

Paper-1

JEE Advanced, 2016

Part III: Mathematics

Read the instructions carefully:

General:



1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.
2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on its left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name and roll number in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at 9:00 am, verify that the booklet contains 36 pages and that all the 54 questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

Optical Response Sheet

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). The Candidate's Sheet is a carbon – less copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with or mutilate the ORS. Do not use the ORS for rough work.

14. Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. Do not write any of these details anywhere else on the ORS. Darken the appropriate bubble under each digit of your roll number.

Darken the Bubbles on the ORS

15. Use a Black Ball Point Pen to darken the bubbles on the ORS.
16. Darken the bubble  completely.
17. The correct way of darkening a bubble is as: 
18. The ORS is machine – gradable. Ensure that the bubbles are darkened in the correct way.
19. Darken the bubbles only if you are sure of the answer. There is no way to erase or “un-darken” a darkened bubble.

PART - III : MATHEMATICS

SECTION-1 : (Maximum Marks : 15)

- This section contains **Five** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories :
Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : -1 In all other cases

37. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is
(A) 380 (B) 320 (C) 260 (D) 95

Ans. (A)

Sol. $({}^6C_4 + {}^6C_3 \cdot {}^4C_1) \cdot {}^4C_1 = 380$

38. The least value of $\alpha \in \mathbb{R}$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is -

- (A) $\frac{1}{64}$ (B) $\frac{1}{32}$ (C) $\frac{1}{27}$ (D) $\frac{1}{25}$

Ans. (C)

Sol. $f(x) = 4\alpha x^2 + \frac{1}{x}; x > 0$

$$f'(x) = 8\alpha x - \frac{1}{x^2}$$

$$= \frac{8\alpha x^3 - 1}{x^2}$$

$f(x)$ attains its minimum at $x = \left(\frac{1}{8\alpha}\right)^{1/3}$

$$f\left(\left(\frac{1}{8\alpha}\right)^{1/3}\right) = 1$$

$$\Rightarrow 4\alpha \left(\frac{1}{8\alpha}\right)^{2/3} + (8\alpha)^{1/3} = 1$$

$$\Rightarrow 3\alpha^{1/3} = 1 \Rightarrow \alpha = \frac{1}{27}$$

39. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals
 (A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

Ans. (C)

Sol. $\alpha_1 = \frac{2\sec\theta + \sqrt{4\sec^2\theta - 4}}{2}$ $\beta_2 = \frac{-2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$ $\{\because \alpha_2 > \beta_2\}$

$\alpha_1 = \sec\theta + |\tan\theta|$ $\{\because \alpha_1 > \beta_1\}$ $\beta_2 = -\tan\theta - \sec\theta$

$\alpha_1 = \sec\theta - \tan\theta$ $\left(\because \theta \in \left(-\frac{\pi}{6}, -\frac{\pi}{12}\right)\right)$

$\alpha_1 + \beta_2 = -2\tan\theta$

40. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solution of the equation

$\sqrt{3}\sec x + \operatorname{cosec}x + 2(\tan x - \cot x) = 0$ in the set S is equal to -

- (A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

Ans. (C)

Sol. $\sqrt{3}\sin x + \cos x = 2\cos 2x$

$\Rightarrow \cos 2x = \cos\left(x - \frac{\pi}{3}\right)$

$\Rightarrow 2x = 2n\pi \pm \left(x - \frac{\pi}{3}\right)$

$x = (6n - 1)\frac{\pi}{3}$ or $(6n + 1)\frac{\pi}{9}$

$\Rightarrow x = -\frac{\pi}{3}, \frac{\pi}{9}, \frac{7\pi}{9}$ and $-\frac{5\pi}{9}$ in $(-\pi, \pi)$

$\Rightarrow \text{sum} = 0$

41. A computer producing factory has only two plants T_1 and T_2 . Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

$P(\text{computer turns out to be defective given that is produced in plant } T_1)$

