MATHEMATICS

1. If $(27)^{999}$ is divided by 7, then the remainder is :

(1) 3 (2) 1 (3) 6 (4) 2

Sol. 3

$\frac{(28-1)^{999}}{7} = \frac{28\lambda-1}{7} \Rightarrow \frac{28\lambda-7+1-1}{7} = \frac{7(4\lambda-1)+6}{7}$

$\therefore$ Rem = 6

2. If $S=\left\{x\in[0,2\pi]:\begin{vmatrix}0 & \cos x & -\sin x\\ \sin x & 0 & \cos x\\ \cos x & \sin x & 0\end{vmatrix}=0\right\}$, Then $\sum_{x\in S}\tan\left(\frac{\pi}{3}+x\right)$

(1) $4+2\sqrt{3}$ (2) $-2-\sqrt{3}$ (3) $-2+\sqrt{3}$ (4) $-4-2\sqrt{3}$

Sol. 2

0(0 – cosx) – cosx(0 – $\cos^2$x) – sinx ($\sin^2$x – 0) = 0

$\cos^3$x – $\sin^3$x = 0

$\tan^3$ = 1 ⇒ tanx = 1

$\sum\frac{\sqrt{3}+\tan x}{1-\sqrt{3}}$

$\sum\frac{\sqrt{3}+1}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}} \Rightarrow \sum\frac{1+3+2\sqrt{3}}{-2} = \sum\frac{4^2}{-2}-\frac{2\sqrt{3}}{3}$

$\sum -2-\sqrt{3}$

3. Let A be any 3 × 3 invertible matrix. Then which one of the folliwing is not Always true ?

(1) adj(adj(1)) = $|A|^2$. $(\text{adj}(1))^{-1}$ (2) adj(adj(1)) = |A|. $(\text{adj}(1))^{-1}$

(3) ajd(adj(1)) = |A|.A (4) adj(1) = |A|.A^{-1}

Sol. 3

See theorey

4. The value of $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, $|x|<\frac{1}{2}$, $x\neq 0$, is equal to :

(1) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$ (2) $\frac{\pi}{4}-\cos^{-1}x^2$ (3) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$ (4) $\frac{\pi}{4}+\cos^{-1}x^2$

Sol. 1

$x^2 = \cos 2\theta$; $\theta = \frac{1}{2}\cos x^2$

$\tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right]$

$\tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right]$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$

$= \frac{\pi}{4}+\frac{1}{2}\cos x^2$

5. The area (in sq. units) of the parallelogram whosed diagonals are along the vectors $8\hat{i}-6\hat{j}$ and $3\hat{i}+4\hat{j}-12\hat{k}$, is :

(1) 20 (2) 65 (3) 52 (4) 26

Sol. **2**

$$d_1 \times d_2 = \begin{vmatrix} i & j & k \\ 8 & -6 & 0 \\ 3 & 4 & -12 \end{vmatrix}$$

$= 72\hat{i}-(-96)\hat{j}+50\hat{k}$

= 5178 + 7056 + 2500

$|d_1 \times d_2| = \sqrt{16900} = 130$

$A = \frac{1}{2}|d_1 \times d_2| = \frac{1}{2}\times 130$

= 65

6. The integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x}{(\tan x + \cot x)^3}$ dx equals :

(1) $\frac{13}{256}$ (2) $\frac{15}{64}$ (3) $\frac{13}{32}$ (4) $\frac{15}{128}$

Sol. **4**

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{\cos 2x}{\left(\frac{1}{\sin 2x}\right)^3} = \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \cos 2x \times \sin 2x.\sin^2(2x)dx$$

$$= \frac{1}{4}\int \sin 4x.(1-\cos 4x)dx$$

$$= \frac{1}{4}\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sin 4x - \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sin 8x$$

7. Let $z \in C$, the set of complex numbers. Then the equation, 2|z + 3i| – |z – i| = 0 a circle with radius $\frac{8}{3}$.

