

# IIT-JEE-Physics-Mains–2001

## MAINS

Time : two hours

Max. Marks : 100

### General Instructions :

1. There are ten questions in this paper. Attempt all Questions.
2. Answer each question starting on a new page. The corresponding question number must be written in the left margin. Answer all the parts of a question at one place only.
3. Use only Arabic numerals (0, 1, 2 .....9) in answering the questions irrespective of the language in which your answer.
4. Use of logarithmic tables is not permitted.
5. Use of calculator is not permitted.

1. An ice cube of mass 0.1 Kg at 0°C is placed in an isolated container which is at 2270C. The specific heat  $S$  of the container varies with temperature  $T$  according to the empirical  $S = A + BT$ , where  $A = 100$  cal/Kg-K and  $B = 2 \times 10^{-2}$  cal/K<sup>2</sup>. If the final temperature of the container is 270C, determine the mass of the container.

(Latent heat of fusion for water =  $8 \times 10^4$  cal/Kg,

Specific heat of water = 103 cal/KgK).

2. A small ball of mass  $2 \times 10^{-3}$  Kg having a charge of  $1 \mu$  C is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution.

3. An inductor of inductance  $L = 400$  mH and resistors of resistances  $R_1 = 2\Omega$  and  $R_2 = 2\Omega$  are connected to a battery of e.m.f.  $E = 12$  V as shown in the figure. The internal resistance of the battery is negligible. The switch  $S$  is closed at time  $t = 0$ .



What is the potential drop across  $L$  as a function of time? After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through  $R_1$  as a function of time?

4. A 5 m long cylindrical steel wire with radius  $2 \times 10^{-3}$  m is suspended vertically from a rigid support and carries a bob of mass 100 Kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring radiation losses.

(For the steel wire : Young's modulus =  $2.1 \times 10^{11}$  Pa;

Density = 7860 Kg/m<sup>3</sup>; Specific heat = 420 J/Kg-K)

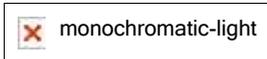
5. The refraction indices of the crown glass for blue and red light are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. An isosceles prism of angle  $60^\circ$  is made of crown glass. A beam of white light is incident at a small angle on the prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light. Determine the angle of the flint glass prism. Calculate the net dispersion of the combined system.

6. In a nuclear reactor  $^{235}\text{U}$  undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required.

7. A nucleus at rest undergoes a decay emitting an  $\alpha$ -particle of de-Broglie wavelength,  $\lambda = 5.76 \times 10^{-15}$  m. If the mass of the daughter nucleus is 223.610 a.m.u. and that of the  $\alpha$ -particle is 4.002 a.m.u., determine the total kinetic energy in the final state. Hence, obtain the mass of the parent nucleus in a.m.u.

(1 a.m.u. = 931.470 MeV/c<sup>2</sup>)

8. A vessel ABCD of 10 cm width has two small slits  $S_1$  and  $S_2$  sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of  $S_1$  and  $S_2$ . A monochromatic light source is kept at S, 40 cm



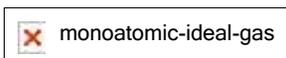
below P and 2 m from the vessel, to illuminate the slits as shown in the figure alongside. Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now, a liquid is poured into the vessel and filled up to OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid.

9. A thin biconvex lens of refractive index  $3/2$  is placed on a horizontal plane mirror as shown in the figure. The space between the lens and the mirror is then filled with water of refractive index  $4/3$ . It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid.



10. A radioactive nucleus X decays to a nucleus Y with a decay constant  $\lambda_x = 0.1 \text{ sec}^{-1}$ . Y further decays to a stable nucleus Z with a decay constant  $\lambda_y = 1/30 \text{ sec}^{-1}$ . Initially, there are only X nuclei and their number is  $N_0 = 1020$ . Set up the rate equations for the populations of X, Y and Z. The population of the Y nucleus as a function of time is given by  $N_Y(t) = \{N_0 \lambda_x / (\lambda_x - \lambda_y)\} \{ \exp(-\lambda_y t) - \exp(-\lambda_x t) \}$ . Find the time at which  $N_Y$  is maximum and determine the populations X and Z at that instant.

11. A monoatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in the figure. The volume ratios are  $V_B/V_A = 2$  and in the figure. The volume ratios are  $V_D/V_A = 4$ . If the temperature  $T_A$  at A is 27°C, calculate,



- the temperature of the gas at point B,
- heat absorbed or released by the gas in each process,
- the total work done by the gas during the complete cycle.

Express your answer in terms of the gas constant R.

**12.** A boat is travelling in a river with a speed 10 m/sec along the stream flowing with a speed 2 m/sec. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.

(a) What will be the frequency detected by a receiver kept inside the river downstream?

(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/sec in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water = 20°C; Density of river water = 1030 Kg/m<sup>3</sup>;

Bulk modulus of water =  $2.088 \times 10^9$  Pa; Gas constant  $R = 8.31$  J/mol-K;

Mean molecular mass of air =  $28.8 \times 10^{-3}$  Kg/mol;  $CP/CV$  for air = 1.4)

**13.** A current of 10 A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure, the circuit consists of eight alternating arcs of radii  $r_1 = 0.08$  m and  $r_2 = 0.12$  m. Each subtends the same angle at the centre.



(a) Find the magnetic field produced by this circuit at the centre.

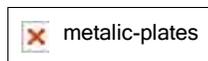
(b) An infinitely long straight wire carrying a current of 10 A is passing through the centre of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current at the centre?

**14.** A car P is moving with a uniform speed of 5 m/s towards a carriage of mass 9 Kg at rest kept on the rails at a point B as shown in figure. The height AC is 120 m. Cannon balls of 1 Kg are fired from the car with an initial velocity 100 m/s at an angle 30° with the horizontal. The first cannon ball hits the stationary carriage after a time  $t_0$  and sticks to it. Determine  $t_0$ .



At  $t_0$ , the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage, what will be the horizontal velocity of the carriage just after the second impact?

**15.** Two heavy metallic plates are joined together at 90° to each other. A laminar sheet of mass 30 Kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 Kg m<sup>2</sup>. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6 N-s,



(a) Find the location of the centre of mass of the laminar sheet from AB.

(b) At what angular velocity does the laminar sheet come back after the first impact?

(c) After how many impacts, does the laminar sheet come to rest?