JEE Main - 2018 (CBT)
Exam Test Date: 15/04/2018
Test Time: 9:30 AM - 12:30 PM
Set - II

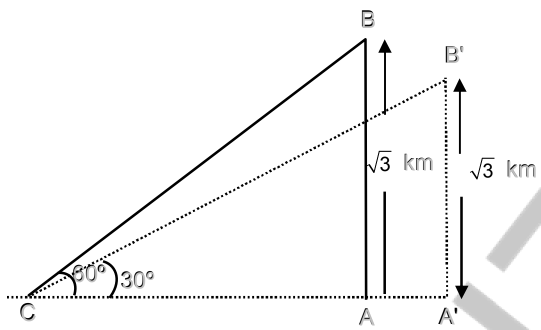
Part - C(Mathematics)

1. An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of 60º from a point on the ground. If, after five seconds , its elevation from the same point, is 30º, then the speed (in km/hr) of the aeroplane, is :

(1) 720 (2) 1500 (3) 750 (4) 1440

Ans. **(4)**

Sol.



Let from point C the angle of elevation of plane at B is 60º
and after 5 seconds it reach at B'

In ΔABC AC = $\sqrt{3}$ cot 60º = 1

In $\Delta CA'B'$ A'C = $\sqrt{3}$ cot 30º = 3

Hence distance AA' = 2km

$$\text{Speed} = \frac{\text{Distance}}{\text{time}} = \frac{2}{\frac{5}{60\times 60}} = \frac{2\times 60\times 60}{5} = \frac{7200}{5} = 1440 \text{ km/hr}$$

2. A box 'A' contains 2 white , 3 red and 2 black balls. Another box 'B' contains 4 white , 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly, selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is :

(1) $\frac{7}{8}$ (2) $\frac{9}{16}$ (3) $\frac{7}{16}$ (4) $\frac{9}{32}$

Sol. Probability that box A is selected P(A) = $\frac{1}{2}$

Probability that box B is selected P(B) = $\frac{1}{2}$

E be event that one ball is white while the other is red

P(E) =P(A) . P(E/A) + P(B) P(E/B)

$$= \frac{1}{2}\left[\frac{2.3}{^7C_2} + \frac{4.2}{^9C_2}\right] = \frac{1}{2}\left[\frac{6}{21} + \frac{8}{36}\right] = \frac{1}{2}\left[\frac{2}{7} + \frac{2}{9}\right] = \frac{16}{63}$$

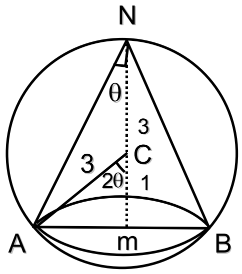
$$P(B/E) = \frac{P(B)P(E/B)}{P(E)} = \frac{1/9}{16/63} = \frac{7}{16}$$

3. If a right circular cone, having maximum volume , is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is :

(1) $8\sqrt{2}\pi$ (2) $6\sqrt{2}\pi$ (3) $8\sqrt{3}\pi$ (4) $6\sqrt{3}\pi$

Ans. **(3)**

Sol.



$V = \frac{1}{3}\pi r^2 h$

where r is radius and h is height of coin

$\Rightarrow V = \frac{1}{3}\pi(3\sin 2\theta)^2(3 + 3\cos 2\theta)$

$= 72\pi \sin^2\theta \cos^4\theta$

$\frac{dv}{d\theta} = 72\pi[2\sin\theta\cos^5\theta - 4\sin^3\theta\cos^3\theta] = 0 \Rightarrow \tan^2\theta = \frac{1}{2}$

V_{max} if $\tan\theta = \frac{1}{\sqrt{2}}$

Hence curved surface area $S = \pi r\ell$

$= \pi r\sqrt{(3+3\cos 2\theta)^2 + (3\sin 2\theta)^2}$

$= \pi(3\sin 2\theta)\sqrt{36\sin^2\theta} = 18\pi(2\sin\theta\cos^2\theta) = 36\pi.\frac{1}{\sqrt{3}}.\frac{2}{3} = \frac{24\pi}{3} = 8\sqrt{3}\pi$