(1) a cirlce with radius $\frac{8}{3}$ (2) an ellipse with length of minor axis $\frac{16}{9}$

(3) an ellipse with length of major axis $\frac{16}{3}$ (4) a circle with diameter $\frac{10}{3}$

Sol. **1**

$2|x+i(y+3)| = |x+i(y-1)|$

= $2\sqrt{x^2+(y+3)^2}$ $\sqrt{x^2+(y-1)^2}$

= $4x^2 + 4(y + 3)^2 = x^2 + (y - 1)^2$

= $3x^2 = y^2 - 2y + 1 - 4y^2 - 24y - 36$

= $3x^2 + 3y^2 + 26y + 35 = 0$

= $x^2 + y^2 + \frac{26}{3}y + \frac{35}{3} = 0$

= $r = \sqrt{0+\frac{169}{9}-\frac{35}{3}}$

= $\sqrt{\frac{64}{9}} = \frac{8}{3}$

8. If the um of the first n terms of the series $\sqrt{3} + \sqrt{75} + \sqrt{243} + \sqrt{507} + \ldots\ldots$ is $435\sqrt{3}$, then n equals :

(1) 13 (2) 15 (3) 29 (4) 18

Sol. **2**

$\sqrt{3}\left[1+\sqrt{25}+\sqrt{81}+\sqrt{69}+\ldots\ldots\right] = 435\sqrt{3}$

$\sqrt{3}$ [1 + 5 + 9 + 13 +.....T_n] = $435\sqrt{3}$

= $\sqrt{3}\times\frac{n}{2}\left[2+(n-1)4\right] = 435\sqrt{3}$

$2n + 4n^2 - 4n = 870$

= $4n^2 - 2n - 870 = 0$

= $2n^2 - n - 435 = 0$

$n = \frac{1\pm\sqrt{1+4\times2\times435}}{4}$

= $\frac{1\pm59}{4}$

= $\frac{1+59}{4} = 4;\ \frac{1-59}{4}$

9. The tangent at the point (2,–2) to the curve $x^2y^2 - 2x = 4(1 - y)$ does not pass through the point :

(1) (–2,–7) (2) (8,5) (3) (–4,–9) (4) $\left(4,\frac{1}{3}\right)$

Sol. **1**

$x^2y^2 - 2x = 4 - 4y$

$2xy^2 + 2y.x^2.\frac{dy}{dx} - 2 = -4.\frac{dy}{dx}$

$\frac{dy}{dx}\left(2y.x^2+4\right) = 2 - 2x.y^2$

$\left.\frac{dy}{dx}\right|_{2,-2} = \frac{2-2\times2\times4}{2(-2)\times4+4} = \frac{+14}{+12} = \frac{7}{6}$

$(y + 2) = \frac{7}{6}(x-2) \Rightarrow 7x - 6y = 26$

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10. Let f(x) = 2^{10}dx + 1 and g(x) = 3^{10}x – 1. If (fog)(x) = x, then x is equal to :

(1) $\frac{2^{10}-1}{2^{10}-3^{-10}}$ (2) $\frac{1-2^{-10}}{3^{10}-2^{-10}}$ (3) $\frac{3^{10}-1}{3^{10}-2^{-10}}$ (4) $\frac{1-3^{-10}}{2^{10}-3^{-10}}$

Sol. **2**

f(g(x)) = x

f(3^{10}x – 1) = 2^{10}(3^{10}.x – 1) = x

= $\frac{1}{3^{10}-2^{-110}}$

2^{10}(3^{10}x –1) + 1 = x

x(6^{10} –1) = 2^{10} – 1

x = $\frac{2^{10}-1}{6^{10}-1}$ = $\frac{1-2^{-10}}{3^{10}-2^{-10}}$

11. The proposition (~p)V(P^~q) is equalvalent to :

(1) P→ ~q (2) P^~q (3) q → p (4) p v~ q

Sol. **2**

(~p) V (P ^ ~q)

p	q	~ p	~ q	p^ ~ q	(~ p)v(p^ ~ q)
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	F	F

