ANSWERS & HINTS
for
WBJEE - 2013
SUB : MATHEMATICS

CATEGORY - I

Q. 1 – Q. 60 carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of 1/3 mark

1. A point P lies on the circle $x^2 + y^2 = 169$. If Q = (5,12) and R = (–12,5), then the a gle QPR is

(A) $\frac{\ }{6}$ (B) $\frac{\ }{4}$ (C) $\frac{\ }{3}$ (D) $\frac{\ }{2}$

Ans : (B)

Hints : Q (5,12) R (–12,5)

0 (0,0) $m_{OQ} = \frac{12}{5}$, $m_{OR} = \frac{5}{12}$

$m_{OQ} . m_{OR} = -1$, so QOR = /2 Hence QPR = /4

2. A circle passing through (0,0), (2,6), (6,2) cuts the x axis a the point P (0,0). Then the length of OP, where O is origin, is

(A) $\frac{5}{2}$ (B) $\frac{5}{\sqrt{2}}$ (C) 5 (D) 10

Ans : (C)

Hints : Circle passes through (0,0)

so, $x^2+y^2+2gx+2fy = 0$

(2,6) & (6,2) lies on it so, $2^2+6^2+4g+12f = 0$ — (1)

$2^2+6^2+12g+4f = 0$ — (2)

From (1) & (2),

$g = f = -5/2$

Eqn. of circle is $x^2+y^2-5x-5y=0$

For y = 0, x(x–5)=0 x=0, x=5

OP = 5

3. The locus of the midpoints of the chords of an ellpse $x^2+4y^2 = 4$ that are drawn form the positive end of the minor axis, is

(A) a circle with centre $\frac{1}{2},0$ and radius 1

(B) a parabola with focus $\frac{1}{2},0$ and directrix x = –1

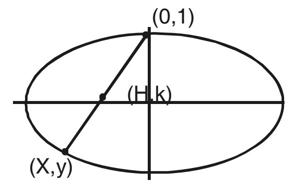
(C) an ellipse with centre $0,\frac{1}{2}$, major axis 1 and minor axis $\frac{1}{2}$

(D) a hyperbola with centre $0,\frac{1}{2}$, transverse axis 1 and conjugate axis $\frac{1}{2}$

Ans : No option is correct

Hints : Positive end of minor axis (0,1) but mid-pt be (h,k) x=2h, y=2k–1 lies on ellipse

$4h^2 + 4(2k–1)^2 = 4 \qquad \frac{h^2}{1} \quad \frac{\left(k \quad \frac{1}{2}\right)^2}{\frac{1}{4}} \quad 1$



Here on ellipse of centre $(0, \frac{1}{2})$, major axis 2, minor axis 1

4. A point moves so that the sum of squares of its distances from the points (1,2) and (–2,1) is always 6. Then its locus is

(A) the straight line $y \quad \frac{3}{2} \quad 3 \ x \quad \frac{1}{2}$

(B) a circle with centre $\frac{1}{2},\frac{3}{2}$ and radius $\frac{1}{\sqrt{2}}$

(C) a parabola with focus (1,2) and directix pssing through (–2,1)

(D) an ellipse with foci (1,2) and (–2,1)

Ans : (B)

Hints : Let (h,k) be co-ordinates of the point

$(h–1)^2 + (k–2)^2 + (h+2)^2 + (k–1)^2 = 6$

$h^2+k^2+h – 3k + 2 = 0$

Circle with centre $\frac{1}{2}, {}^{3}\!/_{2}$ radius = $\frac{\ }{\sqrt{2}}$

5. For the variable t, the locus of the points of intersection of lines x–2y = t and x+2y= $\frac{1}{t}$ is

(A) the straight line x=y

(B) the circle with centre at the origin and radius 1

(C) the ellipse with centre at the origin and one focus $\frac{2}{\sqrt{5}},0$

(D) the hyperbola with centre at the origin and one focus $\frac{\sqrt{5}}{2},0$

Ans : (D)

Hints : (x–2y) (x+2y) = 1 $\quad x^2 - 4y^2 = 1$

$\frac{x^2}{1} \quad \frac{y^2}{\frac{1}{4}} \quad 1$

$a = 1,\ b = \frac{1}{2}\ \ e = \frac{\sqrt{5}}{2}$ focus $\ \frac{\sqrt{5}}{2}, 0$

6. Let $P = \begin{array}{cc} \cos\frac{}{4} & \sin\frac{}{4} \\ \sin\frac{}{4} & \cos\frac{}{4} \end{array}$ and $X = \begin{array}{c} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{array}$. Then P^3X is equal to

(A) $\begin{array}{c} 0 \\ 1 \end{array}$ (B) $\begin{array}{c} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{array}$ (C) $\begin{array}{c} 1 \\ 0 \end{array}$ (D) $\begin{array}{c} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{array}$

Ans : (C)

Hints : $P^2 = \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}$

$P^3 = \begin{array}{cc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array}$

$P^3 X = \begin{array}{c} 1 \\ 0 \end{array}$

7. The number of solutions of the equation x+y+z = 10 in positive integers x,y,z is equal to

(A) 36 (B) 55 (C) 72 (D) 45

Ans : (A)

Hints : ${}^{10-1}C_{3-1} = {}^9C_2 = 36$

8. For 0 P,Q $\frac{}{2}$, if sin P + cos Q=2, then the value of tan $\frac{P\quad Q}{2}$ is equal to

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

Ans : (A)

Hints : P = $/2$, Q = 0

9. If and are the roots of $x^2 - x + 1 = 0$, then the value of $^{2013} + \ ^{2013}$ is equal to

(A) 2 (B) –2 (C) –1 (D) 1

Ans : (B)

Hints : $= - \ , - \ ^2$

$- \ ^{2013} - \ ^{2x2013} = -(\ ^3)^{671} - (\ ^3)^{2x671} = -2$

10. The value of the integral

$$ {}_{1}^{1} \frac{x^{2013}}{e^{|x|} \quad x^{2} \quad \cos x} \quad \frac{1}{e^{|x|}} \ dx $$

is equal to

(A) 0 (B) $1-e^{-1}$ (C) $2e^{-1}$ (D) $2 \ 1 \ \ e^{\ 1}$

Ans : (D)

Hints : $\frac{x^{2013}}{e^{|x|} \quad x^{2} \quad \cos x}$ is odd

$$ I = {}_{1}^{1} \frac{1}{e^{|x|}} dx \quad 2 {}_{0}^{1} e^{\ x} dx \quad 2 \ 1 \quad e^{\ 1} $$

11. Let

$f(x) = 2^{100} x+1,$

$g(x) = 3^{100} x+1.$

Then the set of real numbers x such that $f(g(x)) = x$ is

(A) empty (B) a singleton (C) a finite se with more than one element

(D) infinite

Ans : (B)

Hints : $f(x) = 2^{100}x+1$; $g(x) = 3^{100} x+1$

$$ f(g(x)) = x \qquad x = \frac{1 \quad 2^{100}}{6^{100} \quad 1} $$

12. The limit of $x \sin(e^{1/x})$ as $x \quad 0$

(A) is equal to 0 (B) is equal to 1 (C) is equal to e/2 (D) does not exist

Ans : (A)

