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B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

MATHEMATICS

1. If the line, $\frac{x\quad 3}{1}=\frac{y\quad 2}{1}=\frac{z\quad \lambda}{-2}$ lies in the plane, 2x– 4y +3z=2, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is :

(1) 1 (2) 2 (3) 3 (4) 0

Sol. **4**

pt (3, –2, λ) on pline 2x – 4y +3z –2 = 0
= 6 + 8 – 3λ –2 = 0

= 3λ = 12 $\boxed{\lambda = 4}$

Now

$$\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+4}{-2}=k_1 \qquad ...(1)$$

$$\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}=k_2 \qquad ...(2)$$

pt on give 1 = $(k_1+3, -k_1-2, -2k_1-4)$
pt on give 2= $(12k_2+1, 9k_2, 4k_2)$
$k_1+3 = 12k_2+1 \mid -k_1-2 = 9k_2 \mid -2k_1-4 = 4k_2$

$$\left.\begin{matrix} k_2 = 0 \\ k_1 = -2 \end{matrix}\right]$$

p (1,0,0)
gives are ditersech – thortest distance = 0

2. The coefficient of x^{-5} in the binomial expansion of $\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$ where x≠0,1 is :

(1) –1 (2) 4 (3) 1 (4) –4

Sol. **3**

$$\left[\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{(x^{2/3}-x^{1/3}+1)}-\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

= $(x^{1/3} + 1 - 1 - 1/x^{1/2})^{10}$
= $(x^{1/3} - 1/x^{1/2})$ r = 1/3, b = 1/2

$$r = \frac{\frac{10}{3}-(-5)}{1/3+\frac{1}{2}}$$

$$r = \frac{25/3}{(5\frac{1}{2})}=10$$

cos. = $10c_{10}$ (1) $(-1)^{10}$ = 1

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3. The equation $\text{Im}\left(\frac{iz-2}{z-i}\right)+1=0$, z∈C, z ≠i. represents a part of a circle having radius equal to :

(1) 1 (2) 2 (3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Sol. **3**

Let = x +y

$$\text{Im}\left[\left(\frac{x-y-2}{x+(y-1)}\right)\left(\frac{x-(y-1)}{x-(y-1)}\right)\right]+1=0$$

$$+\ \frac{(y-1)(y+2)+x^2}{x2+(y-1)^2}+1=0$$

$= 2x^2 + 2y^2 - y - 1 = 0$
$= x^2+y^2 - 1/2y - 1/2 = 0$
cot x = 10, 1/4
$= \sqrt{1/2+1/2}=\sqrt{9/16}=3/4$

4. The value of K for which the function $f(x)=\begin{cases}\left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0<x<\frac{\pi}{2}\\ k+\frac{2}{5}, & x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, is

(1) $\frac{2}{5}$ (2) $-\frac{2}{5}$ (3) $\frac{17}{20}$ (4) $\frac{3}{5}$

Sol. **4**

$k+2/5=(4/5)$

$= k+\frac{2}{4}=1$

$K = \frac{3}{5}$

5. If $\int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{4}}}=\frac{k}{k+5}$, then k is equal to :

(1) 4 (2) 2 (3) 3 (4) 1

Sol. **4**

$$\int_1^2 \frac{dx}{(x-1)+3)^{3/2}}$$

$x-1=\sqrt{3}\tan Q$

$= \sqrt{3}\sec^2 Q$

$$\int_0^{\pi/6}\frac{\sqrt{3}\sec dQ}{3\sqrt{3}\sec.3Q}$$

$= \frac{1}{3}\int_0^{\pi/6} \cos esQ = \frac{1}{3}(\tan Q)_0^{\pi/6}$

$= \frac{1}{6} = \frac{k}{k+5} = k+5 = 6k$

$= \boxed{k=1}$

6. For two 3×3 matrices A and B, let A+B =2B' and 3A+2B=I_3, where B' is the transpose of B and I_3 is 3×3 identity matrix. Then :

(1) $10A+5B = 3I_3$ (2) $3A+6B=2I_3$ (3) $5A+10B-2I_3$ (4) $B+2A=I_3$

Sol. **1**

$A^T + B^T = 2B$

$\Rightarrow B = \frac{A^T + B^T}{2}$

$= A + \left(\frac{B^T + A^T}{2}\right) = 2B^T$

$2A + A^T = 2B^T$

$\Rightarrow A = \frac{3B^T - A^T}{2}$

$3A + 2B = I_3$ ...(i)

