Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

JEE MAIN 2016 Online CBT MATHEMATICS Solutions (10/04/2016)

1. Let C be a curve given by $y(x) = 1 + \sqrt{4x-3}$, $x > \frac{3}{4}$. If P is a point on C, such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is

(1) (3, –4) (2) (1, 7) (3) (4, –3) (4) (2, 3)

Ans. **(2)**

Sol. $y(x) = 1 + \sqrt{4x-3}$, $x > \frac{3}{4}$

Let $P(\alpha, 1 + \left(\sqrt{4\alpha - 3}\right)$ be the point.

at which

$\frac{dy}{dx}_{ATP} = \frac{2}{3}$

$\Rightarrow \frac{2}{\sqrt{4\alpha-3}} = \frac{2}{3}$

$\Rightarrow 4\alpha - 3 = 9$

$\Rightarrow \alpha = 3$ Hence P(3,4)

slope of normal at P(3,4) is $= -\frac{3}{2}$

equation of normal

$Y - 4 = -\frac{3}{2}(X - 3)$

$2y - 8 = -3x + 9$

$3x + 2y = 17$

clearly it is passes through (1,7)

2. Let a,b $\in$ R, ($a \neq 0$). If the function f defined as

$$f(x) = \begin{cases} \frac{2x^2}{a}, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \frac{2b^2-4b}{x^3}, & \sqrt{2} \le x < \infty \end{cases}$$

(1) $\left(\sqrt{2}, 1-\sqrt{3}\right)$ (2) $\left(-\sqrt{2}, 1-\sqrt{3}\right)$ (3) $\left(\sqrt{2}, -1+\sqrt{3}\right)$ (4) $\left(-\sqrt{2}, 1+\sqrt{3}\right)$

Ans. **(1)**

Sol. $$f(x) = \begin{cases} \frac{2x^2}{a} & 0 \le x < 1 \\ a & 1 \le x < \sqrt{2} \\ \frac{2b^2-4b}{x^3} & \sqrt{2} \le x < \infty \end{cases}$$

is continuous in $[0,\infty)$

$\Rightarrow$ continuous at x = 1 and x = $\sqrt{2}$

$\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$

$\Rightarrow \frac{2}{a} = a \Rightarrow a^2 = 2$ ------------ (1)

and $\lim_{x\to\sqrt{2}^-} f(x) = \lim_{x\to\sqrt{2}^+} f(x) = f\left(\sqrt{2}\right)$

$\Rightarrow a = \frac{2b^2 - 4b}{2\sqrt{2}}$

$\Rightarrow b^2 - 2b = \sqrt{2}\,a$

If a = $\sqrt{2}$ then $b^2 - 2b - 2 = 0 \Rightarrow b = 1 \pm \sqrt{3}$

If a = $-\sqrt{2}$ then $b^2 - 2b + 2 = 0 \Rightarrow$ b is imaginary which is not possible

$\Rightarrow (a, b) = \left(\sqrt{2}, 1+\sqrt{3}\right)$ or $\left(\sqrt{2}, 1-\sqrt{3}\right)$

3. Let $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$ then the sum of its first 17 terms is equal to

(1) 153 (2) 306 (3) 612 (4) 204

Ans. **(2)**

Sol. $a_1, a_2, a_3 \ldots\ldots\ldots\ a_n \ldots\ldots\ldots\ldots$ are in A.P.

$a_3 + a_{15} = a_7 + a_{11} = a_1 + a_{17} = 36$

sum of first 17 term $= \frac{17}{2}(a_1 + a_{17})$

$= \frac{17}{2} \times 36$

$= 17 \times 18$

$= 306$

4. If A > 0, B > 0 and $A + B = \frac{\pi}{6}$, then the minimum value of tanA + tanB is

(1) $2 - \sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\sqrt{3} - \sqrt{2}$ (4) $4 - 2\sqrt{3}$

Ans. **(4)**

Sol. A, B > 0 and $A + B = \frac{\pi}{6}$

Let y = tanA + tanB

$\frac{dy}{dA} = \sec^2 A - \sec^2\left(\frac{\pi}{6} - A\right)$

Hence tanA + tanB $\uparrow\ \forall\ A \in \left[\frac{\pi}{12}, \frac{\pi}{6}\right]$

and $\tan A + \tan B \downarrow \forall A \in \left[0, \frac{\pi}{12}\right]$

clearly tanA + tanB is minimum when

$A = B = \frac{\pi}{12}$

$\Rightarrow y_{min} = 2\tan\frac{\pi}{12}$

$= \left(2 - \sqrt{3}\right) \times 2 = 4 - 2\sqrt{3}$

5. The contrapositive of the following statement,

" If the side of a square doubles, then its area increases four times", is

(1) If the area of a square does not increase four times, then its side is not doubled.