Hints : $-1 \quad \sin e^{1/x} \quad 1, -x \quad x\sin(e^{1/}) \quad x$

$$ \lim_{x \quad 0} x\sin(e^{1/x}) = \lim_{x \quad 0} x = 0, $$

13. Let $I = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$ and $P = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{matrix}$. Then the matrix P^3+2P^2 is equal to

(A) P (B) I–P (C) 2I+P (D) 2I–P

Ans : (C)

Hints : $|P \quad I| \quad 0$, characteristics equation of P is $P^3+2P^2-P-2I = 0$

$$ P^3+2P^2=P+2I $$

14. If , are the roots of the quadratic equation $x^2+ax+b=0$, (b 0); then the quadratic equation whose roots are $\frac{1}{}$, $\frac{1}{}$ is

(A) $ax^2+a(b-1)x+(a-1)^2=0$ (B) $bx^2+a(b-1)x+(b-1)^2=0$ (C) $x^2+ax+b = 0$ (D) $abx^2+bx+a = 0$

Ans : (B)

Hints : + = –a, = b

$= -\frac{1}{}, \quad \frac{1}{}$

$+ \quad = \frac{a\ b\ \ 1}{b} \qquad \frac{b\ \ 1^{2}}{b}$

Equation is $x^2-(\ +\ )\ x +\ \ = 0$

$bx^2 + a(b-1)x + (b-1)^2 = 0$

15. The value of $1000\ \frac{1}{1x2}\ \frac{1}{2x3}\ \frac{1}{3x4}\ \cdots\cdots\ \frac{1}{999x1000}$ is equal to

(A) 1000 (B) 999 (C) 1001 (D) 1/999

Ans : (B)

Hints : $1000\ \ 1\ \ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{3} \qquad \frac{1}{999}\ \frac{1}{1000}$

$= 1000\ \ 1\ \ \frac{1}{1000}$

= 999

16. The value of the determinant

$$\begin{vmatrix} 1\ \ a^2\ \ b^2 & 2ab & 2b \\ 2ab & 1\ \ a^2\ \ b^2 & 2a \\ 2b & -2a & 1\ \ a^2\ \ b^2 \end{vmatrix}$$

is equal to

(A) 0 (B) $(1+a^2+b^2)$ (C) $(1+a^2+b^2)^2$ (D) $(1+a^2+b^2)^3$

Ans : (D)

Hints : $(1+a^2+b^2)^3$

$C_1 \quad C_1 - bC_3,\ C_2 \quad aC_3 + C_2$

$$\begin{matrix} 1\ \ a^2\ \ b^2 & 0 & 2b \\ 0 & 1\ \ a^2\ \ b^2 & 2a \\ b\ 1\ \ a^2\ \ b^2 & a\ 1\ \ a^2\ \ b^2 & 1\ \ a^2\ \ b^2 \end{matrix}$$

$$= (1+a^2+b^2)^2 \begin{matrix} 1 & 0 & 2b \\ 0 & 1 & 2a \\ b & a & 1\ \ a^2\ \ b^2 \end{matrix}$$

$= (1+a^2+b^2)^3$

17. If the distance between the foci of an ellipse is equal to the length of the latus rectum, then its eccentricity is

(A) $\frac{1}{4}\ \sqrt{5}\quad 1$ (B) $\frac{1}{2}\ \sqrt{5}\quad 1$ (C) $\frac{1}{2}\ \sqrt{5}\quad 1$ (D) $\frac{1}{4}\ \sqrt{5}\quad 1$

Ans : (C)

Hints : $2ae = \frac{2b^2}{a}$

$e = \frac{b^2}{a^2} = 1 - e^2$

$e^2 + e - 1 = 0$

$e = \frac{\sqrt{5}\quad 1}{2}$

18. For the curve $x^2+4xy+8y^2=64$ the tangents are parallel to the x-axis only at the points

(A) $(0,2\sqrt{2})$ and $(0,-2\sqrt{2})$

(B) (8,–4) and (–8,4)

(C) $8\sqrt{2},\quad 2\sqrt{2}$ and $8\sqrt{2},2\sqrt{2}$

(D) (8,0) and (–8,0)

Ans : (B)

Hints : $x^2+4xy+8y^2 = 64$

$2x+4xy +4y+16yy =0$

$(4x+16y)y = -(2x+4y)$

$2x + 4y = 0$

19. The value of $I = \ {}_{0}^{\overline{\ \ }4}\ \tan^{n\ \ 1} x\ \ dx\quad \frac{1}{2}\ {}_{0}^{\overline{\ \ }2}\tan^{n\ \ 1}\ \ x/2\ \ dx$ is equal to

(A) $\frac{1}{n}$ (B) $\frac{n\quad 2}{2n\quad 1}$ (C) $\frac{2n\quad 1}{n}$ (D) $\frac{2n\quad 3}{3n\quad 2}$

Ans : (A)

Hints : $I = \ {}_{0}^{/4}\tan^{n\ \ 1} x.dx\quad \frac{\ }{2}\ {}_{0}^{/2}\tan^{n\ \ 1}\ \frac{x}{2}\ \ dx$

For second integral substitute $\frac{x}{2}\quad y$

$I = \ {}_{0}^{/4}\ \tan^{n\ \ 1} x\quad \tan^{n\ \ 1} x\ \ dx$

${}_{0}^{/4}\tan^{n\ \ 1} x.\sec^2 x\,dx\quad \frac{\tan^n x}{n}\Big|_0^{/4}\quad \frac{1}{n}$

20. Let f( ) = (1+sin^2)(2–sin^2). Then for all values of

(A) f( ) > $\frac{9}{4}$ (B) f( ) <2 (C) f( )> $\frac{11}{4}$ (D) 2 f( ) $\frac{9}{4}$

Ans : (D)

Hints : f( ) = (1+sin^2) (2–sin^2)

f( )=(1+sin^2)(1+cos^2)

=2+sin^2 cos^2

=2+$\frac{1}{4}$ sin^2

2 f( ) $\frac{9}{4}$

21. Let f(x) = $\begin{cases} x^3 \quad 3x \quad 2, & x \quad 2 \\ x^3 \quad 6x^2 \quad 9x \quad 2, & x \quad 2 \end{cases}$

Then

(A) $\lim_{x \quad 2} f(x)$ does not exist

(B) f is not continuous at x = 2

(C) f is continuous but not differentiable at x = 2

(D) f is continuous and differentiable at x = 2

Ans : (C)

Hints : $\lim_{x \quad 2}$ f(x) = 4

$\lim_{x \quad 2}$ f(x) = 4

f (x) = $\begin{cases} 3x^2 \quad 3, & x \quad 2 \\ 3x^2 \quad 12x \quad 9, & x \quad 2 \end{cases}$

so L.H.D at x = 2 is 9, R.H.D at x = 2 is –3

so f(x) is continuous but not differentiable at x = 2

22. The limit of $\sum_{n \quad 1}^{1000} (\quad 1)^n x^n$ as x

(A) does not exist (B) exists and equals to 0 (C) exists and approaches (D) exists and approaches

Ans : (C)

Hints : $\lim_{x} (\quad x \quad x^2 \quad x^3 \quad x^4 .... \quad .... \quad x^{1000})$

= $\lim_{x} (\quad x).\frac{(\quad x)^{1000} \quad 1}{x \quad 1}$ = $\lim_{x} \frac{x^{1001} \quad x}{x \quad 1}$ =