12. If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is :

(1) 47^{th} (2) 45^{th} (3) 46^{th} (4) 44^{th}

Sol. **3**

E,E, N, Q,U

(i) E......... = 24

(ii) N .... = $\frac{4!}{2}=12$

(iii) Q E........ = 3! = 6

(iv) Q N ......= $\frac{3!}{2!}$ = 3

(v) Q U E E N = 1

Total = (i) + (ii) + (iii) + (iv) + (v) = 46^{th}

13. The curve ssatisfying the differential equation, ydx – (x + $3y^2$)dy = 9 and passing through the point (1,1), also passes through the point :

(1) $\left(\frac{1}{4},-\frac{1}{2}\right)$ (2) $\left(-\frac{1}{3},\frac{1}{3}\right)$ (3) $\left(\frac{1}{4},\frac{1}{2}\right)$ (4) $\left(\frac{1}{3},-\frac{1}{3}\right)$

Sol. **2**

ydx – xdy – $3y^2$ dy = 0

$\frac{dx}{dy}$ = $\frac{x}{y}+3y$

$\frac{dx}{dy}-\frac{x}{y}$ = 3y

I.f. = $e^{-\int \frac{1}{y}dy}$ = $e^{-\ln y}$ = $\frac{1}{y}$

$\therefore$ solution is

$\frac{x}{y} = \int 3y.\frac{1}{y}dy$

$\frac{x}{y}$ = 3y + c

pass through (1,1) $\therefore$ 1 = 3 + c ; c = –2

x = $3y^2 - 2y$

(i) $\left(\frac{1}{4}, -\frac{1}{2}\right)$ = $\frac{1}{4} = \frac{3}{4} + 1$

(ii) $-\frac{1}{3} = \frac{1}{3} - \frac{2}{3}$ = $-\frac{1}{3}$

14. An unbiased coin is tossed eight times. The probabiity of obtaining at least one head and at least one tail is :

(1) $\frac{127}{128}$ (2) $\frac{63}{64}$ (3) $\frac{255}{256}$ (4) $\frac{1}{2}$

Sol. **1**

$1 - \int P(\text{All Head}) + P(\text{All Tail})$

$1 - \left\{\frac{1}{2^8} + \frac{1}{2^8}\right\}$

= $1 - \frac{1}{2^7}$

= $1 - \frac{1}{128}$ = $\frac{127}{128}$

15. If the arithmetic mean of two nubers and b,a, > b > 0. is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to :

(1) $\frac{7\sqrt{3}}{12}$ (2) $\frac{3\sqrt{2}}{4}$ (3) $\frac{\sqrt{6}}{2}$ (4) $\frac{5\sqrt{6}}{12}$

Sol. **4**

$\frac{a+b}{2} = 5\sqrt{ab}$

$\frac{a+b}{\sqrt{ab}}$ = 10

$\therefore$ $\frac{a}{b}$ = $\frac{10+\sqrt{96}}{10-\sqrt{96}}$ = $\frac{10+4\sqrt{6}}{10-4\sqrt{6}}$

Use C and D

$\frac{a+b}{a-b} = \frac{20}{8\sqrt{6}}$ = $\frac{5}{2\sqrt{6}}$ = $\frac{5\sqrt{6}}{12}$

16. $\lim_{x\to3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{3}$

Sol. **1**

$\lim_{x\to3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$

Rationalize

$\lim_{x\to3}\frac{(3x-9)\times\left(\sqrt{2x-4}+\sqrt{2}\right)}{\{(2x-4)-2\}\times\left(\sqrt{3x}+3\right)}$

$= \lim_{x\to3}\frac{3(x-3)}{2(x-3)}\times\frac{\sqrt{2x-4}+\sqrt{2}}{\left(\sqrt{3x}+3\right)}$