$\Rightarrow 3\left(\frac{3B^T - A^T}{2}\right) + 2\left(\frac{A^T + B^T}{2}\right) = I_3$

$\Rightarrow \left(\frac{3B^T + 2B^T}{2}\right) + \left(\frac{2A^T - 3A^T}{2}\right) = I_3$

$\Rightarrow 11B^T - A^T = 2I_3$ ...(ii)

Equation (i) + (ii)

$35B = 7I_3$

$\Rightarrow B = \frac{I_3}{5}$

$11\frac{I_3}{5} - A = 2I_3$

$\Rightarrow 11\frac{I_3}{5} - 2I_3 = A$

$\Rightarrow A = \frac{I_3}{5}$

$\because 5A = 5B = I_3$

$\Rightarrow 10A + 5B = 3I_3$

7. If y = mx+c is the normal at a point on the parabola $y^2=8x$ whose focal distance is 8 units, then |c| is equal to :

(1) $8\sqrt{3}$ (2) $10\sqrt{3}$ (3) $2\sqrt{3}$ (4) $16\sqrt{3}$

Sol. **2**

$c = -29m - 9m^3$

$a = 2$

Given $(at^2 - a)^2 + 4a^2t^2 = 64$

$\rightarrow (a(t^2 + 1)) = 8$
$\Rightarrow t^2 + 1 = 4 = t^2 = 3$
$\Rightarrow t = \sqrt{3}$

$\therefore C = -a[-2t - t^3] = 2at(2 + t^2)$
$= 2\sqrt{3}(5)$
$|C| = 10\sqrt{3}$

8. A line drawn through the point P(4,7) cuts the circle $x^2+y^2 = 9$ at the points A and B. Then PA.PB is equal to :
(1) 74 (2) 53 (3) 56 (4) 65

Sol. **3**
P(4,7) it midpoint the circle
PA PB = S_1^2 = PT^2
$S_1 = \sqrt{16+49-9} = \sqrt{56}$
$S_1^2 = 56$; PA –PB = 56

9. The sum of all the real values of x satisfying the equation $2^{(x-1)2(x2+5x-50)}=1$ is :
(1) 16 (2) –5 (3) –4 (4) 14

Sol. **3**
$(x - 5)(x^2 + 5x - 50) = 0$
$\Rightarrow (x - 5)(x + 10)(x - 5) = 0$
$\Rightarrow x = 1, 5, -10$ sum = – 4

10. The function $f : N \rightarrow N$ defined by $f(x) = x - 5\left[\frac{x}{5}\right]$, where N is the set of natural numbers and [x] denotes the greatest integer less than or equal to x, is :
(1) one-one but not onto (2) one-one and onto
(3) neither one-one nor onto (4) onto but not one-one

Sol. **3**
$$\left.\begin{array}{l} f(1) = 1 - 5(1/5) = 1 \\ f(6) = 6 - 6[6/5] = 1 \end{array}\right\} \rightarrow \text{Many one}$$
f(10) = 10 – 5(2) = 0 which is not in coduman
many one + into

11. Let f be a polynomial function such that f(3x)=f'(x). f"(x), for all x∈**R**.Then :
(1) f(2)+f'(2)=28 (2) f"(2)–f'(2) =0
(3) f(2)–f'(2)+f"(2)=10 (4) f"(2)–f(2)=4

Sol. **1**
$f(x) = ax^3 + bx^2 + cx + l$
$f(3x) = 27ax^3 + 9bx^2 + 3cx + d$
$f'(x) = 3ax^2 + 2bx + c$
$f''(x) = 6ax + 2b$
$f(3x) = f'(x) f''(x)$
$27a = 18a^2$
$a = \frac{3}{2}$, b = 0, c = 0, d = 0

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$f(x) = \frac{3}{2}x^3$

$f'(x) = \frac{9}{2}x^2$, $f''(x) = 9x$

12. If three positive numbers a, b and c are in A.P. such that abc=8, then the minimum possible value of b is :

(1) $4^{\frac{2}{3}}$ (2) 2 (3) $4^{\frac{1}{3}}$ (4) 4

Sol. **2**

a + c = 2b

$a.c.\left(\frac{a+c}{2}\right) = 8$

13. If $\lim_{n\to\infty} \frac{1^a + 2^a + \ldots n^a}{(n+1)^{a-1}[(na+1)+(na+2)+\ldots+(na+n)]} = \frac{1}{60}$ for some positive real number a, then a is equal to