23. If f(x) = e^x (x – 2)^2 then

(A) f is increasing in ,0 and 2, and decreasing in (0, 2)

(B) f is increasing in ,0 and decreasing in 0,

(C) f is increasing in 2, and decreasing in ,0

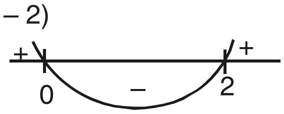
(D) f is increasing in (0, 2) and decreasing in ,0 and 2,

Ans : (A)

Hints : f (x) = e^x [(x –2)^2 + 2(x – 2)]

= e^x [x^2 – 2x] = e^x.x(x – 2)

sign scheme of f (x) will be



so f is increasing in (– , 0) and (2,) and decreasing in (0, 2)

24. Let f : $\mathbb{R}$ $\mathbb{R}$ be such that f is injective and f(x)f(y) = f(x +y) for all x, y $\mathbb{R}$. If f(x), f(y), f(z) are in G.P., then x, y, z are in

(A) A.P. always

(B) G.P. always

(C) A.P. depending on the values of x, y, z

(D) G.P. depending on the values of x, y, z

Ans : (A)

Hints : f (x + y) = f(x).f(y), so $f(x) = a^{kx}$

a^{kx}, a^{ky}, a^{kz} are in G.P

$a^{2ky} = a^{k(x+z)}$

2y = x + z, so x, y, z are in A.P

25. The number of solutions of the equation

$\frac{1}{2}\log_{\sqrt{3}} \frac{x \quad 1}{x \quad 5} \quad \log_9 (x \quad 5)^2 \quad 1$ is

(A) 0 (B) 1 (C) 2 (D) infinite

Ans : (B)

Hints : $\log_3 \frac{x \quad 1}{x \quad 5} \quad \log_3 (x \quad 5) \quad 1$

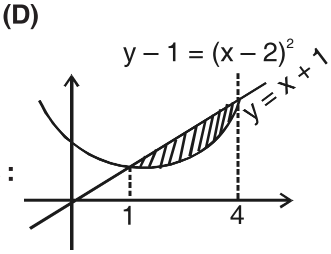
(x + 1) = 3, x = 2 so only one solution

26. The area of the region bounded by the parabola $y = x^2 - 4x + 5$ and the straight line y = x + 1 is

(A) 1/2 (B) 2 (C) 3 (D) 9/2

Ans : (D)

Hints :



Required Area = $\int_1^4 (x \quad 1) \quad (x^2 \quad 4x \quad 5) \; dx$

$= \frac{9}{2}$ sq.unit

27. The value of the integral

$\int_1^2 e^x \quad \log_e x \quad \frac{x \quad 1}{x} \quad dx$ is

(A) $e^2(1 + \log_e 2)$ (B) $e^2 - e$ (C) $e^2(1 + \log_e 2) - e$ (D) $e^2 - e(1 + \log_e 2)$

Ans : (C)

Hints : $\int_1^2 e^x \quad \log_e x \quad 1 \quad \frac{1}{x} \quad dx = \int_1^2 e^x.dx \quad \int_1^2 e^x \quad \log_e x \quad \frac{1}{x} \quad dx$

$= (e^2 - e^1) + \quad e^x \log_e x \Big|_1^2$

$= e^2 - e + e^2\log_e 2$

$= e^2(1 + \log_e 2) - e$

28. Let P = $1 \quad \frac{1}{2 \quad 2} \quad \frac{1}{3 \quad 2^2}$ ....

and Q = $\frac{1}{1 \quad 2} \quad \frac{1}{3 \quad 4} \quad \frac{1}{5 \quad 6}$ ....

Then

(A) P = Q (B) 2P = Q (C) P = 2Q (D) P = 4Q

Ans : (C)

Hints : $\frac{P}{2} \quad \frac{1}{2} \quad \frac{\frac{1}{2}^{2}}{2} \quad \frac{\frac{1}{2}^{3}}{3}$ ....to

$= -\log_e \; 1 \; \frac{1}{2}$ so P = 2 $\log_e$ 2

Q = $\frac{1}{1} \quad \frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6}$ ....to

= $\log_e(1 + 1) = \log_e 2$

P = 2Q

29. Let f(x) = sin x + 2 $\cos^2$ x, $\frac{}{4}$ x $\frac{3}{4}$. Then f attains its

(A) minimum at x = $\frac{}{4}$ (B) maximum at x = $\frac{}{2}$

(C) minimum at x = $\frac{}{2}$ (D) maximum at x = $\sin^{1} \frac{1}{4}$

Ans : (C)

Hints : f(x) = sinx + $2\cos^2$x

$= -2\sin^2 x + \sin x + 2$

$= -2(\sin^2 x - ½ \sin x) + 2$

$= -2 \quad \sin x \quad \frac{1}{4}^{2} \quad 2 \quad \frac{1}{8}$

$= \frac{17}{8} \quad 2 \quad \sin x \quad \frac{1}{4}^{2}$; nder the given domain

f(x) will be minimum when $\sin x \quad \frac{1}{4}^{2}$ is maximum which is at x = $\diagup 2$

30. Each of a and b can take values 1 or 2 with equal probability. The probability that the equation $ax^2 + bx + 1 = 0$ has real roots, is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

Ans : (B)

Hints : $ax^2 + bx + 1 = 0$ has real roots for $b^2 - 4a \quad 0$

So a has to be 1 and b has to be 2

so probability is = $\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{4}$

31. There are two coins, one unbiased with probability $\frac{1}{2}$ of getting heads and the other one is biased with probability $\frac{3}{4}$ of getting heads. A coin is selected at random and tossed. It shows heads up. Then the probability that the unbiased coin was selected is

(A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$

Ans : (D)

Hints : H Event of head showing up

B Event of biased coin chosen

UB Event of unbiased coin chosen

$$P\ \frac{UB}{H}\ \ \frac{P(UB).P\ \frac{H}{UB}}{P(UB).P\ \frac{H}{UB}\ \ P(B).P\ \frac{H}{B}}$$

$$\frac{\frac{1}{2}\ \frac{1}{2}}{\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{3}{4}}\ \ \frac{2}{5}$$

32. For the variable t, the locus of the point of intersection of the line $3tx - 2y + 6t = 0$ and $3x + 2ty - 6 = 0$ is

(A) the ellipse $\frac{x^2}{4}\ \frac{y^2}{9}\ \ 1$ (B) the ellipse $\frac{x^2}{9}\ \frac{y^2}{4}\ \ 1$

(C) the hyperbola $\frac{x^2}{4}\ \frac{y^2}{9}\ \ 1$ (D) the hyperbola $\frac{x^2}{9}\ \frac{y^2}{4}\ \ 1$

Ans : (A)

Hints : The point of intersection of $3tx - 2y + 6t = 0$ and $3x + 2ty - 6 = 0$ is

$$x = \frac{2(1\ \ t^2)}{(1\ \ t^2)},\ y = \frac{6t}{(1\ \ t^2)}$$

Considering $t = \tan\ $, $x = 2\cos 2\ $, $y = 3.\sin 2$

so locus of point of intersection is the ellipse

$$\frac{x^2}{4}\ \frac{y^2}{9}\ \ 1$$

33. Cards are drawn one-by-one without replacement from a well shuffled pack of 52 cards. Then the probability that a face card (Jack, Queen or King) will appear for the first time on the third turn is equal to