$= \frac{3}{2}\times\frac{2\sqrt{2}}{6} = \frac{1}{\sqrt{2}}$

17. The locus of the point of intersection of the straigh lines,
tx – 2y – 3t = 0
x – 2ty + 3 = 0 $(t\in R)$, is :
(1) A hyperbola with the length of conjugate axis 3
(2) a hyperbola with eccentricity $\sqrt{5}$
(3) an ellipse with the length of major axis 6
(4) an ellipse with eccentricity $\frac{2}{\sqrt{5}}$

Sol. **1**
tx – 2y – 3t = 0
x – 2ty + 3 = 0

$$\begin{array}{l} tx - 2y - 3t = 0 \\ tx - 2t^2y + 3t = 0 \\ \quad - \quad + \quad - \\ \hline y(2t^2-2) = 6t \end{array} \qquad \begin{array}{l} t^2x - 2ty - 3t^2 = 0 \\ x - 2ty + 3 = 0 \\ - \quad + \quad - \\ \hline (t^2-1)x = (3t^2+1) \end{array}$$

$y=\frac{6t}{2t^2-2} = \frac{3t}{t^2-1}$ $\qquad x = -3\sec2\theta$

$2y = 3(-\tan2\theta)$
$\sec^2 2\theta - \tan^2 2\theta = 1$

$\frac{x^2}{9}-\frac{y^2}{9/4} = 1$

$a^2 = 9$; $\qquad b^2 = 9/4$

λ(T.A) = 6 ; $e^2 = 1+\frac{9/4}{9} = 1+\frac{1}{4}$; e $e=\frac{\sqrt{5}}{2}$

18. If $y=\left[x+\sqrt{x^2-1}\right]^{15}$ + $y=\left[x-\sqrt{x^2-1}\right]^{15}$, then $(x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is equal to :

(1) $224y^2$ (2) $125y$ (3) $225y$ (4) $225\,y^2$

Sol. 3

$$y = \left\{x+\sqrt{x^2-1}\right\}^{15}+\left\{x-\sqrt{x^2-1}\right\}^{15}$$

$$\frac{dy}{dx} = 15\left(x+\sqrt{x^2-1}\right)^{14}\ 15\left(x-\sqrt{x^2-1}\right)^{14}\left(1-\frac{x}{\sqrt{x^2-1}}\right)$$

$$\frac{dy}{dx}=\frac{15}{\sqrt{x^2-1}}.y \qquad \text{...(i)}$$

$$\sqrt{x^2-1}.\frac{dy}{dx} = 15y$$

$$\frac{x}{\sqrt{x^2-1}}.\frac{dy}{dx} + \sqrt{x^2-1}\frac{d^2y}{dx^2} = 15\frac{dy}{dx}$$

$$x\frac{dy}{dx}+\left(x^2-1\right)\frac{d^2y}{dx^2} = 15\sqrt{x^2-1}.\frac{15}{\sqrt{x^2-1}}.y = 225y$$

19. The area (in sq. units) of the smaller portion enclosed between the curves, $x^2 + y^2 = 4$ and $y^2 = 3x$, is :

(1) $\frac{1}{\sqrt{3}}+\frac{4\pi}{3}$ (2) $\frac{1}{\sqrt{3}}+\frac{2\pi}{3}$ (3) $\frac{1}{2\sqrt{3}}-\frac{\pi}{3}$ (4) $\frac{1}{2\sqrt{3}}+\frac{2\pi}{3}$