4. If β is one of the angles between the normals to the ellipse $x^2 + 3y^2 = 9$ at the points $(3\cos\theta, \sqrt{3}\sin\theta)$ and $(-3\sin\theta, \sqrt{3}\cos\theta)$; $\theta \in \left(0, \frac{\pi}{2}\right)$; then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to :

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\sqrt{2}$

Ans. **(3)**

Sol. $\frac{x^2}{9} + \frac{y^2}{3} = 1$

Normal at $(3\cos\theta, \sqrt{3}\sin\theta)$ is

$3\sec\theta . x - \sqrt{3}\,\text{cosec}\theta\, y = 6$ ……..(i)

normal at $(-3\sin\theta, \sqrt{3}\cos\theta)$ is

$-3\text{cosec}\theta .x - \sqrt{3}\sec\theta\, y = 6$ ……..(ii)

Angle between normal is β

$\Rightarrow \tan\beta = \left|\frac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1-3}\right| = \left|-\frac{\sqrt{3}}{2\sin\theta\cos\theta}\right| \Rightarrow \tan\beta = \frac{\sqrt{3}}{\sin 2\theta} \Rightarrow \frac{2\cot\beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$

5. If $\left(f\,\dfrac{x-4}{x+2}\right) = 2x + 1,\ (x \in R - \{1, -2\})$, then $\int f(x)dx$ is equal to : (where C is a constant of integration)

(1) $12\log_e|1-x| - 3x + C$
(2) $-12\log_e|1-x| - 3x + C$
(3) $12\log_e|1-x| + 3x + C$
(4) $-12\log_e|1-x| + 3x + C$

Ans. **(2)**

Sol. $f\left(\dfrac{x-4}{x+2}\right) = 2x + 1$

$\Rightarrow f(x) = 2\left\{1 - 3\left(\dfrac{x+1}{x-1}\right)\right\} + 1$

$= 3 - \dfrac{6x+6}{x-1} = \dfrac{-3x-9}{x-1}$

$\Rightarrow f(x) = \dfrac{3(x+3)}{(1-x)}$

$\Rightarrow \int f(x)dx = 3\int\left(\dfrac{x+3}{1-x}\right)dx = 3\int\dfrac{4-(1-x)}{1-x}dx = 3\left\{\int\dfrac{4}{1-x}dx - \int dx\right\}$

$= 3\{-4\ell n|1-x - x| + C = -12\ell n|1-x| - 3x + C$

6. If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2-\lambda)x + (10-\lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is :

(1) $4\sqrt{2}$
(2) 20
(3) $2\sqrt{5}$
(4) $2\sqrt{7}$

Ans. **(3)**

Sol. $x^2 + (2-\lambda)x \times + (10-\lambda) = 0$

Let roots are α & β

$\Rightarrow \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= (\lambda-2)^3 - 3(10-\lambda)(\lambda-2)$

$= \lambda^3 - 6\lambda^2 + 12\lambda - 8 - 3(10\lambda - \lambda^2 - 20 + 2\lambda)$

$= \lambda^3 - 3\lambda^2 - 24\lambda + 52$

$\dfrac{dz}{d\lambda} = 3\lambda^2 - 6\lambda - 24$ where $= \alpha^3 + \beta^3 = 0$

$\Rightarrow \lambda^2 - 2\lambda - 8 = 0$

$\Rightarrow (\lambda-4)(\lambda+2) = 0$

$\Rightarrow \lambda = -2, 4$

$\dfrac{d^2z}{d\lambda^2} = 12\lambda - 6$

$\dfrac{d^2z}{d\lambda^2}(\lambda = -2) < 0 \Rightarrow \alpha^3 + \beta^3$ max if $\lambda = -2$