$= 10P(\text{computer turns out to be defective given that it is produced in plant } T_2)$

where $P(E)$ denotes the probability of an event E. A computer produces in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

- (A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{93}$

Ans. (C)

Sol. $P(T_1) = \frac{20}{100}$ $P(T_2) = \frac{80}{100}$

Let $P\left(\frac{D}{T_2}\right) = x$

$P\left(\frac{D}{T_1}\right) = 10x$

$P(D) = \frac{7}{100}$ (given)

$P(T_1)P\left(\frac{D}{T_1}\right) + P(T_2)P\left(\frac{D}{T_2}\right) = \frac{7}{100}$

$\frac{20}{100} \times 10x + \frac{80}{100} \times x = \frac{7}{100}$

$x = \frac{1}{40}$

$P\left(\frac{D}{T_2}\right) = \frac{1}{40} \Rightarrow P\left(\frac{\bar{D}}{T_2}\right) = \frac{39}{40}$

$P\left(\frac{D}{T_1}\right) = \frac{10}{40} \Rightarrow P\left(\frac{\bar{D}}{T_1}\right) = \frac{30}{40}$

$P\left(\frac{T_2}{\bar{D}}\right) = \frac{\frac{80}{100} \times \frac{39}{40}}{\frac{20}{100} \times \frac{30}{40} + \frac{80}{100} \times \frac{39}{40}} = \frac{78}{93}$

SECTION-2 : (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to the correct option(s) is (are) darkened.

Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, Provided NO incorrect option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : -2 In all other cases.
- for example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened

42. A solution curve of the differential equation $(x^2 + xy + 4x + 2y + 4) \frac{dy}{dx} - y^2 = 0, x > 0$, passes through the point (1,3). The the solution curve-
- (A) intersects $y = x + 2$ exactly at one point
 (B) intersects $y = x + 2$ exactly at two points
 (C) intersects $y = (x + 2)^2$
 (D) does NOT intersect $y = (x + 3)^2$

Ans. (A,D)

Sol. $(x^2 + xy + 4x + 2y + 4) \frac{dy}{dx} - y^2 = 0$

$$((x+2)^2 + y(x+2)) \frac{dy}{dx} = y^2$$

Let $x + 2 = X, y = Y$

$$(X)(X + Y) \frac{dY}{dX} = Y^2$$

$$-X^2 dY = XY dY - Y^2 dX$$

$$-X^2 dY = Y(X dY - Y dX)$$

$$-\frac{dY}{Y} = \frac{X dY - Y dX}{X^2}$$

$$-\ln |Y| = \left(\frac{Y}{X} \right) + C$$

$$-\ln |y| = \frac{y}{x+2} + C$$

\therefore it is passing through (1, 3) $-\ln 3 = 1 + C$

$$C = -1 - \ln 3$$

\therefore curve $\frac{y}{x+2} + \ln |y| - 1 - \ln 3 = 0, x > 0$ (i)

put $y = x + 2$ in equation (i)

then $\frac{x+2}{x+2} + \ln |x+2| - 1 - \ln 3 = 0$

$x = 1, -5$ (reject)

\therefore curve intersect $y = x + 2$ at point (1, 3)

for option (C), put $y = (x + 2)^2$, we will get

$$x + 2 + 2 \ln(x + 2) = 1 + \ln 3$$

Clearly left hand side is an increasing function

Hence, it is always greater than $2 + 2\ln 2$

therefore no solution

for option (C) put $y = (x + 3)^2$ in equation (i)

$$\frac{(x+3)^2}{x+2} + \ln(x+3)^2 - 1 - \ln 3 = 0$$

$$\frac{(x+3)^2}{x+2} + \ln \frac{(x+3)^2}{3} - 1 = 0$$

$$\because x > 0 \Rightarrow x + 3 > x + 2$$

$$\text{and } x + 3 > 3$$

$$\text{So } \frac{(x+3)^2}{x+2} + \ln \frac{(x+3)^2}{3} > 1$$

$$\therefore \frac{(x+3)^2}{x+2} + \ln \frac{(x+3)^2}{3} - 1 = 0 \text{ has no solution}$$