(1) $\frac{17}{2}$ (2) $\frac{15}{2}$ (3) 7 (4) 8

Sol. **3**

$$\lim_{n\to\infty} \frac{\frac{1}{(a+1)}.n^{a+1} + a_1 n^a + a_2 n^{a-1} + \ldots..}{(n+1)^{a-1}.n^2\left(a + \frac{1+\frac{1}{n}}{2}\right)} = \frac{1}{60}$$

$$\Rightarrow \lim_{n\to\infty} \frac{\frac{1}{a+1} + \frac{a_1}{n} + \frac{a_2}{n^2} + \ldots.}{\left(1+\frac{1}{n}\right)^a \left(a + \frac{1+\frac{1}{n}}{2}\right)} = \frac{1}{60}$$

$$\Rightarrow \frac{\frac{1}{a+1}}{\left(a+\frac{1}{2}\right)} = \frac{1}{60} \Rightarrow (a+1)(2a+1) = 120$$

$2a^2 + 3a - 119 = 0$

$2a^2 + 17a - 14a - 119 = 0$

$\Rightarrow (a - 7)(2a + 17) = 0$

$a = 7, -\frac{17}{2}$

14. If $f\left(\frac{3x-4}{3x+4}\right) = x+2, x \neq -\frac{4}{3}$, and $\int f(x)dx = A\log|1-x| + Bx + C$, then the ordered pair (A,B) is equal to :(where c is a constant of integration)

(1) $\left(-\frac{8}{3}, -\frac{2}{3}\right)$ (2) $\left(-\frac{8}{3}, \frac{2}{3}\right)$ (3) $\left(\frac{8}{3}, \frac{2}{3}\right)$ (4) $\left(\frac{8}{3}, -\frac{2}{3}\right)$

Sol. **2**

$f\left(\frac{3x-4}{3x+4}\right) = x + 2,\ x \neq -\frac{4}{3}$

Let $\frac{3x-4}{3x+4} = t$

$3x - 4 = 3tx + 4t$

$x = \frac{4t+4}{3-3t} + 2$

$f(t) = \frac{10-2t}{3-3t}$

$f(x) = \frac{2x-10}{3x-3}$

$\int f(x)dx = \int \frac{2x-10}{3x-3}dx$

$= \int \frac{2x}{3x-3}dx - 10\int \frac{dx}{3x-3}$

$= \frac{2}{3}\int \frac{x-1}{x-1}dx + \frac{2}{3}\int \frac{dx}{x-1} - \frac{10}{3}\int \frac{dx}{x-1} = \frac{2x}{3} - \frac{8}{3}\ln(x-1) + C$

15. A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30^0 with the positive direction of the x-axis, then the sum of the x-coordinates of the vertices of the square is :

(1) $2\sqrt{3} - 2$ (2) $\sqrt{3} - 2$ (3) $2\sqrt{3} - 1$ (4) $\sqrt{3} - 1$

Sol. **1**

$\frac{x}{\cos 30°} = \frac{y}{\sin 30°} = 2$

$x = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$y = 1$

$\frac{x}{\cos 120°} = \frac{y}{\sin 120°} = 2$

$x = -1,\ y = \sqrt{3}$

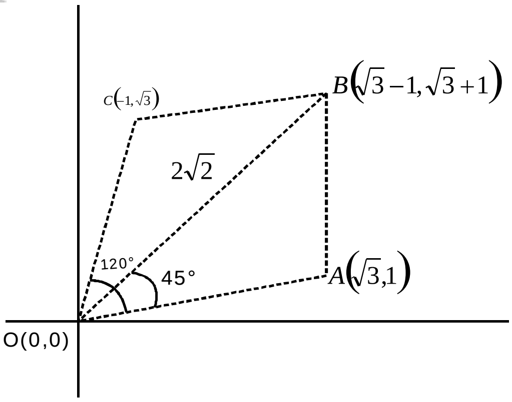
$\frac{x}{\cos 75°} = \frac{y}{\sin 75°} = 2\sqrt{2}$