(A) $\frac{300}{2197}$ (B) $\frac{36}{85}$ (C) $\frac{12}{85}$ (D) $\frac{4}{51}$

Ans : (C)

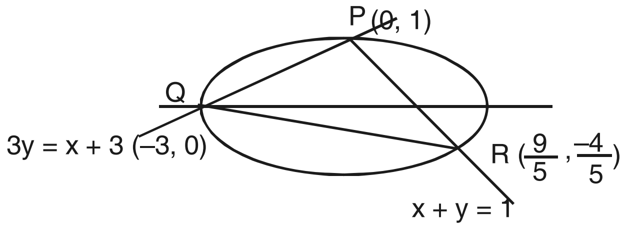
Hints : P (face card on third turn) = P (no face card in first turn) × P (no face card in 2nd turn) × P (face card in 3rd turn)

$$= \frac{40}{52}\ \frac{39}{51}\ \frac{12}{50}\ \frac{12}{85}$$

34. Lines x + y = 1 and 3y = x + 3 intersect the ellipse $x^2 + 9y^2 = 9$ at the points P,Q,R. The are of the triangle PQR is

(A) $\frac{36}{5}$ (B) $\frac{18}{5}$ (C) $\frac{9}{5}$ (D) $\frac{1}{5}$

Ans : (B)

Hints : 

Area of PQR = $\left|\frac{1}{2}\begin{vmatrix} 0 & 3 & \frac{9}{5} \\ 1 & 0 & 4/5 \\ 1 & 1 & 1 \end{vmatrix}\right|$

$= \frac{18}{5}$ sq. units

35. The number of onto functions from the set {1, 2, ..., 11} to set {1, 2, ... 10} is

(A) 5 $\underline{|11}$ (B) $\underline{|10}$ (C) $\frac{\underline{|11}}{2}$ (D) 10 $\underline{|11}$

Ans : (D)

Hints : No. of onto function = ${}^{11}C_{10}$ $\underline{|10}$ 10

$= 10 \times \underline{|11}$

36. The limit of $\frac{1}{x^2} \quad \frac{(2013)^x}{e^x \quad 1} \quad \frac{1}{e^x \quad 1}$ as x 0

(A) approaches + (B) approaches – (C) is equal to $\log_e$ (2013) (D) does not exist

Ans : (A)

Hints : $\lim_{x \quad 0} \frac{1}{x^2} \quad \frac{(2013)^x}{e^x \quad 1} \quad \frac{1}{e^x \quad 1}$

$= \lim_{x \quad 0} \frac{1}{x^2} \quad \lim_{x \quad 0} \frac{(2013)^x \quad 1}{x} \quad \frac{x}{e^x \quad 1}$

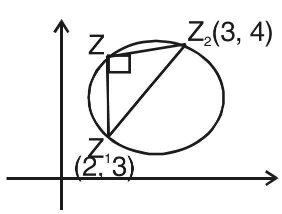
$= \quad \log_e(2013)$

=

37. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$ be two points on the complex plane. Then the set of complex numbers z satisfying $|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$ represents

(A) a straight line (B) a point (C) a circle (D) a pair of straight lines

Ans : (C)

Hints : 

Clearly the locus of Z is a circle with Z_1 & Z_2 as end point of diameter.

38. Let p(x) be a quadratic polynomial with constant term 1. Suppose p(x) when divided by x – 1 leaves remainder 2 and when divided by x + 1 leaves remainder 4. Then the sum of the roots of p(x) = 0 is

(A) –1 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{2}$

Ans : (D)

Hints : $P(x) = ax^2 + bx + 1$

$P(1) = a + b + 1 = 2$

$P(-1) = a - b + 1 = 4$

so $b = -1, a = 2$

sum of roots of P(x) is $\frac{b}{a} = \frac{1}{2}$

39. Eleven apples are distributed among a girl and a boy. Then which one of the following statements is true ?

(A) At least one of them will receive 7 apples

(B) The girl receives at least 4 apples or the boy receives at least 9 apples

(C) The girl receives at least 5 apples or the boy receives at least 8 apples

(D) The girl receives at least 4 apples or the boy receives at least 8 apples

Ans : ()

Hints :

40. Five numbers are in H.P. The middle term is 1 and the ratio of the second and the fourth terms is 2:1. Then the sum of the first three terms is

(A) 11/2 (B) 5 (C) 2 (D) 14/3

Ans : (A)

Hints : Let a, b, 1, c, d are H.P

so $\frac{1}{a}, \frac{1}{b}, 1, \frac{1}{c}, \frac{1}{d}$ are A.P, b = 2C.

$\frac{1}{b} \quad \frac{1}{c} \quad 2$ so $b = \frac{3}{2}, c = \frac{3}{4}, a = 3.$

so, sum of the first three terms = $3 \quad \frac{3}{2} \quad 1 \quad \frac{11}{2}$

41. The limit of $\frac{1}{x}\sqrt{1 \quad x} \quad \sqrt{1 \quad \frac{1}{x^2}}$ as x 0

(A) does not exist (B) is equal to 1/2 (C) is equal to 0 (D) is equal to 1

Ans : (A)

Hints : R.H.L. $\lim_{x \quad 0}\frac{\sqrt{1 \quad x}}{x} \quad \frac{\sqrt{x^2 \quad 1}}{x} = \lim_{x \quad 0}\frac{\sqrt{1 \quad x} \quad \sqrt{x^2 \quad 1}}{x} \quad \frac{\sqrt{1 \quad x} \quad \sqrt{x^2 \quad 1}}{\sqrt{1 \quad x} \quad \sqrt{x^2 \quad 1}}$

$= \lim_{x \quad 0}\frac{1 \quad x \quad 1 \quad x^2}{x \sqrt{1 \quad x} \quad \sqrt{1 \quad x^2}}$

$= \lim_{x \quad 0}\frac{x(1 \quad x)}{x \sqrt{1 \quad x} \quad \sqrt{1 \quad x^2}} = \frac{1}{2}$. L.H.L. $= \lim_{x \quad 0}\frac{\sqrt{1 \quad x} \quad \sqrt{x^2 \quad 1}}{x}$

R.H.L. L.H.L.