$\dfrac{d^2z}{d\lambda^2}(\lambda = 4) > 0 \Rightarrow \alpha^3 + \beta^3$ min. if $\lambda = 4$

$\Rightarrow$ Equation is $x^2 - 2x + 6 = 0$ $\left\langle \begin{array}{l} 1+\sqrt{5}\,i \\ 1-\sqrt{5}\,i \end{array}\right.$

$|\alpha - \beta| = 2\sqrt{5}$

7. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3 , then the equation of the common tangent to the two parabolas is :

(1) $3(x + y) + 4 = 0$ (2) $8 (2x + y) + 3 = 0$ (3) $x + 2y + 3 = 0$ (4) $4(x + y) + 3 = 0$

Ans. **(4)**

Sol. Equation two parabola are $y^2 = 3x$ and $x^2 = 3y$

Let equation of tangent to $y^2 = 3x$ is $y = mx + \dfrac{3}{4m}$

is also tangent to $x^2 = 3y$

$\Rightarrow x^2 = 3mx + \dfrac{9}{4m}$

$\Rightarrow 4mx^2 - 12m^2x - 9 = 0$ have equal roots

$\Rightarrow D = 0$

$\Rightarrow 144\, m^4 = 4\,(4m)\,(-9)$

$\Rightarrow m^4 + m = 0 \Rightarrow m = -1$

Hence common tangent is $y = -x - \dfrac{3}{4}$

$4(x + y) + 3 = 0$

8. If $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then

$\lim\limits_{x \to 0} \dfrac{f'(x)}{x}$

(1) does not exist (2) exists and is equal to – 2

(3) exists and is equal to 0 (4) exists and is equal to 2.

Ans. **(2)**

Sol. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

$= -x^2 \cos x + \tan x \,.\, x^2$

$= x^2 (\tan x - \cos x)$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{f'(x)}{x} = \lim\limits_{x \to 0} \dfrac{2x(\tan x - \cos x) + x^2(\sec^2 x + \sin x)}{x}$

$= \lim\limits_{x \to 0} 2\,(\tan x - \cos x) + x(\sec^2 x + \sin x)$

$= -2$

9. The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx$ is :

(1) $\frac{3}{4}$ (2) $\frac{3}{8}\pi$ (3) 0 (4) $\frac{3}{16}\pi$

Ans. **(2)**

Sol. $I = \int_{-\pi/2}^{\pi/2} \sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx$ ....(i)

Use proerties $\int_0^b f(x)dx = \int_a^b f(a+b-x)dx$

$= \int_{-\pi/2}^{\pi/2} \sin^4 x\left(1+\log\left(\frac{2-\sin x}{2+\sin x}\right)\right)dx$ ....(ii)

by (i) + (ii)

$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} 2\sin^4 x dx$

$\Rightarrow I = 2\int_{0}^{\pi/2} \sin^4 x dx =$

$= x. \frac{3.1}{4.2}.\frac{\pi}{2} = \frac{3\pi}{8}$

10. n-digit number are formed using only three digit 2,5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is :

(1) 9 (2) 7 (3) 8 (4) 6

Ans. **2**

Sol. n-digit number are formed using only three digits
2, 5 and 7 with repetition is = 3^n
$3^6 < 900$
$3^7 > 900$
so n = 7

11. If the tangents drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinates axes at the distinct points A and B, then the locus of the mid point of AB is :

(1) $4x^2 - y^2 + 16x^2y^2 = 0$ (2) $x^2 - 4y^2 + 16x^2y^2 = 0$
(3) $x^2 - 4y^2 - 16x^2y^2 = 0$ (4) $4x^2 - y^2 - 16x^2y^2 = 0$

Ans. **(3)**

Sol. Let tangent drawn at point (x, y) to the hyperbola $4y^2 = x^2 + 1$ is : $4yy_1 = xx_1 + 1$