(2) If the area of a square increases four times, then its side is not doubled.

(3) If the area of a square increases four times, then its side is doubled.

(4) If the side of a square is not doubled, then its area does not increase four times.

Ans. **(1)**

Sol. $p \equiv$ The side of a square doubles

$q \equiv$ Area of square increases four time

so the contrapositive of $p \to q$ is $\sim q \to \sim P$

6. Let A be a 3 × 3 matrix such that $A^2 - 5A + 7I = 0$.

Statement – I : $A^{-1} = \frac{1}{7}(5I - A)$.

Statement – II : The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$.

Then

(1) Statement-I is false, but Statement-II is true.

(2) Both the statements are false.

(3) Both the statements are true.

(4) Statement-I is true, but Statement-II is false.

Ans. **(3)**

Sol. $A^2 - 5A + 7I = 0 \quad |A| \pm 0$

$\Rightarrow \quad A - 5I = -7A^{-1}$

$\Rightarrow \quad A^{-1} = \frac{1}{7}(5I - A)$

Hence statement 1 is true

Now $\quad A^3 - 2A^2 - 3A + I = A(A^2) - 2A^2 - 3A + I$

$= A(5A - 7I) - 2A^2 - 3A + I$

$= 3A^2 - 10A + I$

$= 5A - 20I \qquad = 3((5A - 7I) - 10A + I$

$= 5(A - 4I)$

Statement 2 also correct

7. Equation of the tangent to the circle, at the point (1, –1), whose centre is the point of intersection of the straight lines $x - y = 1$ and $2x + y = 3$ is

(1) $3x - y - 4 = 0$ (2) $x + 4y + 3 = 0$ (3) $x - 3y - 4 = 0$ (4) $4x + y - 3 = 0$

Ans. **(2)**

Sol. Centre of circle is $\left(\frac{4}{3}, \frac{1}{3}\right)$

$\Rightarrow$ equation of circle is $\left(x - \frac{4}{3}\right)^2 + \left(y - \frac{1}{3}\right)^2 = \left(1 - \frac{4}{3}\right)^2 + \left(-1 - \frac{1}{3}\right)^2$

$\Rightarrow x^2 - \frac{8}{3}x + \frac{16}{9} + y^2 - \frac{2}{3}y + \frac{1}{9} = \frac{1}{9} + \frac{16}{9}$

$\Rightarrow x^2 + y^2 - \frac{8}{3}x - \frac{2}{3}y = 0$

$\Rightarrow 3x^2 + 3y^2 - 8x - 2y = 0$

Equation of tangent at (1, –1) is $3x - 3y - 4(x + 1) - (y - 1) = 0$

$\Rightarrow -x - 4y - 3 = 0$

$\Rightarrow x + 4y + 3 = 0$

8. The sum $\sum_{r=1}^{10} (r^2 + 1) \times (r!)$ is equal to

(1) $10 \times (11!)$ (2) $101 \times (10!)$ (3) $(11!)$ (4) $11 \times (11!)$

Ans. **(1)**

Sol. $\sum_{r=1}^{10} (r^2 + 1).r!$

$= \sum_{r=1}^{10} \left\{(r+1)^2 - 2r\right\} r\ !$

$= \sum_{r=1}^{10} (r+1)(r+1)! - 2\sum_{r=1}^{10} r.r!$

$= \sum_{r=1}^{10} \{(r+1)(r+1)! - r(r!)\} - \sum_{r=1}^{10} r.r!$

$= (11.11! - 1) - \sum_{r=1}^{10} ((r+1)! - r!)$

$= (11.11! - 1 - (11! - 1!)$

$= 10.11!$

9. Let ABC be a triangle whose circumcentre is at P. If the position vectors of A,B,C and P are $\vec{a}, \vec{b}, \vec{c}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocentre of this triangle, is

(1) $\vec{0}$ (2) $-\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$ (3) $\vec{a}+\vec{b}+\vec{c}$ (4) $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