42. The maximum and minimum values of $\cos^6 \quad + \sin^6$ are respectively

(A) 1 and 1/4 (B) 1 and 0 (C) 2 and 0 (D) 1 and 1/2

Ans : (A)

Hints : $\sin^6 \quad + \cos^6 \quad = 1 - 3\sin^2 \quad .\cos^2$

$= 1 - \frac{3}{4}\sin^2 2$

43. If a, b, c are in A.P., then the straight line ax + 2by + c = 0 will always pass through a fixed point whose co-ordinates are

(A) (1, –1) (B) (–1, 1) (C) (1, –2) (D) (–2, 1)

Ans : (A)

Hints : a(x + y) + c(y + 1) = 0

x = 1, y = – 1

44. If one end of a diameter of the circle $3x^2 + 3y^2 - 9x + 6y + 5 = 0$ is (1, 2) then the other end is

(A) (2, 1) (B) (2, 4) (C) (2, –4) (D) (–4, 2)

Ans : (C)

Hints : Center $\frac{3}{2},\ 1$

Let the other point be (h, k) $\frac{h \quad 1}{2} \quad \frac{3}{2} \quad h \quad 2$

$\frac{k \quad 2}{2} \quad 1 \quad k \quad 4$

45. The value of $\cos^2 75^0 + \cos^2 45^0 + \cos^2 15^0 - \cos^2 30^0 - \cos^2 60^0$ is

(A) 0 (B) 1 (C) 1/2 (D) 1/4

Ans : (C)

Hints : cos15º = sin75º

$\cos^2 75º + \cos^2 45º + \cos^2 15º - \cos^2 30º - \cos^2 60º$

$= \cos^2 75º + \sin^2 75º + \cos^2 45º - \cos^2 30º - \cos^2 60º$

$= 1 \quad \frac{1}{2} \quad \frac{3}{4} \quad \frac{1}{4}$

$= \frac{1}{2}$

46. Suppose z= x + iy where x and y are real numbers and $i \quad \sqrt{1}$. The points (x, y) for which $\frac{z \quad 1}{z \quad i}$ is real, lie on

(A) an ellipse (B) a circle (C) a parabola (D) a straight line

Ans : (D)

Hints : $\frac{(x \quad 1) \quad iy}{x \quad i(y \quad 1)} \quad k$

$\frac{(x \quad 1) \quad iy}{x \quad i(y \quad 1)} \frac{x \quad i(y \quad 1)}{x \quad i(y \quad 1)} \quad k$

Imaginary part = 0 x + y = 1

47. The equation $2x^2 + 5xy - 12y^2 = 0$ represents a

(A) circle

(B) pair of non-perpendicular intersecting straight lines

(C) pair of perpendicular straight lines

(D) hyperbola

Ans : (B)

Hints : $2x^2 + 5xy - 12y^2 = 0$

(x + 4y) (2x – 3y) = 0

48. The line y = x intersects the hyperbola $\frac{x^2}{9}\quad\frac{y^2}{25}\quad 1$ at the points P and Q. The eccentricity of ellipse with PQ as major axis and minor axis of length $\frac{5}{\sqrt{2}}$ is

(A) $\frac{\sqrt{5}}{3}$ (B) $\frac{5}{\sqrt{3}}$ (C) $\frac{5}{9}$ (D) $\frac{25}{9}$

Ans : ()

Hints : For y = x, $x^2\quad\frac{1}{9}\quad\frac{1}{25}\qquad 1\qquad x^2\qquad\frac{5\quad 3}{4}^{\;2}\qquad\frac{15}{4}^{\;2}$

$a\quad\frac{15\sqrt{2}}{4}\quad\frac{15}{2\sqrt{2}},\quad b\quad\frac{5}{2\sqrt{2}}$

$e^2\quad 1\quad\frac{1}{9}\quad\frac{8}{9}$

$e\quad\frac{2\sqrt{2}}{3}$

49. The equation of the circle passing through the point (1, 1) and the points of intersection of $x^2 + y^2 - 6x - 8 = 0$ and $x^2 + y^2 - 6 = 0$ is

(A) $x^2 + y^2 + 3x - 5 = 0$ (B) $x^2 + y^2 - 4x + 2 = 0$ (C) $x^2 + y^2 + 6x - 4 = 0$ (D) $x^2 + y^2 - 4y - 2 = 0$

Ans : (A)

Hints : Circle passing through point of intersection of ircles is $x^2 + y^2 - 6x - 8 + \quad (x^2 + y^2 - 6) = 0$

It passes through (1, 1) so, $\quad = -3$

Circle is $x^2 + y^2 + 3x - 5 = 0$

50. Six positive numbers are in G.P., such that the r product is 1000. If the fourth term is 1, then the last term is

(A) 1000 (B) 100 (C) 1/100 (D) 1/1000

Ans : (C)

Hints : $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r},$ ar, ar^3, ar^5

$a^6 = 1000\qquad a^2 = 10$

given ar = 1, $\quad a^2r^2 = 1,\; r^2\quad\frac{1}{10}$

$ar^5 = \frac{1}{100}$

51. In the set of all 3×3 real matrices a relation is defined as follows. A matrix A is related to a matrix B if and only if there is a non-singular 3×3 matrix P such that $B = P^{-1}AP$. This relation is

(A) Reflexive, Symmetric but not Transitive
(B) Reflexive, Transitive but not Symmetric
(C) Symmetric, Transitive but not Reflexive
(D) an Equivalence relation

Ans : (D)

Hints : R = {(A, B) | $B = P^{-1}AP$}

$A = I^{-1}AI\qquad (A, A)\quad R\qquad$ R is reflexive

Let (A, B) $\quad$ R , $B = P^{-1}AP$

$PB = AP\qquad PBP^{-1} = A\qquad A = (P^{-1})^{-1} B(P^{-1})$

(B, A) R, R is symmetric
Let (A, B) R, (B, C) R
$A = P^{-1}BP$ and $B = Q^{-1}CQ$
$A = P^{-1}Q^{-1}C\,QP = (QP)^{-1}\,C(QP)$ (A, C) R

52. The number of lines which pass through the point (2, –3) and are at the distance 8 from the point (–1, 2) is

(A) infinite (B) 4 (C) 2 (D) 0

Ans : (D)

Hints : The maximum distance of the line passing through (2, –3) from (–1, 2) is $\sqrt{34}$. So there is no possible line

53. If are the roots of the quadratic equation $ax^2 + bx + c = 0$ and $3b^2 = 16ac$ then

(A) = 4 or = 4 (B) = –4 or = –4 (C) = 3 or = 3 (D) = –3 or = –3

Ans : (C)

Hints : $3b^2 = 16ac$

$$3\,\frac{b}{a}^{\;2}\quad 16\frac{c}{a}$$

$$3\qquad{}^{2}\quad 16\quad ,\ 3\ {}^{2}\quad 3\ {}^{2}\quad 10$$

$$3—\quad 3—\quad 10\text{, Let }—\quad y$$

$3y^2 - 10y + 3 = 0$, $(3y - 1)(y - 3) = 0$

$y\ \ \frac{1}{3}$ or $y = 3$

3 = or = 3

54. For any two real numbers a and b, we define a R b if and only if $\sin^2 a + \cos^2 b = 1$. The relation R is

(A) Reflexive but not Symmetric (B) Symmetric but not transitive
(C) Transitive but not Reflexive (D) an Equivalence relation

Ans : (D)

Hints : $\sin^2 a + \cos^2 b = 1$
Reflexive : $\sin^2 a + \cos^2 a = 1$
aRa
$\sin^2 a + \cos^2 b = 1$, $1 - \cos^2 a + 1 - \sin^2 b = 1$
$\sin^2 b + \cos^2 a = 1$
bRa
Hence symmetric Let aRb bR
$\sin^2 a + \cos^2 b = 1$ .... (1)
$\sin^2 b + \cos^2 c = 1$ .... (2)
(1) + (2)
$\sin^2 a + \cos^2 c = 1$
Hence transitive therefore equivalence relation.

