Paper-1

JEE Advanced, 2016

Part I: Physics

Read the instructions carefully:

General:

1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.
2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on its left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name and roll number in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at 9:00 am, verify that the booklet contains 36 pages and that all the 54 questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

Optical Response Sheet

9. The ORS (top sheet) will be provided with an attached Candidate’s Sheet (bottom sheet). The Candidate’s Sheet is a carbon – less copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate’s Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate’s Sheet at the end of the examination.
13. Do not tamper with of mutilate the ORS. Do not use the ORS for rough work.

14. Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. Do not write any of these details anywhere else on the ORS. Darken the appropriate bubble under each digit of your roll number.

Darken the Bubbles on the ORS

15. Use a Black Ball Point Pen to darken the bubbles on the ORS.

16. Darken the bubble  completely.

17. The correct way of darkening a bubble is as: 

18. The ORS is machine – gradable. Ensure that the bubbles are darkened in the correct way.

19. Darken the bubbles only if you are sure of the answer. There is no way to erase or “un–darken” a darkened bubble.

PART-I : PHYSICS

SECTION–1 : (Maximum Marks : 15)

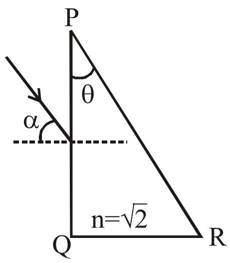
- This section contains **Five** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –1 In all other cases

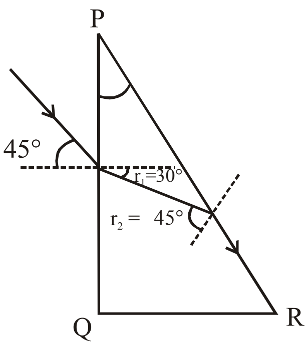
1. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is :



(A) 15° (B) 22.5° (C) 30° (D) 45°

Ans. **(A)**

Sol.



$1 \sin 45° = \sqrt{2} \sin r_1$

$r_2 - r_1 = \theta$

$\theta = 45° - 30°$

$\Rightarrow \theta = 15°$

2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

$\lambda(\mu m)$	$V_0(Volt)$
0.3	2.0
0.4	1.0
0.5	0.4

Given that c = 3×10^8 ms^{-1} and e = 1.6×10^{-19}C, Planck's constant (in units of J s) found from such an experiment is :

(A) 6.0×10^{-34} (B) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34}

Ans. (B)

Sol. $KE_{max} = \frac{hC}{\lambda} - \phi$

$$eV_s = \frac{hC}{\lambda} - \phi$$

$$1.6\times10^{-19} \times 2 = \frac{h\times3\times10^8}{3000\times10^{-10}} - \phi \quad \text{... (i)}$$

$$1.6\times10^{-19} \times 1 = \frac{h\times3\times10^8}{4000\times10^{-10}} - \phi \quad \text{... (ii)}$$

From (ii) $\phi = \frac{h\times3\times10^8}{4000\times10^{-10}} - 1.6\times10^{-19}$

$$1.6\times10^{-19} \times 2 = \frac{h\times3\times10^8}{3000\times10^{-10}} - \frac{h\times3\times10^8}{4000\times10^{-10}} + 1.6\times10^{-19}$$

$$1.6\times10^{-19} = \frac{h\times3\times10^8}{10^{-7}}\left(\frac{1}{3}-\frac{1}{4}\right) = \frac{h\times3\times10^8}{10^{-7}}\left[\frac{4-3}{12}\right]$$

$$1.6\times10^{-19} = \frac{h\times3\times10^8}{10^{-7}} \times \frac{1}{12}$$

$$1.6\times4\times\frac{10^{-19}\times10^{-7}}{10^8} = h$$

6.4×10^{-34} Js = h

3. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h ($<\ell$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are:

($g = 10\ ms^{-2}$)

(A) $\frac{h}{\ell} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} N$

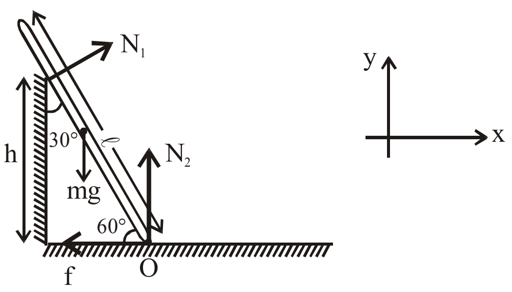
(B) $\frac{h}{\ell} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3} N$

(C) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3} N$

(D) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} N$

Ans. (D)

Sol.