$x = \sqrt{3} - 1$

$y = \sqrt{3} + 1$

sum $= 0 + \sqrt{3} + \sqrt{3} - 1 + (-1)$

$= 2\sqrt{3} - 2$

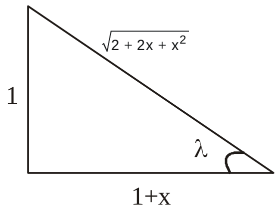


16. A value of x satisfying the equation $\sin[\cot^{-1}(1+x)] = \cos[\tan^{-1}x]$, is :

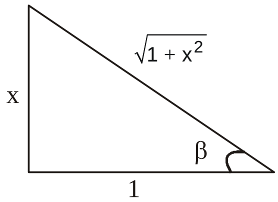
(1) $-\frac{1}{2}$ (2) 0 (3) -1 (4) $\frac{1}{2}$

Sol. **1**

$$\sin\left[\frac{\cot^{-1}(1+x)}{\lambda}\right] = \cos\left(\frac{\tan^{-1}x}{\beta}\right)$$



$\cot\lambda = 1 + x$



$$\frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{1\sqrt{1+x^2}}$$

$x^2 + 2x + 2 = x^2 + 1$

$\boxed{x = -1/2}$

17. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one women. Then the probability for these committees to have more women than men, is :

(1) $\frac{3}{11}$ (2) $\frac{2}{23}$ (3) $\frac{1}{11}$ (4) $\frac{21}{220}$

Sol. **3**

18. The function f defined by $f(x)=x^3-3x^2+5x+7$, is :

(1) decreasing in **R** (2) increasing in **R**

(3) increasing in $(0, \infty)$ and decreasing in $(-\infty,0)$

(4) decreasing in $(0, \infty)$ and increasing in $(-\infty,0)$

Sol. **2**

$f(x) = x^3 - 3x^2 + 5x + 7$

$f'(x) = 3x^2 - 6x + 5 > 0$

$f'(x) = 3x^2 - 6x + 5 < 0$

$x \in \phi$

19. The ecentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points (4, –1) and (–2,2) is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{1}{2}$

Sol. **1**

e = ?, centre at (0,0)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{16}{a^2} + \frac{1}{b^2} = 1$$

$16b^2 + a^2 = a^2b^2$ (1)

$$\frac{4}{a^2} + \frac{4}{b^2} = 1$$

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$4b^2 + 4a^2 = a^2b^2$ (2)

From (1) & (2)

$$16b^2 + a^2 = 4a^2 + 4b^2$$

$$3a^2 = 12b^2 = \boxed{a^2 = 4b^2}$$

20. A tangent to the curve, y = f(x) at P(x,y) meets x-axis at A and y-axis at B. If AP : BP = 1 : 3 and f(1)= 1, then the curve also passes through the point :

(1) $\left(\frac{1}{3}, 24\right)$ (2) $\left(\frac{1}{2}, 4\right)$ (3) $\left(2, \frac{1}{8}\right)$ (4) $\left(3, \frac{1}{28}\right)$

Sol. **3**

21. If x = a, y = b, z = c is a solution of the system of linear equations

$x + 8y + 7z = 0$

$9x + 2y + 3z = 0$

$y + y + z = 0$

such that the point (a,b,c) lies on the plane x + 2y + z = 6, then 2a+b+c equals :

(1) 2 (2) –1 (3) 1 (4) 0

Sol. **1**

$$\left.\begin{aligned} x + 8y + 7z &= 0 \\ 9x + 2y + 3z &= 0 \\ x + y + z &= 0 \end{aligned}\right\} \begin{aligned} &7y + 6z = 0 \\ \\ &7x + z = 0 \end{aligned}$$

$$x = \lambda \quad \left| y = \frac{-6(-7\lambda)}{7} \right| \quad z = -7\lambda$$

$$\boxed{x = \lambda} \quad \boxed{y = 6\lambda} \quad \boxed{z = -7\lambda}$$

$$\lambda + 12\lambda - 7\lambda = 6 \quad \Big| \quad 2\lambda + 6\lambda - 7\lambda$$

$$6\lambda = 6 \quad \Big| \quad = 2\lambda$$

$$\boxed{\lambda = 1} \quad \Big| \quad = \boxed{2}$$

22. Let $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + \ldots + \frac{1+2+\ldots,+n}{1^3+2^3+\ldots+n^3}$. If 100 S_n = n, then n is equal to :