\Rightarrow curve $y = (x + 3)^2$ does not intersect

43. Consider a pyramid OPQRS located in the first octant ($x \geq 0, y \geq 0, z \geq 0$) with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP= 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then-

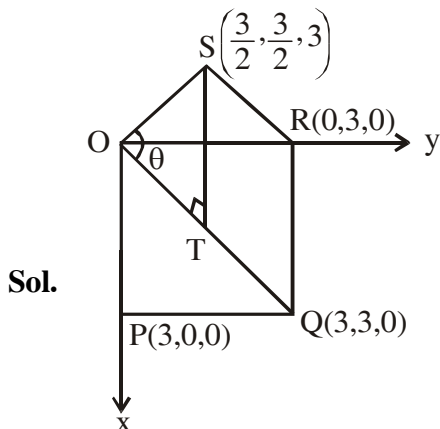
(A) the acute angle between OQ and OS is $\frac{\pi}{3}$

(B) the equation of the plane containing the triangle OQS is $x - y = 0$

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Ans. (B,C,D)



Given $OP = OR = 3$ and $OPQR$ is a square

$$\Rightarrow OQ = 3\sqrt{2} \Rightarrow OT = \frac{3}{\sqrt{2}} \text{ and } ST = 3$$

$$\text{using } \Delta SOT, \tan \theta = \frac{ST}{OT} = \sqrt{2} \Rightarrow \theta = \tan^{-1} \sqrt{2}$$

clearly, equation of plane containing triangle OQS is $Y - X = 0$

Also, length of perpendicular from P to the plane containing the triangle OQS is $PT = \frac{3}{\sqrt{2}}$

$$\text{Also equation of RS is } \vec{r} = 3\hat{j} + t\left(\frac{3}{2}\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$$

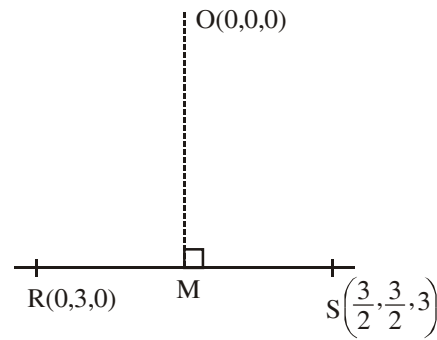
$$= \left(\frac{3t}{2}, 3 - \frac{3t}{2}, 3t\right)$$

$$\text{Let co-ordinates of } M = \left(\frac{3t}{2}, 3 - \frac{3t}{2}, 3t\right)$$

$$\therefore \overline{OM} \cdot \overline{RS} = 0$$

$$\Rightarrow \frac{9}{4}t - \frac{3}{2}\left(3 - \frac{3t}{2}\right) + 9t = 0 \Rightarrow \frac{9t}{2} + 9t = \frac{9}{2} \Rightarrow t = \frac{1}{3}$$

$$\therefore M = \left(\frac{1}{2}, \frac{5}{2}, 1\right) \Rightarrow OM = \sqrt{\frac{1}{4} + \frac{25}{4} + 1} = \sqrt{\frac{30}{4}} = \sqrt{\frac{15}{2}}$$



44. In a triangle XYZ , let x, y, z be the lengths of sides opposite to the angles X, Y, Z , respectively and

$2s = x + y + z$. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then-

(A) area of the triangle XYZ is $6\sqrt{6}$

(B) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(C) $\sin \frac{X}{2} \sin \frac{Y}{2} \sin \frac{Z}{2} = \frac{4}{35}$

(D) $\sin^2 \left(\frac{X+Y}{2}\right) = \frac{3}{5}$

Ans. (A,C,D)

Sol.