Ans. **(4)**

Sol. Position vector of the centroid of ΔABC is = $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$

Now we known that centroid divides the line joining orthocentre to circum centre divided by centriod divided by centroid in the ratio in 2 : 1

$\Rightarrow$ orthocentre = $3\left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right) - 2\left(\frac{\vec{a}+\vec{b}+\vec{c}}{4}\right) = \left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

10. Let f(x) = $\sin^4x + \cos^4x$. Then f is an increasing function in the interval

(1) $\left]\frac{\pi}{4}, \frac{\pi}{2}\right[$ (2) $\left]\frac{5\pi}{8}, \frac{3\pi}{4}\right[$ (3) $\left]0, \frac{\pi}{4}\right[$ (4) $\left]\frac{\pi}{2}, \frac{5\pi}{8}\right[$

Ans. **(1)**

Sol. $f(x) = \sin^4x + \cos^4x$

$f'(x) = 4\sin^3x \cos x - 4\cos^3x \sin x$

$= 4\sin x\cos x(\sin^2x - \cos^2x)$

$= -2\sin 2x.\cos 2x$

$= -\sin 4x > 0$

$\Rightarrow \sin 4x < 0$

$\Rightarrow \pi < 4x < 2\pi$

$\frac{\pi}{4} < x < \frac{\pi}{2}$

11. Let z = 1 + ai be a complex number, a > 0 such that z^3 is a real number. Then the sum $1 + z + z^2 + ... + z^{11}$ is equal to

(1) $-1250\sqrt{3}\,i$ (2) $1250\sqrt{3}\,i$ (3) $-1365\sqrt{3}\,i$ (4) $1365\sqrt{3}\,i$

Ans. **(3)**

Sol. z = 1 + ai, a > 0

$z^3 = 1 - 3a^2 + (3a - a^3)i$ is a real number

$\Rightarrow 3a - a^3 = 0$

$\Rightarrow a^2 = 3$

$\Rightarrow a = \sqrt{3}$, a > 0

$\Rightarrow z = 1 + \sqrt{3}\,i$

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Now $1 + z + z^2 + ... + z^{11} = \frac{1(1-z^{12})}{1-z} = \frac{1-2^{12}(\cos 4\pi + i\sin 4\pi)}{1-(1+i\sqrt{3})} = \frac{1-2^{12}}{-i\sqrt{3}} = \frac{4095}{i\sqrt{3}} = -1365\sqrt{3}\,i$

12. Let P = {θ : $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$} and Q = {$\theta$: $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$} be two sets. Then

(1) $Q \not\subset P$ (2) $P \not\subset Q$ (3) $P \subset Q$ and $Q - P \neq \phi$ (4) P = Q

Ans. **(4)**

Sol. For Let P

$\sin\theta = \cos\theta(\sqrt{2} + 1)$

$(\sqrt{2} - 1)\sin\theta = \cos\theta$ ..... (i)

For Let Q

$\cos\theta = (\sqrt{2} - 1)\sin\theta$ .... (ii)

(i) & (ii) are same P = Q

13. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1,2 and 6, then the mean deviation from the mean of the data is

(1) 2.5 (2) 2.8 (3) 2.6 (4) 2.4

Ans. **(Bonus)**

Sol. **This question is wrong**

$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 5$

$\sum_{i=1}^{5} x_i = 25$ ....(i)

Also $\sigma^2 = 124$

$\Rightarrow \frac{\sum x_1^2}{5} - (\bar{x})^2 = 124$

$\Rightarrow \frac{\sum x_1^2}{5} = 124 + 25 = 149$

$\Rightarrow (x_1^2 + x_2^2 + ... + x_5^2) = 745$

$\Rightarrow x_1^2 + x_2^2 = 704$ ....(ii)

by (i) $x_1 + x_2 = 16$............(iii)

$2x_1 x_2 + 704 = 256$

$x_1 x_2 = \frac{256 - 704}{2}$

$x_1 x_2 = 128 - 352 = -224$...........(iv)

Now $\frac{\sum |x_1 - 5|}{5} = \frac{|x_1 - 5| + |x_2 - 5| + 4 + 3 + 1}{5}$

$= \frac{8 + |x_1 - 5| + |11 - x_1|}{5}$

$= \frac{8 + 6}{5} = 2.8$ Ans

14. The number of distinct real values of λ for which the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{\lambda^2}$ and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$ are coplanar is