This tangent intersect co-ordinate axes at A and B respectively then A $\left(-\frac{1}{x_1},0\right)$ and B $\left(0,\frac{1}{4y_1}\right)$

Let mid point is M (h,k) then of AB

$2h = -\frac{1}{x_1} \Rightarrow x_1 = -\frac{1}{2h}$ ....(i)

$2k = \frac{1}{4y_1} \Rightarrow y_1 = \frac{1}{8k}$ ....(ii)

Since point $P(x_1, y_1)$ lies on the hyperbola so
$4y_1^2 = x_1^2 + 1$
from (i) & (ii)

$$4\left(\frac{1}{8k}\right)^2 = \left(-\frac{1}{2h}\right)^2 + 1 \Rightarrow \quad \frac{1}{16k^2} = \frac{1}{4h^2} + 1$$

$4h^2 = 16k^2(1 + 4h^2)$
$x^2 = 4y^2 + 16x^2y^2$
$x^2 - 4y^2 - 16x^2y^2 = 0$
$x^2 - 4y^2 - 16x^2y^2 = 0$
locus of M

12. If tan A and tanB are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of $3\sin^2(A + B) - 10\sin(A + B).\cos(A + B) - 25\cos^2(A + B)$ is :

(1) – 25 (2) 10 (3) – 10 (4) 25

Ans. **(1)**

Sol. Since tanA and tanB are roots of the equation $3x^2 - 10x - 25 = 0$

so $\tan A + \tan B = \frac{10}{3}$

$\tan B.\tan B = -\frac{25}{3}$

$$\therefore \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B} = \frac{10/3}{1 + \frac{25}{3}} = \frac{10}{28} = \frac{5}{14}$$

$$= \text{so } \sin(A + B) = \frac{5}{\sqrt{221}} \text{ and } \cos(A + B) = \frac{14}{\sqrt{227}}$$

$\therefore 3\sin^2(A + B) - 10\sin(A + B)\cos(A + B) - 25\cos^2(A + B)$

$$= 3 \times \frac{25}{221} - \frac{10 \times 5 \times 14}{221} - 25 \times \frac{14^2}{221} = \frac{25}{221}(3 - 28 - 196) = -25$$

13. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$, where $f(x) = \begin{cases} 1 & , \quad x \in [0,1] \\ 0 & , \quad \text{otherwise} \end{cases}$

If y(0) = 0, then $y\left(\frac{3}{2}\right)$ is :

(1) $\frac{e^2 - 1}{e^3}$ (2) $\frac{1}{2e}$ (3) $\frac{e^2 + 1}{2e^4}$ (4) $\frac{e^2 - 1}{2e^3}$

Ans. **(4)**

Sol. $\frac{dy}{dx} + 2y = f(x)$ is a linear differential equation

$If = e^{\int 2dx} = e^{2x}$

solution of the above equation is

$y.e^{2x} = \int f(x).e^{2x}dx + C$

$y(x) = e^{-2x}\int_0^x f(x)e^{2x}dx + ce^{-2x}$

$y(0) = 0 \Rightarrow C = 0$

$\Rightarrow y(x) = e^{-2x}\int_0^x f(x)e^{2x}dx$

$$y(3/2) = e^{-3}\left[\int_0^1 e^{2x}dx + \int_1^{3/2} 0.dx\right] = \frac{e^{-3}}{2}[e^2 - 1] = \frac{e^2 - 1}{2e^3}$$

14. If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval :

(1) $[10, \infty)$ (2) $(-\infty, -10]$ (3) $(-10,0)$ (4) $(0, 10)$

Ans. **(4)**

Sol. If b is the first term and r is the common ratio of an infinite G.P. then sum is 5

$5 = \frac{b}{1-r}]$

$1 - r = \frac{b}{5}$

$r = 1 - \frac{b}{5}$

$r = \frac{5-b}{5}$

$\therefore -1 < r < 1$

$\therefore -1 < \frac{5-b}{5} < 1$

$-5 < 5 - b < 5$

$-5 < 5 - b < 5$

$-10 < -b < 0$

$0 < b < 10$

$b \in (0,10)$

15. Consider the following two binary relations on the set A = {a, b, c} :

R_1 = {(c,a), (b,b) , (a,c), (c,c), (b,c), (a,a)} and R_2 = {(a,b), (b,a), (c,c), (c,a), (a,a), (b,b), (a,c)}.