$$\text{Let } \frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2} = k$$

$$s-x = 4k \Rightarrow y+z-x = 8k$$

$$s-y = 3k \Rightarrow x+z-y = 6k$$

$$s-z = 2k \Rightarrow x+y-z = 4k$$

$$\Rightarrow x = 5k, y = 6k, z = 7k$$

$$\Rightarrow 3s - (x + y + z) = 9k$$

$$s = 9k$$

Let r be inradius

$$\Rightarrow \pi r^2 = \frac{8\pi}{3}$$

$$\pi \left(\frac{\Delta}{s} \right)^2 = \frac{8\pi}{3}$$

$$\Delta = \sqrt{\frac{8}{3}} \cdot s$$

$$\sqrt{s(s-x)(s-y)(s-z)} = \sqrt{\frac{8}{3}} \cdot s$$

$$\sqrt{9k \cdot 4k \cdot 3k \cdot 2k} = \sqrt{\frac{8}{3}} \cdot 9k$$

$$\sqrt{24 \cdot 9k^2} = \sqrt{\frac{8}{3}} \cdot 9k$$

$$k = 1$$

$$\Rightarrow x = 5, y = 6, z = 7$$

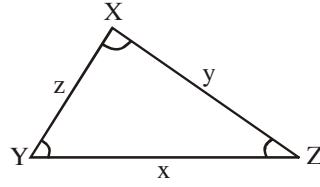
$$\Delta = \sqrt{\frac{8}{3}} \cdot 9k = \sqrt{\frac{8}{3}} \cdot 9 = 6\sqrt{6}$$

$$R = \text{circumradius} = \frac{xyz}{4\Delta} = \frac{5 \cdot 6 \cdot 7}{4 \cdot 6\sqrt{6}} = \frac{35}{4\sqrt{6}}$$

$$\sin \frac{X}{2} \sin \frac{Y}{2} \sin \frac{Z}{2} = \frac{r}{4R} = \frac{\frac{\Delta}{s}}{\frac{xyz}{4\Delta}} = \frac{\Delta^2}{s \cdot xyz} = \frac{36 \cdot 6}{9 \cdot 5 \cdot 6 \cdot 7} = \frac{4}{35}$$

$$\cos Z = \frac{x^2 + y^2 - z^2}{2xy} = \frac{25 + 36 - 49}{2 \cdot 5 \cdot 6} = \frac{1}{5}$$

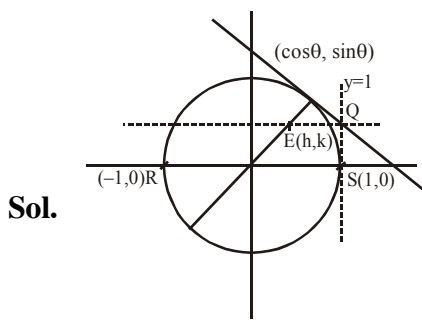
$$\sin^2 \left(\frac{X+Y}{2} \right) = \cos^2 \frac{Z}{2} = \frac{1 + \cos Z}{2} = \frac{1 + \frac{1}{5}}{2} = \frac{3}{5}$$



45. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1,0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. then the locus of E passes through the point(s)-

- (A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

Ans. (A,C)



Tangent at P : $x \cos\theta + y \sin\theta = 1$... (i)

Tangent at S : $x = 1$... (ii)

\therefore By (i) & (ii) : $Q\left(1, \frac{1 - \cos\theta}{\sin\theta}\right)$

Line through Q parallel to RS :

$y = \frac{1 - \cos\theta}{\sin\theta} \Rightarrow y = \tan \frac{\theta}{2}$ (iii)

Normal at P : $y = \frac{\sin\theta}{\cos\theta} x \Rightarrow y = \tan\theta \cdot x$ (iv)

Point of intersection of equation (iii) and (iv), E : $h = \frac{1 - \tan^2 \frac{\theta}{2}}{2}; k = \tan \frac{\theta}{2}$

eliminating θ : $h = \frac{1 - k^2}{2} \Rightarrow y^2 = 1 - 2x$

Options (A) and (C) satisfies the locus.