(1) 3 (2) 2 (3) 1 (4) 4

Ans. **(1)**

Sol. $\begin{vmatrix} 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \\ 2 & 0 & 4 \end{vmatrix} = 0$

$4\lambda^2 - 2(0) + \lambda^2(-2\lambda^2) = 0$

$2\lambda^2[2-\lambda^2] = 0$

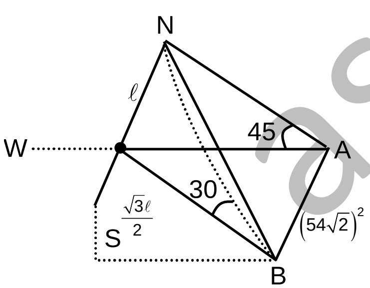
$\lambda = 0 \quad \lambda = \pm\sqrt{2}$

15. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45º. The angle of elevation of the top of the same tower from a point B, due south of A is 30º. If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in metres), is

(1) 54 (2) 108 (3) $54\sqrt{3}$ (4) $36\sqrt{3}$

Ans. **(2)**

Sol.



$\frac{\ell^2}{4} + (54v_2)^2 = \frac{3\ell^2}{4}$

$(54)^2 \times 2 \times 2 = \ell^2$

$\ell = 54 \times 2 = 108$

16. $\lim\limits_{x\to 0} \frac{(1-\cos 2x)^2}{2x\tan x - x\tan 2x}$ is

(1) 2 (2) $-\frac{1}{2}$ (3) $\frac{1}{2}$ (4) –2

Ans. **(1)**

Sol. $$\frac{(2\sin^2 x)^2}{x\left[2\left(x-\frac{x^3}{3}+\frac{2x^5}{15}.....\right)-\left(2x-\frac{8x^3}{3}....\right)\right]}$$

$$= \frac{4\sin^4 x}{x^4\left[-\frac{2}{3}+\frac{8}{3}\right]} = \frac{4}{2} = 2$$

17. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}$, where $0 \le x < \frac{\pi}{2}$ and y(0) = 1, is given by

(1) $y^2 = 1 - \frac{x}{\sec x + \tan x}$

(2) $y^2 = 1 + \frac{x}{\sec x + \tan x}$

(3) $y = 1 + \frac{x}{\sec x + \tan x}$

(4) $y = 1 - \frac{x}{\sec x + \tan x}$

Ans. **(4)**

Sol. $2y\frac{dy}{dx} + y^2\sec x = \tan x$

put $y^2 = t$ $\Rightarrow$ $2y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + t\sec x = \tan x$

I.F. $= e^{\int \sec x dx} = e^{\ell n(\sec x + \tan x)} = \sec x + \tan x$

$t.(\sec x + \tan x) = \int (\sec x + \tan x)\tan x \, dx$

$= \int \sec x \tan x dx + \int \tan^2 x dx$

$y^2(\sec x + \tan x) = \sec x + \tan x - x + c$

$y(0) = 1$ $\Rightarrow$ $c = 0$

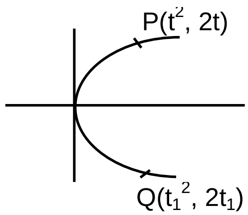
$\Rightarrow$ $y^2 = 1 - \frac{x}{\sec x + \tan x}$

18. P and Q are two distinct points on the parabola, $y^2 = 4x$, with parameters t and t_1 respectively. If the normal at p passes through Q, then the minimum value of t_1^2 is

(1) 4 (2) 6 (3) 8 (4) 2

Ans. **(3)**

Sol.



$t_1 = -t - \frac{2}{t}$

$t_1^2 = t^2 + \frac{4}{t^2} + 4$

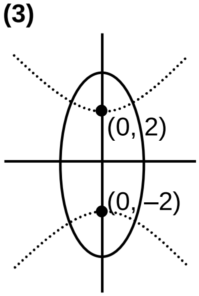
min of $t_1^2 = 8$

19. A hyperbola whose transverse axis is along the major axis of the conic, $\frac{x^2}{3} + \frac{y^2}{4} = 4$ and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does NOT lie on it ?

(1) $\left(\sqrt{5}, 2\sqrt{2}\right)$ (2) $\left(5,\ 2\sqrt{3}\right)$ (3) (0, 2) (4) $\left(\sqrt{10},\ 2\sqrt{3}\right)$

Ans. **(3)**

Sol.