(1) 200 (2) 199 (3) 99 (4) 19

Sol. **2**

$$T_n = \frac{\frac{n+(n+1)}{2}}{\left(\frac{n+(n+1)}{2}\right)^2}$$

$$T_n = \frac{2}{n(n+1)}$$

$$S_n = 2\sum_{n=1}^{n}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= 2\left\{\begin{matrix} 1-\frac{1}{2} \\ \frac{1}{2}-\frac{1}{3} \end{matrix}\right.$$

$$\left.\frac{1}{n}-\frac{1}{n+1}\right\}$$

$$= 2\left\{1-\frac{1}{n+1}\right\}$$

$$\boxed{S_n = \frac{2n}{n+1}}$$

$$100\times\frac{2n}{n+1} = n$$

$n + 1 = 200$

$n = 199$

23. If the vector $\vec{b} = 3\hat{j}+4\hat{k}$ is written as the sum of a vector $\vec{b}_1$, parallel to $\vec{a} = \hat{i}+\hat{j}$ and a vector $\vec{b}_2$, perpendicular to $\vec{a}$, then $\vec{b}_1\times\vec{b}_2$ is equal to :

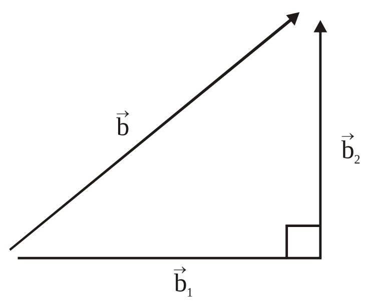
(1) $6\hat{i}-6\hat{j}+\frac{9}{2}\hat{k}$ (2) $-3\hat{i}+3\hat{j}-9\hat{k}$ (3) $-6\hat{i}+6\hat{j}-\frac{9}{2}\hat{k}$ (4) $3\hat{i}-3\hat{j}+9\hat{k}$

Sol. **1**

$$\vec{b}_1 = \frac{(\vec{b}.\vec{a})\hat{a}}{1}$$

$$= \left\{\frac{(3\hat{j}+4\hat{k}).(\hat{i}+\hat{j})}{\sqrt{2}}\right\}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$$

$$= \frac{3(\hat{i}+\hat{j})}{\sqrt{2}\times\sqrt{2}} = \frac{3(\hat{i}+\hat{j})}{2}$$



$\vec{b}_1+\vec{b}_2 = \vec{b}$

$\vec{b}_2 = \vec{b}-\vec{b}_1$

$$= \left(3\hat{i}+4\hat{k}\right)-\frac{3}{2}\left(\hat{i}+\hat{j}\right)$$

$$\boxed{\vec{b}_2 = -\frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}+4\hat{k}}$$

$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{3}{2} & 0 \\ -\frac{3}{2} & \frac{3}{2} & 4 \end{vmatrix}$$

$$\hat{i}(6)-\hat{j}(6)+\hat{k}\left(-\frac{9}{4}+\frac{9}{4}\right)\ ;\ 6\hat{i}-6\hat{j}+\frac{9}{2}\hat{k}$$

24. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60^0. If the area of the quadrilateral is $4\sqrt{3}$, then the perimeter of the quadrilateral is :

(1) 12.5 (2) 13 (3) 13.2 (4) 12

Sol. **4**

$\cos 60 = \frac{4+25-c^2}{2.2.5}$

$10 = 29 - c^2$

$c^2 = 19$

$\boxed{c = \sqrt{19}}$

$-\frac{1}{2} = \frac{a^2+b^2-19}{2ab}$

$a^2 + b^2 - 19 = -ab$

$a^2 + b^2 + ab = 19$

Area $= \frac{1}{2}\times 2\times 5\sin 60 + \frac{1}{2}ab\sin x = 4\sqrt{3}$

$\frac{5\sqrt{3}}{2} + \frac{ab\sqrt{3}}{4} = 4\sqrt{3}$

$\frac{ab}{4} = 4 - \frac{5}{2} = \frac{3}{2}$

$\boxed{ab = 6}$

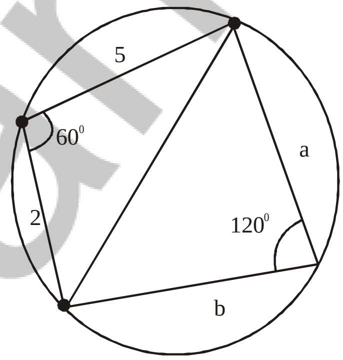
$a^2+b^2 = 13$

$\boxed{a = 2, b = 3}$

Perimeter

= 2 + 5 + 2+ 3

= 12



25. The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is :

(1) 7! (2) $5\times6!$ (3) $6\times6!$ (4) $5\times7!$

Sol. **2**

4 boy and 2 girls in circle

$5!\times {}^6c_2 \times 2!$

$5!\times \frac{6!}{4!2!}\times 2!$

$5\times 6!$

26. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A,B and C, then the locus of the centroid of ΔABC is .