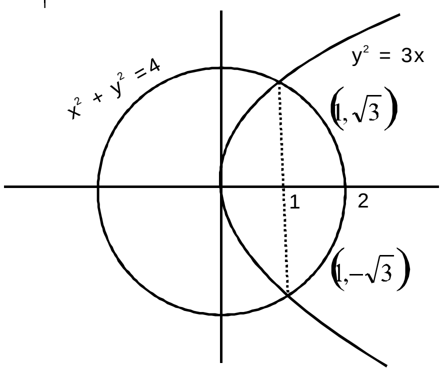
Sol. 1

$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$

$x = -4,\ x = 1$

$$\text{Area} = \left(\int_0^1\sqrt{3}.\sqrt{x}dx\right) + \int_0^1\left(\int_0^1\sqrt{3}.\sqrt{x}dx+\int_1^2\sqrt{4-x^2}.dx\right)\times 2$$



$$= \left(\sqrt{3}\left(\frac{x^{3/2}}{3/2}\right)_0^1+\left(\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}\right)_1^2\right)\times 2$$

$$= \left(\sqrt{3}\left(\frac{2}{3}\right)+\left\{2.\frac{\pi}{2}-\left(\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right)\right\}\right)\times 2$$

$$\left(\frac{2}{\sqrt{3}}-\frac{\sqrt{3}}{2}+\frac{2\pi}{3}\right)\times 2$$

$$= \left(\frac{1}{2\sqrt{3}}+\frac{2\pi}{3}\right)\times 2 = \frac{1}{\sqrt{3}}+\frac{4\pi}{3}$$

d

20. If the common tangents to the parabola, $x^2 + 4y$ and the circle, $x^2 + y^2 = 4$ intersect at the point P, then the distance of P from the origin, is :

(1) $2\left(\sqrt{2}+1\right)$ (2) $3+2\sqrt{2}$ (3) $\sqrt{2}+1$ (4) $2\left(3+2\sqrt{2}\right)$

Sol. **4**

tangent to $x^2 + y^2 = 4$

$y = mx \pm 2\sqrt{1+m^2}$

$x^2 = 4y$

$x^2 = 4mx + 8\sqrt{1+m^2}$

$x^2 = 4mx - 8\sqrt{1+m^2} = 0$

$D = 0$

$16m^2 + 4.8\sqrt{1+m^2} = 0$

$m^2 + 2\sqrt{1+m^2} = 0$

or $m^2 = \sqrt{1+m^2}$

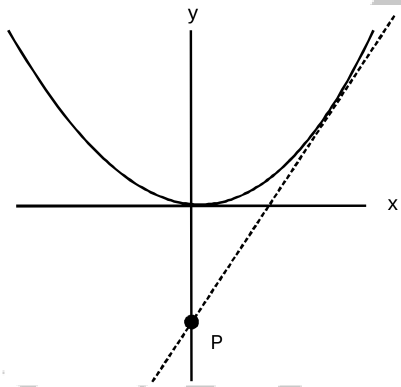
$m^4 = 4 + 4m^2$

$m^4 - 4m^2 - 4 = 0$

$m^2 = \dfrac{4 \pm \sqrt{16+16}}{2}$

$= \dfrac{4 \pm 4\sqrt{2}}{2}$

$m^2 = 2 + 2\sqrt{2}$



21. The line of intersection of the planes

$\vec{r}.(3\hat{i} - \hat{j} + \hat{k}) = 1$ and

$\vec{r}.(\hat{i} + 4\hat{j} - 2\hat{k}) = 2$, is :

(1) $\dfrac{x - \frac{6}{13}}{2} = \dfrac{y - \frac{5}{13}}{7} = \dfrac{z}{-13}$

(2) $\dfrac{x - \frac{4}{7}}{2} = \dfrac{y}{-7} = \dfrac{z + \frac{5}{7}}{13}$

(3) $\dfrac{x - \frac{6}{13}}{2} = \dfrac{y - \frac{5}{13}}{-7} = \dfrac{z}{-13}$

(4) $\dfrac{x - \frac{4}{7}}{-2} = \dfrac{y}{7} = \dfrac{z - \frac{5}{7}}{13}$

Sol. **3**

$\vec{n} = \vec{n}_1 \times \vec{n}_2$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 1 & 4 & -2 \end{vmatrix}$$