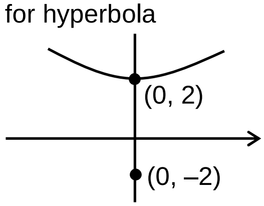


ellipse $\frac{x^2}{12} + \frac{y^2}{16} = 1$

foci (0, ± be)

$e_e = \sqrt{1 - \frac{12}{16}} = \frac{1}{2}$

for hyperbola



$h_H = 2$

$e_H = \frac{3}{2}$

equation $\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

$e_H = \frac{3}{2} = \sqrt{1+\frac{a^2}{b^2}} \quad \Rightarrow \quad \frac{9}{4} - 1 = \frac{a^2}{b^2}$

$\frac{a^2}{b^2} = \frac{5}{4} \quad \Rightarrow \quad a^2 = 5$

$$\frac{x^2}{5} - \frac{y^2}{4} = -1$$

20. For $x \in R$, $x \neq 0$, if y(x) is a differentiable function such that $x\int_1^x y(t)\,dt = (x+1)\int_1^x t y(t)\,dt$, then y(x) equals (where C is a constant)

(1) $Cx^3 e^{\frac{1}{x}}$ (2) $\frac{C}{x}e^{-\frac{1}{x}}$ (3) $\frac{C}{x^2}e^{-\frac{1}{x}}$ (4) $\frac{C}{x^3}e^{-\frac{1}{x}}$

Ans. **(2)**

Sol. $x\int_1^x y(t)dt = (x+1)\int_1^x ty(t)dt$ ....(i)

differentiate equation (1)

$xy(1) + \int_1^x y(t)dt = (x+1)\, xy(x) - \int_1^x ty(t)dt$

$\int_1^x y(t)dt = x^2 y(x) - \int_1^x ty(t)dt$

again differentiate

$y(x) = 2xy(x) + x^2 y(x) - xy(x)$

$y = xy + x^2 \frac{dy}{dx}$

$y(1-x) = x^2 \frac{dy}{dx}$

$\frac{(1-x)}{x^2} dx = \frac{dy}{y}$

solve differential equation

$$-\frac{1}{x} - \ell nx = \ell ny + \ell nc$$

$$-\frac{1}{x} = \ell n\, xy + \ell nc$$

$$xyc = e^{-1/x}$$

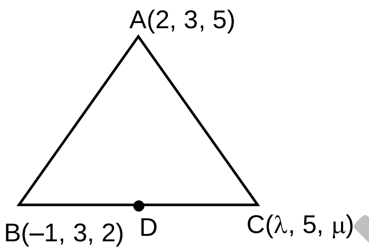
$$y = \frac{c}{x}\, e^{-1/x}$$

21. ABC is a triangle in a plane with vertices A(2,3,5), B(–1,3,2) and C(λ, 5, μ). If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3 + \mu^3 + 5)$ is

(1) 676 (2) 1130 (3) 1348 (4) 1077

Ans. **(3)**

Sol.



$$D \equiv \left(\frac{-1+\lambda}{2}, 4, \frac{2+\mu}{2}\right)$$

direction cosine of AD = $\left\{\frac{-1+\lambda}{2} - 2, 4-3, \frac{2+\mu}{2} - 5\right\}$

$$\left\{\frac{-1+\lambda}{2} - 2, 4-3, \frac{2+\mu}{2} - 5\right\}$$

$$\overrightarrow{AD} = \frac{\lambda-5}{2} i + j + \frac{\mu-8}{2}\hat{k}$$

$$\Rightarrow \frac{\left(\frac{\lambda-5}{2}\right)}{\sqrt{\left(\frac{\lambda-5}{2}\right)^2 + 1^2 + \left(\frac{\mu-8}{2}\right)^2}} = \frac{1}{\sqrt{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}} = \frac{\left(\frac{\mu-8}{2}\right)}{\sqrt{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}}$$

$$\overrightarrow{AD}.i = \overrightarrow{AD}.\hat{j} = \overrightarrow{AD}.\hat{k}$$

$\lambda = 7$, $\mu = 10$

$\lambda^3 + \mu^3 + 5 = 343 + 1000 + 5 = 1348$

22. A ray of light is incident along a line which meets another line, $7x - y + 1 = 0$, at the point (0, 1). The ray is then reflected from this point along the line, $y + 2x = 1$. Then the equation of the line of incidence of the ray of light is