Then :

(1) R_2 is symmetric but it is not transitive

(2) both R_1 and R_2 are not symmetric

(3) both R_1 and R_2 are transitive.

(4) R_1 is not symmetric but it is transitive

Ans. **(1)**

Sol. $R_1 \in (b, c)$ but $R_1 \notin (c,b)$

Example R_1 is not symmetric

in R_1 ; $(b,c) \in R_1$ and $(c,a) \in R_1$ but $(b,a) \notin R_1$

So R_1 is not transitive

R_2 is symmetric

is R_2 ; $(b,a) \in R_2$ and $(a,c) \in R_2$ but $(b,c) \notin R_2$

So R_2 is not transitive

16. A circle passes through the points (2,3) and (4,5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to :

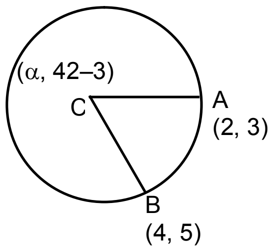
(1) $\sqrt{5}$ (2) $\sqrt{2}$ (3) 1 (4) 2

Ans. **(4)**

Sol. Let centre of circle is $c(\alpha.\beta)$

it lies is line $y - 4x + 3 = 0$ $B = 4\alpha - 3$

$\therefore c(\alpha, 4\alpha - 3)$



$\therefore$ CA = CB

$(\alpha-2)^2 + (4\alpha-6)^2 = (\alpha-4)^2 + (4\alpha-8)^2$

$-4\alpha + 4 - 48\alpha + 36 = -8\alpha + 16 - 64\alpha + 64$

$(64 + 8 - 4 - 48)\alpha = 80 - 40$

$\alpha = \frac{40}{20} = 2$

c(2,5)

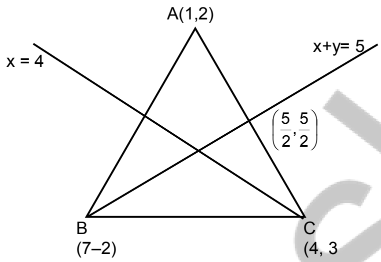
$\therefore r = 2$

17. In a triangle ABC, coordinates of A are (1,2) and the equations of the medians through B and C are respectively, $x + y = 5$ and $x = 4$. Then area of ΔABC (in sq. units) is :

(1) 12 (2) 4 (3) 9 (4) 5

Ans. **(3)**

Sol.



$\therefore$ Area of $\Delta = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & -2 & 1 \\ 4 & 3 & 1 \end{vmatrix} = \frac{1}{2}[18]$ **= 9 sq.unit.**

18. The set of all $\alpha \in R$, for which $w = \frac{1+(1-8\alpha)z}{1-z}$ is a purely imaginary number, for all $z \in C$ satisfying $|z| = 1$ and Re $z \neq 1$, is :

(1) {0} (2) $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$ (3) equal to R (4) an empty set

Ans. **(1)**

Sol. $\frac{1+(1-8\alpha)z}{1-z}+\frac{1+(1-8\alpha)\bar{z}}{1-\bar{z}}=0$

$1-\bar{z}+(1-8\alpha)z-(1-8\alpha)+1-z+(1-8\alpha)\bar{z}-(1-8\alpha)=0$

$2-(z+\bar{z})+(1-8\alpha)(z+\bar{z})-2+16\alpha=0$

$\Rightarrow$ $8\alpha(z+\bar{z})=16\alpha$

$\Rightarrow$ $z+\bar{z}=2$ or $\alpha=0$

For all $z\in C$ we have $\alpha=0$ $\Rightarrow$

19. If $x_1, x_2, \ldots\ldots, x_n$ and $\frac{1}{h_1},\frac{1}{h_2},\ldots\ldots,\frac{1}{h_n}$ are two A.P. such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 \cdot h_{10}$ equals :