$= \hat{i}(-2) - \hat{j}(-7) + \hat{k}(13)$

$\vec{n} = 2\hat{i} + 7\hat{j} + 13\hat{k}$

Now

$3x - y + z = 1$

$x + 4y - 2z = 2$

but $z = 0$

$3x - y = 1 \times 4$

$x + 4y = 2$

$13x = 6$ $\quad\quad$ $x = 6/13$

$y = 5/13$

.... is

$$\boxed{\frac{x-6/13}{-2}=\frac{y-5/13}{7}=\frac{z-0}{13}}$$

or

$$\boxed{\frac{x-6/13}{2}=\frac{y-5/13}{-7}=\frac{z}{-13}}$$

22. If a point P has co - ordinates (0,–2) and Q ia any point on the circle, $x^2 + y^2 - 5x - y + 5 = 0$, then the maximum value of $(PQ)^2$ is :

(1) $8+5\sqrt{3}$ (2) $\frac{47+10\sqrt{6}}{2}$ (3) $14+5\sqrt{3}$ (4) $\frac{25+\sqrt{6}}{2}$

Sol. **3**

$(x - 5/2)^2 \frac{-25}{4} + (y - 1/2)^2 - 1/4 + 5 = 0$

$= (x - 5/2)^2 + (y - 1/2)^2 = 3/2$

on circle Q = $5/2 + \sqrt{3/2}\cos Q$, $\frac{1}{2}+\sqrt{3/2}\sin Q$

$$PQ^2 = \left(\frac{5}{2}+\sqrt{3/2}\cos Q\right)^2 + \left(\frac{5}{2}+\sqrt{3/2}\sin Q\right)^2$$

$PQ^2 = \frac{25}{2}+\frac{3}{2}+5\sqrt{3/2}$ (cos Q + tan Q)

$= 14+5\sqrt{3/2}(\cos Q+\sin Q)$

$man^{mr} = 14+5\sqrt{3/2}\,(\sqrt{2})$

$= 14+5\sqrt{3}$

23. The mean age of 25 teaches in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. if now the mean age of the teachers in this school is 39 years, then the age (in years) of the newly appointed teachers is :

(1) 35 (2) 40 (3) 25 (4) 30

Sol. **1**

$\frac{x_1+x_2+...+x_{25}}{25} = \overline{x} = 40$

$x_1 + x_2 + \ldots + x_{25} = 1000$

$x_2 + x_2 + \ldots + x_{25} - 60 + A = \overline{x} \times 25$

$1000 - 60 + A = 39 \times 25 = 975$

$A = 975 - 940 = 35$

24. Let p(x) be a quadratic polynomial such that p(0) = 1. if p(x) leaves remainder 4 when divided by x – 1 and it leaves remainder 6 when divided by x + 1, then :

(1) p(–2) = 19 (2) p(2) = 19 (3) p(–2) = 11 (4) p(2) = 11

Sol. **1**

$p(x) = ax^2 + bx + c$

p (0) = 1 = c = 1

$$\left.\begin{matrix} p(1)=4 \\ p(-1)=6 \end{matrix}\right]$$

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$\left.\begin{matrix} a+b+c=4 \\ a-b+c=6 \end{matrix}\right] = \left.\begin{matrix} a=4 \\ b=-1 \end{matrix}\right]$

$p(x) = 4x^2 - x + 1$

$p(-2) = 16 + 2 + 1 = 19$

25. consider an ellipse, whose centre is at the origin and its major axis is along the x - axis. if its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area (in sq. units) of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse, is :

(1) 32 (2) 80 (3) 40 (4) 8

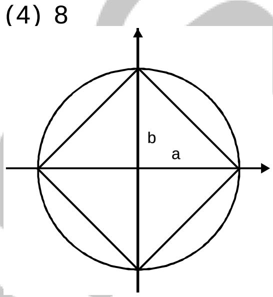
Sol. **3**

e = 3/5, 2ae = 6, a(5) a = 5

$b^2 = a^2 (1-e^2)$

b2 = 25 (1–9/25)

b = 4



area = 4 (1/2 ab)

= 2ab = 40

26. If two parallel chords of the a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the centre respectively, then the distance between these chords, is :

