\log x - x + c$

(C) $(x+1)\log x - (x+1) + c$ (D) $(x+1)\log(x+1) - x + c$

53. **If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$ and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ then θ is equal to :**

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

54. **The values of a for which the points A, B, C with position vectors $2i - j + k$, $i - 3j - 5k$ and $ai - 3j + k$ respectively are the vertices of a right angled triangle with $C = \frac{\pi}{2}$ are :**

(A) 2 and 1

(B) –2 and –1

(C) –2 and 1

(D) 2 and –1

55. **The angle between the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and the plane $3x + 4y + z + 5 = 0$ is :**

(A) $\sin^{-1} \frac{7}{\sqrt{26}}$

(B) $\sin^{-1} \sqrt{\frac{7}{26}}$

(C) $\sin^{-1} \sqrt{\frac{7}{52}}$

(D) $\sin^{-1} \sqrt{\frac{7}{13}}$

56. **The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is :**

(A) 0°

(B) 30°

(C) 45°

(D) 90°

57. **The maximum value of $z = 4x + y$ subject to**

$$x + y \leq 50$$
$$3x + y \leq 90$$
$$x \geq 0, \quad y \geq 0$$

is :

(A) 120

(B) 110

(C) 60

(D) 50

58. **The corner points of a feasible region determined by the following system of linear inequalities**

$$2x + y \leq 10$$
$$x + 3y \leq 15$$
$$x, y \geq 0$$

are (0, 0), (5, 0), (3, 4) and (0, 5). Let $z = px + qy$, $p > 0$, $q > 0$. The maximum of z occurs at both (3, 4) and (0, 5). Then :

(A) $p = q$

(B) $p = 2q$

(C) $p = 3q$

(D) $q = 3p$

59. **A box contains 16 bulbs out of which 4 are defective. 3 bulbs are chosen without replacement. What is the probability that one of the bulbs drawn is defective ?**

(A) $\frac{11}{28}$ (B) $\frac{9}{70}$

(C) $\frac{31}{40}$ (D) $\frac{33}{70}$

60. **Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = p$, $P(A \cup B) = \frac{3}{5}$. The value of p if A and B are independent is :**

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$

(C) $\frac{1}{10}$ (D) $\frac{3}{10}$