(1) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$ (2) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$

(3) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$ (4) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$

Sol. **1**

Let Centroid be (h,k,l)

$\therefore$ x – intp = 3h   Y– intp = 3k, 3–int = 3l

Equ. $\frac{x}{3h}+\frac{y}{3k}+\frac{z}{3\ell}=1$

dist from (0,0,0)

$$\left|\frac{-1}{\sqrt{\frac{1}{9h^2}+\frac{1}{9k^2}+\frac{1}{9l^2}}}\right|=3$$

$$\boxed{\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1}$$

27. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3,4 and 5, were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is :

(1) 8.25   (2) 8.50   (3) 9.00   (4) 8.00

Sol. **3**

$\sum_{i=1}^{100} xi = 400$ $\sum_{i=1}^{100} xi^2 = 2475$

variance

$\boxed{\sigma^2=\frac{\sum xi^2}{N}-\left(\frac{\sum x_1}{N}\right)^2}$ $3^2+4^2+5^2$

$9+16+25=50$

$=\frac{2475}{97}-\left(\frac{388}{97}\right)^2$

$\frac{2425}{97}-16$

$\frac{2425-1552}{97}=\frac{873}{97}$

= (9)

28. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is :

(1) $\frac{4}{3}$ (2) $\frac{1}{3}$ (3) $\frac{3}{2}$ (4) $\frac{5}{12}$

Sol. **1**

$P(\bar{E}\cap\bar{F}) = P(\bar{E}).P(\bar{F}) = \frac{1}{12} = \boxed{xy=\frac{1}{12}}$

$P(\bar{E}\cap\bar{F}) = P(\bar{E}).P(\bar{F}) = \frac{1}{2}$

$\Rightarrow (1 - P(E))(1 - P(F) = \frac{1}{2}$

$= 1 - x - y + xy = \frac{1}{2}$

$1 - x - y + \frac{1}{12} = \frac{1}{2}$

$1 - x - y = \frac{1}{2} - \frac{1}{12} = \frac{5}{12}$

$\boxed{x + y = \frac{7}{12}}$

$x + \frac{1}{12x} = \frac{7}{12}$

$\frac{12x^2 + 1}{12x} = \frac{7}{12}$

$12x^2 - 7x + 1 = 0$

$12x^2 - 4x - 3x + 1 = 0$

$4x(3x-1) - 1(3x-1) = 0$

$x = \frac{1}{3}, \ x = \frac{1}{4}$

$y = \frac{1}{4}, \ y = \frac{1}{3}$

$\therefore \frac{x}{y} = \frac{1/3}{1/4} = \frac{4}{3}$ or $\frac{1/4}{1/3} = \frac{3}{4}$

29. If $2x = y^{\frac{1}{5}} + y^{-\frac{1}{5}}$ and $(x^2 - 1)\frac{d^2y}{dx^2} + \lambda x\frac{dy}{dx} + ky = 0$, then $\lambda + k$ is equal to :

(1) 26 (2) –24 (3) –23 (4) –26

Sol. **2**

$y^{1/5} + y^{-1/5} = 2x$

$\left(\frac{1}{5}y^{-4/5} - \frac{1}{5}y^{-6/5}\right)\frac{dy}{dx} = 2$

$y'(y^{1/5} - y^{-1/5}) = 10y$

$y'\left(2\sqrt{x^2 - 1}\right) = 10y$

$y''\left(2\sqrt{x^2 - 1}\right) + y'2\frac{2x}{2\sqrt{x^2 - 1}} = \sqrt{y'}$

$y''(x^2 - 1) + xy' = 5\sqrt{x^2 - 1}(y')$

$\boxed{y''(x^2 - 1) + xy' - 25y = 0}$

$\lambda = 1, \ k = -25$

30. Contrapositive of the statement
'If two numbers are not equal, then their squares are not equal', is :
(1) If the squares of two numbers are equal, then the numbers are not equal
(2) If the squares of two numbers are not equal, then the numbers are equal
(3) If the squares of two numbers are not equal, then the numbers are not equal
(4) If the squares of two numbers are equal, then the numbers are equal.

Sol. **4**

$\rightarrow q$

contrapositive is

$\sim q \rightarrow \sim p$

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