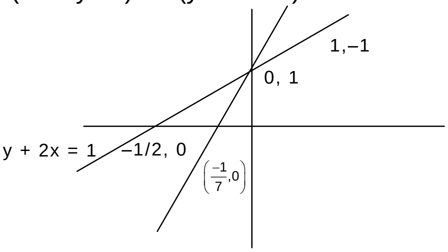
(1) $41x + 38y - 38 = 0$ (2) $41x - 38y + 38 = 0$ (3) $41x + 25y - 25 = 0$ (4) $41x - 25y + 25 = 0$

Ans. **(2)**

Sol. Incidene line

$L_1 + \lambda L_2 = 0$

$(7x - y + 1) + \lambda(y + 2x - 1) = 0$



Let a point (1, –1) on $y + 2x = 1$

And image of (1, –1) lie on incidence line in

$7x - y + 1 = 0$

$$\frac{x-1}{1} = \frac{y+1}{-1} = \frac{-2(7+1+1)}{50} = x = \frac{-38}{25}, \quad y = \frac{-16}{25}$$

$$\left(7\left(\frac{-38}{25}\right) + \frac{16}{25} + 1\right) + \lambda\left(\frac{-16}{25} - \frac{76}{25} - 1\right)$$

$$\lambda = \frac{-225}{117}$$

$$(7x - y + 1)\ \frac{-225}{117}\ (y + 2x - 1) = 0$$

$369x - 342y + 342 = 0$

$41x - 38y + 38 = 0$

23. A straight line through origin O meets the line $3y = 10 - 4x$ and $8x + 6y + 5 = 0$ at points A and B respectively. Then O divides the segment AB in the ratio

(1) 3 : 4 (2) 1 : 2 (3) 2 : 3 (4) 4 : 1

Ans. **(4)**

Sol. Let equation of line thourgh 0(0, 0) is $\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r$ If this line meets $3y = 10 - 4x$ at A then

$3r, \sin\theta = 10 - 4\, r_1 \cos\theta$

$r_1(3\sin\theta + 4\cos\theta) = 10$…….(i)

Again the line meets $8x + 6y + 5 = 0$ at B

$\Rightarrow 8r_2\cos\theta + 6r_2\sin\theta + 5 = 0$

$\Rightarrow 2r_2(3\sin\theta + 4\cos\theta) = -5$……..(ii)

by $\frac{1}{2} \quad \Rightarrow \quad \frac{r_1}{2r_2} = \frac{10}{-5} \quad \Rightarrow \quad \frac{r_1}{r_2} = -\frac{4}{1} = 4$

24. The value of the integral $\int_4^{10} \frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$, where [x] denotes the greatest integer less than or equal to x, is

(1) 3 (2) 7 (3) 6 (4) $\frac{1}{3}$

Ans. **(1)**

Sol. $I = \int_4^{10} \frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$ ....(i)

Use property $\int_a^b f(a+b-x)dx = \int_a^b f(x)dx$

$\Rightarrow I = \int_4^{10} \frac{[x^2-28x+196]dx}{[x^2]+[x^2-28x+196]}$ ....(ii)

by (i) and (ii)

$2I = \int_4^{10} dx = 10 - 4 = 6$

$I = 3$

25. If $\frac{^{n+2}C_6}{^{n-2}P_2} = 11$, then n satisfies the equation

(1) $n^2 + n - 110 = 0$ (2) $n^2 + 5n - 84 = 0$ (3) $n^2 + 3n - 108 = 0$ (4) $n^2 + 2n - 80 = 0$

Ans. **(3)**

Sol. $\frac{^{n+2}C_6}{^{n-2}P_2} = 11 \quad \Rightarrow \quad \frac{(n+2)!}{6!(n-4)!} = 11.\frac{(n-2)!}{(n-4)!}$

$\Rightarrow$ (n+2)! = 11. 6! (n–2)!

$\Rightarrow$ (n+2) (n+1) n(n–1) = 11.6!