55. Let n be a positive even integer. The ratio of the largest coefficient and the 2nd largest coefficient in the expansion of $(1 + x)^n$ is 11:10. The the number of terms in the expansion of $(1 + x)^n$ is

(A) 20 (B) 21 (C) 10 (D) 11

Ans : (B)

Hints : Let n = 2m

$$\frac{{}^{2m}C_m}{{}^{2m}C_{m\ \ 1}}\quad \frac{11}{10}$$

m = 10, n = 20
Total No. of term = 21

56. Let exp(x) denote exponential function e^x. If $f(x) = \exp\ x^{\frac{1}{x}}$, x > 0 then the minimum value of *f* in the interval [2, 5] is

(A) $\exp\ e^{\frac{1}{e}}$ (B) $\exp\ 2^{\frac{1}{2}}$ (C) $\exp\ 5^{\frac{1}{5}}$ (D) $\exp\ 3^{\frac{1}{3}}$

Ans : (C)

Hints : $f(x) \quad e^{x^{1/x}}$

$g(x) = \log f(x) = x^{\frac{1}{x}}$

g(x) increases is (0, e) & decreases in (e,) it will be minimum at either 2 or 5

$2^{\frac{1}{2}} \quad 5^{\frac{1}{5}}$ minimum value of $f(x) = e^{5^{\frac{1}{5}}}$

57. The sum of the series $\frac{1}{1\ \ 2}\ {}^{25}C_0 \quad \frac{1}{2\ \ 3}\ {}^{25}C_1 \quad \frac{1}{3\ \ 4}\ {}^{25}C_2 \quad .... \quad \frac{1}{26\ \ 27}\ {}^{25}C_{25}$

(A) $\frac{2^{27}\ \ 1}{26\ \ 27}$ (B) $\frac{2^{27}\ \ 28}{26\ \ 27}$ (C) $\frac{1}{2}\ \frac{2^{26}\ \ 1}{26\ \ 27}$ (D) $\frac{2^{26}\ \ 1}{52}$

Ans : (B)

Hints : On integrate $(1 + x)^{25}$ twice 1st under the limit 0 to x & then 0 to 1 we get sum = $\frac{2^{27}\ \ 28}{26\ \ 27}$

58. Five numbers are in A.P. with common difference 0 If the 1st, 3rd and 4th terms are in G.P., then

(A) the 5th term is always 0 (B) the 1st term is always 0

(C) the middle term is always 0 (D) the middle term is always –2

Ans : (A)

Hints : Let a, a + d, a + 2d, a + 3d, a + 4d are five number in A.P.

Given $\frac{a\ \ 2d}{a} \quad \frac{a\ \ 3d}{a\ \ 2d}$

a + 4d = 0

59. The minimum value of the function *f*(x) = 2|x – 1| + |x – 2| is

(A) 0 (B) 1 (C) 2 (D) 3

Ans : (B)

Hints : f(x) will be minimum at x = 1

60. If P, Q, R are angles of an isosceles triangle and P $\frac{}{2}$, then the value of $\left(\cos\frac{P}{3} \quad i\sin\frac{P}{3}\right)^3$ + (cosQ + isinQ) (cosR – isinR) + (cosP – isinP) (cosQ – isinQ) (cosR – isinR) is equal to

(A) i (B) –i (C) 1 (D) –1

Ans : (B)

Hints : P $\frac{}{2}$, Q R $\frac{}{4}$

$\left(\cos\frac{P}{3} \quad i\sin\frac{P}{3}\right)^3 \quad \cos Q \quad i\sin Q\ (\cos R \quad i\sin R) \quad (\cos P \quad i\sin P)(\cos Q \quad i\sin Q)(\cos R \quad i\sin R)$

$= e^{ip} \quad e^{iQ}.e^{iR} \quad e^{iP} \quad e^{iQ} \quad {}^{iR}$

$= e^{i /2} \quad e^{i(Q R)} \quad e^{i(P Q R)}$

$= e^{i /2} \quad e^{0} \quad e^{i}$

$= \cos\frac{}{2} \quad i\sin\frac{}{2} \quad 1 \quad \cos \quad i\sin$

$= -i + 1 - 1 - 0 = -i$

CATEGORY - II

Q. 61 – Q. 75 carry two marks each, for which only option is correct. Any wrong answer will lead to deduction of 2/3 mark.

61. A line passing through the point of intersection of $x + y = 4$ and $x - y = 2$ makes an angle $\tan^{-1}(3/4)$ with the x-axis. It intersects the parabola $y^2 = 4(x-3)$ at points (x_1, y_1) and (x_2, y_2) respectively. Then $|x_1 - x_2|$ is equal to

(A) $\frac{16}{9}$ (B) $\frac{32}{9}$ (C) $\frac{40}{9}$ (D) $\frac{80}{9}$

Ans : (B)

Hints : A(3, 1)

$y \quad 1 \quad \frac{3}{4} \quad x \quad 3 \qquad y \quad \frac{3}{4}x \quad 1 \quad \frac{9}{4}$ or, $y \quad \frac{3}{4}x \quad \frac{5}{4}, y^2 \quad 4 \quad x \quad 3$

$\frac{3x \quad 5}{4}^{2} \quad 4 \quad x \quad 3$ or, $9x^2 \quad 30x \quad 25 \quad 64x \quad 64 \quad 3$

$9x^2 \quad 94x \quad 217 \quad 0$

$x_1 \quad x_2 \quad \frac{94}{9}$

$x_1x_2 \quad \frac{217}{9}$

$x_1 \quad x_2 \;{}^2 \quad x_1 \quad x_2 \;{}^2 \quad 4x_1x_2 \quad \frac{94}{9}^{2} \quad 4\frac{217}{9}$

$\frac{94^{2} \quad 4.217.9}{9^2} \quad \frac{32}{9}$

62. Let [a] denote the greatest integer which is less than or equal to a. Then the value of the integral

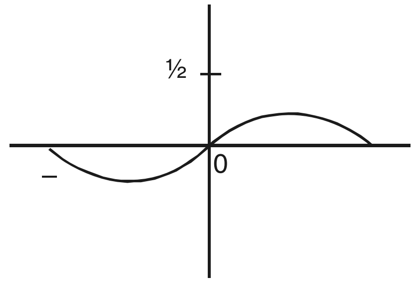
$\int_{\frac{}{2}}^{\frac{}{2}} [\sin x \cos x]dx$ is

(A) $\frac{}{2}$ (B) (C) (D) $/2$

Ans : (D)

Hints : $I \quad \int_{\frac{}{2}}^{\frac{}{2}} \frac{1}{2}\mathrm{Sin}\,2x \quad dx \quad \mathrm{Put}\, 2x \qquad \mathrm{or}, 2dx \quad d$

$\frac{1}{2} \quad \frac{1}{2}\text{Sin} \quad d \quad \frac{1}{2} \, ^{0} \quad 1\,dx \quad _{0} \, 0dx$



$\frac{1}{2} \quad x\,^{0} \quad 0 \quad \frac{1}{2}\,0 \quad \overline{2}$

63. If $P \begin{matrix} 2 & 2 & 4 \\ 1 & 3 & 4 \\ 1 & 2 & 3 \end{matrix}$ then P^5 equals

(A) P (B) 2P (C) –P (D) –2P

Ans : (A)

Hints : $P^2 \begin{matrix} 2 & 2 & 4 \\ 1 & 3 & 4 \\ 1 & 2 & 3 \end{matrix} \begin{matrix} 2 & 2 & 4 \\ 1 & 3 & 4 \\ 1 & 2 & 3 \end{matrix}$