(1) 3200 (2) 1600 (3) 2650 (4) 2560

Ans. **(4)**

$x_1, x_2, x_3, \ldots\ldots x_n$ in AP.

$x_3 = 8$ & $x_8 = 20$ $\Rightarrow x_5 = \frac{64}{5}$

$h_1, h_2, h_3, \ldots\ldots h_n$, in HP

$h_2 = 8$, $h_7 = 20$ $\Rightarrow$ $h_{10} = 200$

$\Rightarrow x_5 h_{10} = \frac{64}{5}\times 200 = \frac{12800}{5} = 2560$

20. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+2\vec{b}+2\vec{c}=0$, then $|\vec{a}\times\vec{c}|$ is equal to :

(1) $\frac{1}{4}$ (2) $\frac{15}{16}$ (3) $\frac{\sqrt{15}}{4}$ (4) $\frac{\sqrt{15}}{16}$

Ans. **(3)**

Sol. $\vec{a}+2\vec{b}+2\vec{c}=\vec{o}$

$\vec{a}+2\vec{c}=-2\vec{b}$

$|\vec{a}|^2+4|\vec{c}|^2+4\vec{a}.\vec{c}=4|\vec{b}|^2$

$1+4+4\cos\theta=-\frac{1}{4}$ $\therefore$ $\sin\theta=\frac{\sqrt{1-\cos^2\theta}}{\sqrt{1-\frac{1}{16}}}=\frac{\sqrt{15}}{4}$

Now $|\vec{a}\times\vec{c}| = |\vec{a}|\ |\vec{c}|\ \sin\theta = \frac{\sqrt{15}}{4}$

21. A variable plane passes through a fixed point (3,2,1) and meets x, y and z axes at A, B and C respectively . A plane is drawn parallel to yz - plane through A, a second plane is drawn parallel zx-plane through B and a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is :

(1) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$ (2) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$ (3) $x+y+z=6$ (4) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$

Ans. **(1)**

Sol. Let plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

it passes through (3,2,1) $\therefore$ $\frac{3}{a}+\frac{2}{b}+\frac{1}{c}=1$

Now A (a,0,0) , B (0, b, 0) , C (0,0,c)

$\therefore$ Locus of point of intersection of planes x = a

y = b, z = c is $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

22. Let A be a matrix such that $A.\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ is a scalar matrix and |3A| = 108. Then A^2 equals :

(1) $\begin{bmatrix}4 & 0\\-32 & 36\end{bmatrix}$ (2) $\begin{bmatrix}36 & -32\\0 & 4\end{bmatrix}$ (3) $\begin{bmatrix}36 & 0\\-32 & -32\end{bmatrix}$ (4) $\begin{bmatrix}4 & -32\\0 & 36\end{bmatrix}$

Ans. **(2)**

Sol. $A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$

Now $A.\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix} = \begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$

$= \begin{bmatrix}a & 2a+3b\\c & 2c+3d\end{bmatrix}$ is scalar

$\therefore$ c = 0, 2a + 3b = 0, a = 2c + 3d a = 3d $\therefore$ $a^2 = 9d^2 = 36$

|3A| = 108

$\therefore$ |A| = 12 = ad – bc = ad

$\therefore$ $d^2 = 4$

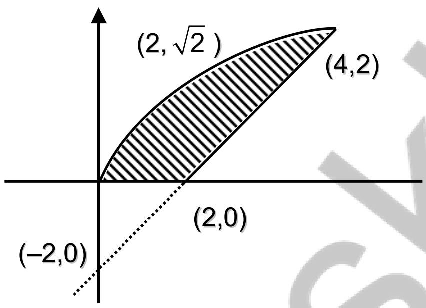
Now $A^2 = \begin{bmatrix}a & b\\0 & d\end{bmatrix}\begin{bmatrix}a & b\\0 & d\end{bmatrix} = \begin{bmatrix}a^2 & ab+bd\\0 & d^2\end{bmatrix}$