(1) $\frac{8}{\sqrt{7}}$ (2) $\frac{16}{7}$ (3) $\frac{4}{\sqrt{7}}$ (4) $\frac{8}{7}$

Sol. **1**

cos 2Q = 1/7 $= 2\cos^2 Q - 1 = 1/7$

$= 2\cos^2 Q = 8/7$

$\cos^2 Q = 4/7$

$= \frac{cp^2}{4} = \frac{4}{7}$

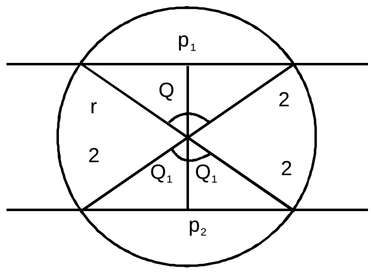
$= Cp = \frac{4}{\sqrt{7}}$

sec2Q = 7 $= \frac{1}{2\cos^2 Q - 1} = 7$

$= 2\left(\frac{Cp_2}{2}\right)^2 - 1 = \frac{1}{7}$

$= 2\left(\frac{Cp_2}{2}\right)^2 = \frac{8}{7}$

$= \boxed{Cp_2 = \frac{4}{\sqrt{7}}}$



$\frac{4}{\sqrt{7}} + \frac{4}{\sqrt{7}} = \frac{8}{\sqrt{7}}$

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27. The number of real values of λ for which the system of linear equations
$2x + 4y - \lambda z = 0$
$4x + \lambda y + 2z = 0$
$\lambda x + 2y + 2z = 0$
has infinitely many solutions, is :
(1) 3 (2) 1 (3) 2 (4) 0

Sol. **2**

$\Delta = 0$

$$\begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & z \\ \lambda & z & 2 \end{vmatrix} \begin{matrix} 2 & 4 \\ 4 & \lambda \\ \lambda & z \end{matrix} = 0$$

$= -(32 + 8 - \lambda^3) = 0$
$= \lambda 3 + 4\lambda - 40 = 0$
=

28. The coodinates of the foot of the perpendicular from the point (1, – 2,1) on the plane containing the lines,

$\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and

$\frac{x-1}{3} = \frac{y-2}{5} = \frac{z-3}{7}$, is :

(1) (2, – 4, 2) (2) (1, 1, 1) (3) (0,0,0) (4) (–1, 2, –`=1)

Sol. **3**

$\bar{n} = \bar{n}_1 \times \bar{n}_2$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}$$

= (9, –18, 9)
= (1, –2, 1)

1(x+1)–2 (y–1) + (2–3) = 0

= $\boxed{x - 2y + z = 0}$

foot to z

$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-1}{1} = -\frac{[1+4+1]}{6}$

$\boxed{x = 0, y = 0, z = 0}$

10,000

29. Three persong P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is :

(1) $\frac{39}{64}$ (2) $\frac{21}{64}$ (3) $\frac{9}{64}$ (4) $\frac{15}{64}$

Sol. **2**

$\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)+\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)$

$= \frac{12+9}{64}$

$= \frac{27}{64}$

30. The integral $\int\sqrt{1+2\cot x(\operatorname{cosec}x+\cot x)}dx \quad \left(0<x<\frac{\pi}{2}\right)$ is equal to :

(where C is a constant of integration)

(1) $2\log\left(\sin\frac{x}{2}\right)+C$ (2) $4\log\left(\sin\frac{x}{2}\right)+C$ (3) $4\log\left(\cos\frac{x}{2}\right)+C$ (4) $2\log\left(\cos\frac{x}{2}\right)+C$

Sol. **1**

$\int\left(\sqrt{+2\cot x\cos ecx+\cos ec^2x+\cot x}\right)dx$

$\int \cos|x+\cot x|\,dx$

$\int (\cos ec+\cot x)dx$

$\int \cos ec\ dx$

$2\log(\log(x_2)+c$