$= \begin{matrix} 2 & 2 & 4 \\ 1 & 3 & 4 \\ 1 & 2 & 3 \end{matrix}$ $\quad P^2 = p$; $p^4 = p$; $p^5 = p^2 = p$

64. If $\sin^2 \quad 3\cos \quad 2$, then $\cos^3 \quad \sec^3 \quad$ is

(A) 1 (B) 4 (C) 9 (D) 18

Ans : (D)

Hints : $\text{Cos}^2 \quad 3\text{Cos} \quad 1 \quad 0$ or, $\text{Cos} \quad \frac{1}{\text{Cos}} \quad 3$,

$C^3 \quad \frac{1}{C^3} \quad 3.C.\frac{1}{C} \quad C \quad \frac{1}{C} \quad 27$ or, $\text{Cos}^3 \quad \sec^3 \quad 3.3 \quad 27$

$\text{Cos}^3 \quad \sec^3 \quad 18$

65. $x \quad 1 \quad \frac{1}{2 \ \lfloor 1} \quad \frac{1}{4 \ \lfloor 2} \quad \frac{1}{8 \ \lfloor 3}$ .... and $y \quad 1 \quad \frac{x^2}{\lfloor 1} \quad \frac{x^4}{\lfloor 2} \quad \frac{x^6}{\lfloor 3}$ .... Then the value of $\log_e y$ is

(A) e (B) e^2 (C) 1 (D) 1/e

Ans : (A)

Hints : $y \quad e^{x^2}, x \quad e^{\frac{1}{2}}$

$\ln y = x^2 = e$

66. The value of the infinite series $\frac{1^2 \quad 2^2}{\lfloor 3} \quad \frac{1^2 \quad 2^2 \quad 3^2}{\lfloor 4} \quad \frac{1^2 \quad 2^2 \quad 3^2 \quad 4^2}{\lfloor 5}$.......... is

(A) e (B) 5e (C) $\frac{5e}{6} \quad \frac{1}{2}$ (D) $\frac{5e}{6}$

Ans : (C)

Hints : $\overset{n}{\underset{r\ 1}{}} \frac{1^2 \quad 2^2 \quad 3^2 \quad .... \ r \quad 1^{\ 2}}{\lfloor r \quad 2} = \frac{1}{6} \quad \frac{2r \quad 3}{\lfloor r} \quad \frac{1}{6}{}_{r\ 1} \quad \frac{2}{\lfloor r \quad 1} \quad \frac{3}{\lfloor r} = \frac{1}{6} \ 2e \quad 3 \ e \quad 1 \quad \frac{5}{6}e \quad \frac{1}{2}$

67. The value of the integral $\overset{\overline{3}}{\underset{\overline{6}}{}} \frac{\sin x \quad x\cos x}{x \ x \quad \sin x} dx$ is equal to

(A) $\log_e \frac{2 \quad 3}{2 \quad 3\sqrt{3}}$ (B) $\log_e \frac{3}{2\ 2 \quad 3\sqrt{3}}$ (C) $\log_e \frac{2 \quad 3\sqrt{3}}{2 \quad 3}$ (D) $\log_e \frac{2\ 2 \quad 3\sqrt{3}}{3}$

Ans : (A)

Hints : $I \quad \frac{x \quad Sinx \quad x \ 1 \quad Cosx}{x \ x \quad Sinx} dx = \frac{1}{x} \quad \frac{1 \quad Cosx}{x \quad Sinx} \ dx$

$= \ln\frac{}{3} \quad \ln\frac{}{6} \quad \ln \ \frac{}{3} \quad \frac{\sqrt{3}}{2} \quad \ln \ \frac{}{6} \quad \frac{1}{2}$

$\ln \ \frac{2 \quad 6}{2 \quad 3\sqrt{3}}$

68. Let $f(x) \quad x \ \frac{1}{x \quad 1} \quad \frac{1}{x} \quad \frac{1}{x \quad 1}$, $x \quad 1$ Then

(A) $f(x) \quad 1$ (B) $1 < f(x) \quad 2$ (C) $2 < f(x) \quad 3$ (D) $f(x)>3$

Ans : (D)

Hints : $f(x) \quad x \ \frac{1}{x \quad 1} \quad \frac{1}{x \quad 1} \quad \frac{1}{x}$

$= x \ \frac{x \quad 1 \quad x \quad 1}{x^2 \quad 1} \quad 1 \quad \frac{2x^2}{x^2 \quad 1} \quad 1 \quad \frac{2}{1 \quad \frac{1}{x^2}} \quad 1$

69. Let F(x) = $\int_0^x \frac{\cos t}{1 \;\; t^2} dt, 0 \;\; x \;\; 2$. Then

(A) F is increasing in $\frac{}{2}, \frac{3}{2}$ and decreasing in $0, \frac{}{2}$ and $\frac{3}{2}, 2$

(B) F is increasing in $0,$ and decreasing in $\;\;, 2$

(C) F is increasing in $\;\;, 2$ and decreasing in $0,$

(D) F is increasing in $0, \frac{}{2}$ and $\frac{3}{2}, 2$ and decreasing in $\frac{}{2}, \frac{3}{2}$

Ans : (D)

Hints : $F \;\; x \;\; \frac{\text{Cos}\, x}{1 \;\; x^2}$

$\text{Cos}\, x \;\; 0 \;\; x \;\; 0, \frac{}{2} \;\;\; \frac{3}{2}, 2$

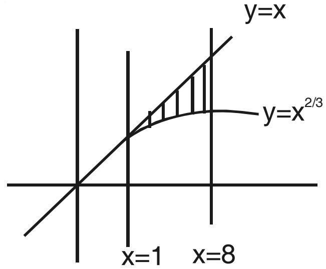
$\text{Cos}\, x \;\; 0 \;\; x \;\; \frac{}{2}, \frac{3}{2}$

70. Let f(x) = $x^{2/3}$, x 0. Then the area of the region enclosed by the c rve y = f(x) and the three lines y = x, x = 1 and x = 8 is

(A) $\frac{63}{2}$ (B) $\frac{93}{5}$ (C) $\frac{105}{7}$ (D) $\frac{129}{10}$

Ans : (D)

Hints :



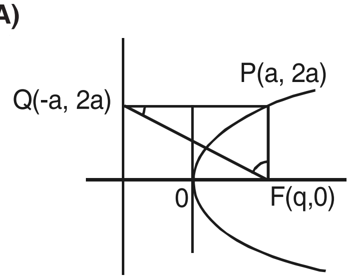
$A \int_1^8 x \;\; x^{\frac{2}{3}} dx \;\; \frac{1}{2} x^2 \Big|_1^8 \;\; \frac{3}{5} x^{5/3} \Big|_1^8 \;\; \frac{1}{2} 64 \;\; 1 \;\; \frac{3}{5} 32 \;\; 1 \;\; \frac{129}{10}$

71. Let P be a point on the parabola $y^2 = 4ax$ with focus F. Let Q denote the foot of the perpendicular from P onto the directrix. Then $\frac{\tan \;\; PQF}{\tan \;\; PFQ}$ is

(A) 1 (B) 1/2 (C) 2 (D) 1/4

Ans : (A)