23. The area (in sq. units) of the region $\{x \in R : x \geq 0, y \geq 0, y \geq x - 2 \text{ and } y \leq \sqrt{x}\}$, is :

(A) $\frac{13}{3}$ (2) $\frac{8}{3}$ (3) $\frac{10}{3}$ (4) $\frac{5}{3}$

Ans. **(3)**

Sol.



$$\int_0^2 \sqrt{x}dx + \int_2^4 (\sqrt{x} - x + 2)dx$$

$$\int_0^4 \sqrt{x}dx + \int_2^4 (2 - x)dx$$

$$\left(\frac{2x^{\frac{3}{2}}}{3}\right)_0^4 + \left(2x - \frac{x^2}{2}\right)_2^4$$

$$= \frac{2}{3}(8) + -(4-2) = \frac{16}{3} - 2 = \frac{10}{3}$$

24. If $x^2 + y^2 + \sin y = 4$, then the value of $\frac{d^2y}{dx^2}$ at the point (–2, 0) is :

(1) – 34 (2) 4 (3) – 2 (4) – 32

Ans. **(1)**

Sol. $x^2 + y^2 + \sin y = 4 \Rightarrow \quad 2x + (2y + \cos y)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{2y + \cos y}$ at (–2, 0) $\Rightarrow \frac{dy}{dx} = \frac{4}{1} = 4$

and $(2y + \cos y)\frac{dy}{dx} + 2x = 0$

$(2y+\cos y)\frac{d^2y}{dx^2} + (2 - \sin y)\left(\frac{dy}{dx}\right)^2 + 2 = 0$

$\frac{d^2y}{dx^2} + (2-0)(4)^2 + 2 = 0$

$\Rightarrow \quad \frac{d^2y}{dx^2} = -34$

25. An angle between the plane, $x + y + z = 5$ and the line of intersection of the planes, $3x + 4y + z - 1 = 0$ and $5x + 8y + 2z + 14 = 0$, is :

(1) $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$ (2) $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (3) $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (4) $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

Ans. **(4)**

Sol. $\begin{vmatrix} i & j & k \\ 3 & 4 & 1 \\ 5 & 8 & 2 \end{vmatrix}$

$i(0) - j(6-5) + k(24-20) = -\hat{j} + 4\hat{k}$

Angle = $\frac{\pi}{2} - \cos^{-1}\left(\frac{-1+4}{\sqrt{3}\sqrt{17}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{\sqrt{3}}{17}\right) = \sin^{-1}\sqrt{\frac{3}{17}}$

26. Let S = $\{(\lambda, \mu) \in R \times R : f(t) = (|\lambda|e^{|t|} - \mu) . \sin(2|t|), t \in R$, is a differentiable function$\}$. Then S is a subset of :

(1) $(-\infty, 0) \times R$ (2) $R \times [0, \infty)$ (3) $[0, \infty) \times R$ (4) $R \times (-\infty,0)$

Ans. **(2)**

Sol. Let $s = \{(\lambda, \mu) \in R\times R\}$

$f(t) = \{|\lambda| e^{|t|} - \mu) \sin 2|t|$ → $0^+ = 0$, $0^- = 0$

RHD = $\lim_{h\to 0} \frac{f(0+h) - 0}{h} = \lim_{h\to 0}(|\lambda| e^h - \mu)\frac{\sin^2 h}{h} = 2(|\lambda| e^h - \mu) = 0$