$\Rightarrow$ (n+2) (n+1) n (n–1) = 11.10.9.8

$\Rightarrow$ n + 2 = 11

$\Rightarrow$ n = 9

Which satifies the $n^2 + 3n - 108 = 0$

26. If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}}+\frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, (x > 0), are m and n respectively, then $\frac{m}{n}$ is equal to

(1) $\frac{5}{4}$ (2) $\frac{4}{5}$ (3) 27 (4) 182

Ans. **(4)**

Sol. $T_{r+1} = {}^{18}C_r(x^{1/3})^{18-r}\left(\frac{1}{2x^{1/3}}\right)^r$

$= {}^{18}C_r\left(\frac{1}{2}\right)^r x^{\frac{18-2r}{3}}$

For coefficient of x^{-2}, $\frac{18-2r}{3} = -2 \Rightarrow r = 12$

For coefficient of x^{-4}, $\frac{18-2r}{3} = -4 \Rightarrow r = 15 \Rightarrow \frac{m}{n} = \frac{{}^{18}C_{12}\left(\frac{1}{2}\right)^{12}}{{}^{18}C_{15}\left(\frac{1}{2}\right)^{15}}$

$\frac{{}^{18}C_6(2)^3}{{}^{18}C_3} = 182$

27. If $A = \begin{bmatrix}-4 & -1\\ 3 & 1\end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is

(1) 2014 (2) 2016 (3) –175 (4) –25

Ans. **(4)**

Sol. $A = \begin{bmatrix}-4 & -1\\ 3 & 1\end{bmatrix}$

$A^2 = \begin{bmatrix}-4 & -1\\ 3 & 1\end{bmatrix}\begin{bmatrix}-4 & -1\\ 3 & 1\end{bmatrix} = \begin{bmatrix}13 & 3\\ -9 & -2\end{bmatrix}$

$A^2 - 2A - I = \begin{bmatrix}13 & 3\\ -9 & -2\end{bmatrix} - \begin{bmatrix}-8 & -2\\ 6 & 2\end{bmatrix} - \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}20 & 5\\ -15 & -5\end{bmatrix}$

And $|A| = -1$

$\Rightarrow \quad |A^{2016} - 2A^{2015} - A^{2014}| = |A|^{2014} \quad |A^2 - 2A - I| = 1\begin{vmatrix}20 & 5\\ -15 & -5\end{vmatrix} = (-100 + 75) = -25$

28. If x is a solution of the equation, $\sqrt{2x+1}-\sqrt{2x-1}=1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to

(1) 2 (2) $\frac{3}{4}$ (3) $2\sqrt{2}$ (4) $\frac{1}{2}$

Ans. **(2)**

Sol. $\sqrt{2x+1} = 1 + \sqrt{2x-1}$

Squaring on both sides

$2x + 1 = 1 + 2x - 1 + 2\sqrt{2x-1}$

$1 = 2\sqrt{2x-1}$

$1 = 4\sqrt{2x-1}$

$x = 5/8$

Now $\sqrt{4x^2-1}$ at $x = 5/8$ $=$ $\sqrt{4x\frac{25}{64}-1} = 3/4$

29. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is

(1) $\frac{192}{729}$ (2) $\frac{256}{729}$ (3) $\frac{240}{729}$ (4) $\frac{496}{729}$

Ans. **(2)**

Sol. Given = p = 2q & we know that p+q = 1 $\Rightarrow$ P = 2/3, q = 1/3

The problem of at least 5 successes

$= {}^6C_5\ P^5\ q + {}^6C_6\ P^6$

$= 6\times\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)+1\times\left(\frac{2}{3}\right)^6 = \frac{256}{729}$

30. The integral $\int \frac{dx}{\left(1+\sqrt{x}\right)\sqrt{x-x^2}}$ is equal to (where C is a constant of integration)

(1) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$ (2*) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$ (3) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$ (4) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

Ans. **(2)**

Sol. $I = \int \frac{dx}{\left(1+\sqrt{x}\right)\sqrt{x-x^2}}$

put $x = \cos^2\theta$

$dx = -2\cos\theta \sin\theta \, d\theta$

$I = \int \frac{-2\sin\theta\cos\theta d\theta}{(1+\cos\theta)\cos\theta\sin\theta}$ $= -2\int \frac{d\theta}{2\cos^2\theta/2}$

$= -\int \sec^2\left(\frac{\theta}{2}\right)d\theta$ $\therefore \cos\theta = \sqrt{x}$

$= -2 \tan \theta/2 + C$ $\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} = \sqrt{x}$

$= -2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + c$ $\Rightarrow$ $\tan^2\left(\frac{\theta}{2}\right) = \frac{1-\sqrt{x}}{1+\sqrt{x}}$