Hints :



PQ = PF so =

72. An objective type test paper has 5 questions. Out of these 5 questions, 3 questions have four options each (A, B, C, D) with one option being the correct answer. The other 2 questions have two options each, namely True and False. A candidate randomly ticks the options. Then the probability that he/she will tick the correct option in at least four questions, is

(A) $\frac{5}{32}$ (B) $\frac{3}{128}$ (C) $\frac{3}{256}$ (D) $\frac{3}{64}$

Ans : (D)

Hints : $n(S) = 4^3 . 2^2$, $n(e) = ({}^3C_1 .3 + {}^2C_1 . 1) + 1$

$$P \quad \frac{3.3 \quad 2 \quad 1}{4^3.2^2} \quad \frac{12}{4.4.4.4} \quad \frac{3}{64}$$

73. A family of curves is such that the length intercepted on the y-axis between the origin and the tangent at a point is three times the ordinate of the point of contact. The family of curves is

(A) $xy = c$, c is a constant
(B) $xy^2 = c$, c is a constant
(C) $x^2y = c$, c is a constant
(D) $x^2y^2 = c$, c is a constant

Ans : (C)

Hints : $y - x\frac{dy}{dx} = 3y$ or, $-x\frac{dy}{dx} = 2y$

or, $\frac{dy}{y} \quad 2\frac{dx}{x}$ (Integrate) or, $\ln y = -2 \ln x + \ln c$ or, $\ln y + \ln x^2 = \ln$ or, $y \quad x^2 = c$

74. The solution of the differential equation $y^2 \quad 2x \frac{dy}{dx} \quad y$ satisfies x = 1, y = 1. Then the solution is

(A) $x = y^2(1 + \log_e y)$ (B) $y = x^2(1 + \log_e x)$ (C) $x = y^2(1 - \log_e y)$ (D) $y = x^2(1 - \log_e x)$

Ans : (A)

Hints : $\frac{dx}{dy} \quad \frac{y^2 \quad 2x}{y}$ or, $\frac{dx}{dy} \quad \frac{2}{y}.x \quad y$ or, IF $e^{\frac{2}{y}dy} \quad e^{\ln y} \quad \frac{1}{y^2}$ or, $x.\frac{1}{y^2} \quad y.\frac{1}{y^2}dy \quad c$

or, $\frac{x}{y^2} \quad \ln y \quad c \qquad y(1) = 1$ or, $1 = 0 + c$

$x = y^2(\ln y + 1)$

75. The solution of the differential equatio y sin (x/y) dx= (x sin(x/y)–y) dy satisfying y / 4 =1 is

(A) $\cos\frac{x}{y} \quad \log_e y \quad \frac{1}{\sqrt{2}}$ (B) $\sin\frac{x}{y} \quad \log_e y \quad \frac{1}{\sqrt{2}}$ (C) $\sin\frac{x}{y} \quad \log_e x \quad \frac{1}{\sqrt{2}}$ (D) $\cos\frac{x}{y} \quad \log_e x \quad \frac{1}{\sqrt{2}}$

Ans : ()

Hints : $\frac{dx}{dy} \quad \frac{\frac{x}{y}\sin\frac{x}{y} \quad 1}{\sin\frac{x}{y}}$ Put $\frac{x}{y}$ or, x = y . then, $\frac{dx}{dy} \quad y\frac{d}{dy}$

or, $y\frac{d}{dy} \quad \frac{Sin \quad 1}{Sin}$ or, $y\frac{d}{dy} \quad \frac{Sin \quad 1}{Sin} \quad \frac{1}{Sin}$ or, $\sin \; d \quad \frac{dy}{y}$

or, $\ln y \quad \cos \quad c \qquad y \quad \frac{}{4} \quad 1 \quad c \quad \frac{1}{\sqrt{2}}$

or, $\ln y \quad \cos\frac{x}{y} \quad \frac{1}{\sqrt{2}}$

CATEGORY - 3

Q. 76 – Q. 80 carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question –irrespective of the number of correct options marked

76. The area of the region enclosed between parabola $y^2 = x$ and the line $y = mx$ is $\frac{1}{48}$. Then the value of m is

(A) –2 (B) –1 (C) 1 (D) 2

Ans : (A, D)

Hints : . $A \quad {}_{0}^{\frac{1}{m}} \quad \frac{y}{m} \quad y^2 \ dy = \left| \frac{1}{2m} \ y^2 \ {}_{0}^{1/m} \quad \frac{1}{3} \ y^3 \ {}_{0}^{\frac{1}{m}} \right|$

$\frac{1}{48} \quad \left| \frac{1}{2m^3} \quad \frac{1}{3m^3} \right|$ or, $\frac{1}{48} \quad \left| \frac{1}{6m^3} \right|$

(1) $m^3 \quad \frac{1}{6}.48 \quad 1.8 \quad 8$ or, $m \quad 2$

(2) $m^3 \quad \frac{1}{6}.48 \quad 1.8$ or, $m \quad 2$

77. Consider the system of equations:

$x + y + z \quad = 0$

$x + \ y + \ z \quad = 0$

${}^{2}x + \ {}^{2}y + \ {}^{2}z \quad = 0$

Then the system of equations has

(A) A unique solution for all values , ,

(B) Infinite number of solutions if any two of , are equal

(C) A unique solution if , , are distinct

(D) More than one, but finite number of solutions depending on values of , ,

Ans : (B, C)

Hints : $\begin{vmatrix} 1 & 1 & 1 \\ & & \\ {}^{2} & {}^{2} & {}^{2} \end{vmatrix}$

78. The equations of the circles which touch both the axes and the line $4x + 3y = 12$ and have centres in the first quadrant, are

(A) $x^2 + y^2 - x - y + 1 = 0$

(B) $x^2 + y^2 - 2x - 2y + 1 = 0$

(C) $x^2 + y^2 - 12x - 12y + 36 = 0$

(D) $x^2 + y^2 - 6x - 6y + 36 = 0$

Ans : (B, C)

Hints : $\left| \frac{4h \quad 3h \quad 12}{5} \right| \quad h$ or, $|7h-12| = 5h$

(i) $7h - 12 = 5h$ or, $2h = 12$ or, $h = 6$ Centre (6, 6)

$x^2 + y^2 - 12x - 12y + 36 = 0$

(ii) $7h - 12 = -5h$ or, $h = 1$ (1, 1) or, $r = 1$

79. Which of the following real valued functions is/are not even functions?

(A) $f(x) \quad x^3 \sin x$

(B) $f(x) = x^2 \cos x$

(C) $f(x) = e^x x^3 \sin x$

(D) $f(x) = x - [x]$, where [x] denotes the greatest integer less than or equal to x

Ans : (C, D)

Hints : (A) f(x) f(x) even

(B) f(x) f(x) even

(C) f(x) f(x) not even

(D) f(x) f(x) not even

80. Let sin , cos be the roots of the equation x^2–bx + c= 0. Then which of the following statements is/are correct?

(A) c $\frac{1}{2}$

(B) b $\sqrt{2}$

(C) c $\frac{1}{2}$

(D) b $\sqrt{2}$

Ans : (A, B)

Hints : Sin Cos b, Sin .Cos c

b $\sqrt{2}$,

c $\frac{1}{2}$sin2

c $\frac{1}{2}$