LHD = $\lim_{h\to 0} \frac{f(0-h) - 0}{-h} = \lim_{h\to 0}(\lambda^{e+h} - \mu)\frac{\sin^2 h}{-h} = -2(|\lambda| e^h - \mu)$

$|\lambda|e^h = \mu$

$|\lambda| = \mu$

$\Rightarrow \mu \geq 0 \ \& \ \lambda \in R$

27. Let S be the set of all real values of k for which the system of linear equations
x + y + z = 2
2x + y – 2 = 3
3x + 2y + kz = 4
Has a unique solution. Then S is :
(1) equal to R – {0} (2) an empty set (3) equal to R (4) equal to {0}

Ans. **(4)**

Sol. $\Delta \neq 0$ for $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$

1(k+2) –1(2k+3) + 1(4–3) = 0 ⇒ k+2–2k – 3 + 1 = 0 ⇒ –k = 0 ⇒ k = 0

28. If n is the degree of the polynomial, $\left[\frac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$ and m is the coefficient of x^n in it, then the ordered pair (n,m) is equal to :
(1) $(8,5(10)^4)$ (2) $(12, 8(10)^4)$ (3) $(12,(20)^4)$ (4) $(24, (10)^8)$

Ans. **(3)**

Sol. $\left[\frac{2\left(\sqrt{5x^3}\quad 1\quad \sqrt{5x^3}\quad 1\right)}{2}\right]^8 + \left[\frac{2\left(\sqrt{5x^3+1}-\sqrt{5x^3-1}\right)}{2}\right]^8$

$= 2\left[{}^8C_0\left(\sqrt{5x^3+1}\right)^8 + {}^8C_2\left(\sqrt{5x^3+1}\right)^6(5x^3-1) + {}^8C_4\left(\sqrt{5x^3+1}\right)^4(5x^3-1)^2 + {}^8C_6\left(\sqrt{5x^3+1}\right)^2(5x^3-1)^3 + {}^8C_8(5x^3-1)^4\right]$

$= 2\left[(5x^3-1)^4 + 28(5x^3+1)^3(5x^3-1) + 70(5x^3+1)^2(5x^3-1)^2 + 28(5x^3+1)(5x^3-1)^3 + (5x^3-1)^4\right]$

h = 12 & m = $2(5^4 + 140.5^3 + 70.5^4 + 140.5^3 + 5^4)$
= 160000 = $(20)^4$

29. The mean of a set of 30 observations is 75. If each observations is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. Then λ is equal to :
(1) $\frac{4}{3}$ (2) $\frac{1}{3}$ (3) $\frac{10}{3}$ (4) $\frac{2}{3}$

Ans. **(1)**

Sol. $x_1 + x_2 + \ldots\ldots + x_{30} = 75 \times 30$
Now given $\lambda(x_1 + x_2 + x_3 + \ldots\ldots + x_{30}) - 25 \times 30 = 75 \times 30$
$\lambda(75 \times 30) = 100 \times 30$
$\lambda = \frac{4}{3}$

30. If $(p \wedge \sim q) \wedge (p \wedge r) \to \sim p \vee q$ is false, then the truth values of p, q and r are, respectively :
(1) T,T,T (2) F,T,F (3) T,F,T (4) F,F,F

Ans. **(3)**

Sol. $(p \wedge \sim q) \wedge (p \wedge \sim r) \to \sim p \vee q$

(1) $(T \wedge F) \wedge (T \wedge T) \to (F \vee T)$
$\equiv (F \wedge T) \to T$
$\equiv F \to T \equiv T$

(2) $(F \wedge F) \wedge (F \wedge F) \to T \vee T$
$F \to T \equiv T$

(3) $(T \wedge T) \wedge (T \wedge T) \to F \vee F$
$T \to F \equiv F$

(4) $(F \wedge T) \wedge (F \wedge F) \to T \vee F$
$F \wedge F \to T$
$F \to T \equiv T$