Force equation in x-direction,

$N_1 \cos 30° - f = 0$ ... (i)

Force equation in y-direction,

$N_1 \sin 30° + N_2 - mg = 0$ ... (ii)

Torque equation about O,

$$mg\frac{\ell}{2}\cos 60° - N_1\frac{h}{\cos 30°} = 0 \quad \text{... (iii)}$$

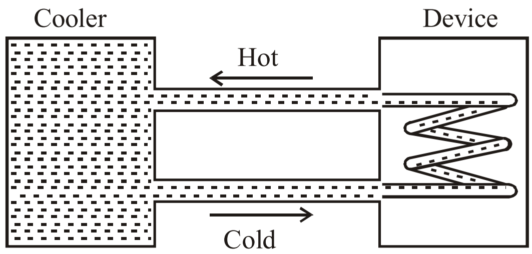
Also, given $N_1 = N_2$ ... (iv)

[Note taking reaction from floor as normal reaction only]

solving (i), (ii), (iii) & (iv) we have

$$\frac{h}{\ell} = \frac{3\sqrt{3}}{16} \ \& \ f = \frac{16\sqrt{3}}{3}$$

4. A water cooler of storage capacity 120 litres can cool water at constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is :



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

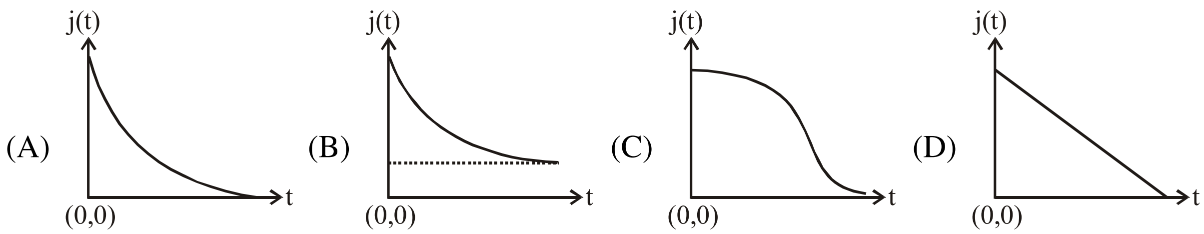
Ans. (B)

Sol. $3000 - P = (120 \times 1)(4.2 \times 10^3)\dfrac{dT}{dt}$

$$\frac{dT}{dt} = \frac{20}{60 \times 60 \times 3}$$

P = 2067 W

5. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material?



Ans. (A)

Sol. This is the problem of RC circuit where the product RC is a constant.

So due to leakage current, charge & current density will exponentially decay & will become zero at infinite time. So correct answer is (A)

for any small element

Resistance $R = \dfrac{dr}{\sigma(2\pi r\ell)}$

Capacitance $C = \dfrac{\in 2\pi r\ell}{dr}$

Product $R \times C = \dfrac{\in}{\sigma}$ = constant

$$q = q_0 e^{-\left(\frac{t\sigma}{\in}\right)}$$

$$I = \frac{dq}{dt} = \frac{q_0\sigma}{\in} e^{-\left(\frac{t\sigma}{\in}\right)}$$

$$\text{Current density} = \frac{I}{A} = \frac{q_0 \frac{\sigma}{\in} e^{-\frac{t\sigma}{\in}}}{2\pi r\ell}$$

$$j \propto e^{-\frac{t\sigma}{\in}}$$

SECTION–2 : (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +4 If only the bubble(s) corresponding to the correct option(s) is (are) darkened.

  Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, Provided NO incorrect option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

  Negative Marks : –2 In all other cases.
- for example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened