Indian National Physics Olympiad – 2011

Roll Number: | P | 1 | 1 | | | |

INPhO – 2011 Date: 30^{th} January 2011

Duration: **Three Hours** Maximum Marks: **65**

Please fill in all the data below correctly. The contact details provided here would be used for all further correspondence.

Full Name (BLOCK letters) Ms. / Mr.: ______________________________

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Male / Female Date of Birth (dd/mm/yyyy): ______________

Name of the school / junior college: ______________________________

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Class: XI/ XII Board: ICSE / CBSE / State Board / Other

Address for correspondence (include city and PIN code): ______________

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______________________________PIN Code: | | | | | | |

Telephone (with area code): ______________Mobile: ______________

Email address: ______________________________

Besides the International Physics Olympiad, do you also want to be considered for the Asian Physics Olympiad? The APhO - 2011 will be held from April 30- May 07 and your presence will be required in Delhi/Israel from April 22 to May 08, 2011.

Yes/No.

I have read the procedural rules for INO and agree to abide by them.

Signature

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(Do not write below this line)

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MARKS

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Total**
Marks								

HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd, Mumbai, 400 088

Instructions:

1. Write your Roll Number on every page of this booklet.
2. Fill out the attached performance card. **Do not detach it from this booklet.**
3. Booklet consists of 26 pages (excluding this sheet) and seven (7) questions.
4. Questions consist of sub-questions. Write your **detailed answer** in the **blank space** provided below the sub-question and **final answer** to the sub-question in the **smaller box** which follows the blank space.
5. Extra sheets are also attached at the end in case you need more space. You may also use these extra sheets for rough work.
6. Computational tools such as calculators, mobiles, pagers, smart watches, slide rules, log tables etc. are **not** allowed.
7. **This entire booklet must be returned.**

Table of Information

Speed of light in vacuum	c	=	3.00×10^{8} m·s^{-1}
Planck's constant	h	=	6.63×10^{-34} J·s
Universal constant of Gravitation	G	=	6.67×10^{-11} N·m^{2}·kg^{-2}
Magnitude of the electron charge	e	=	1.60×10^{-19} C
Mass of the electron	m_e	=	9.11×10^{-31} kg
Stefan-Boltzmann constant	σ	=	5.67×10^{-8} W·m^{-2} · K^{-4}
Permittivity constant	ϵ_0	=	8.85×10^{-12} F·m^{-1}
Permeability constant	μ_0	=	$4\pi \times 10^{-7}$ H·m^{-1}
Acceleration due to gravity	g	=	9.81 m·s^{-2}
Universal Gas Constant	R	=	8.31 J· K^{-1}·mole^{-1}

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1. A long wire of radius 'a' is carrying a direct current I. From its surface at point A, an electron of charge $-e$ $(e > 0)$ escapes with velocity v_0 perpendicular to this surface (see Fig.(1)). Ignore gravity.

[2.5 + 4 + 1.5 = 8]

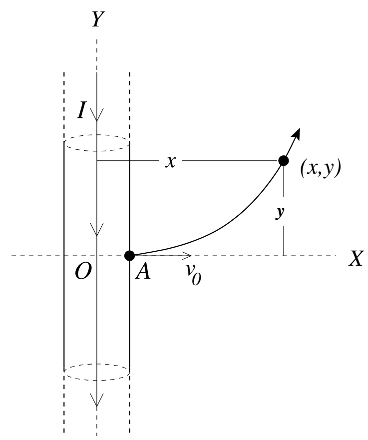

Figure 1:

(a) At x and y the components of the velocity are v_x and v_y respectively. Obtain the components of force F_x and F_y on the electron at any arbitrary point $\{x, y\}$.

$F_x =$
$F_y =$

(b) Integrate the equation of motion to obtain v_x.

$v_x =$

(c) Find the maximum distance x_{max} of electron from the axis of the wire before it turns back.

$x_{max} =$

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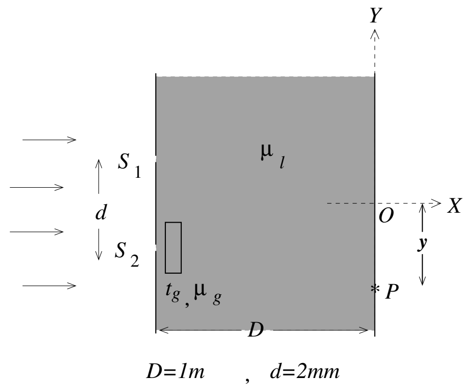
2. In a modified Young's double slit experiment the region between screen and slits is immersed in a liquid whose refractive index varies with time t (in seconds) as $\mu_l = 2.50 - 0.25t$ until it reaches a steady state value 1.25. The distance between the slits and the screen is $D = 1.00$ m and the distance between the slits S_1 and S_2 is $d = 2.00 \times 10^{-3}$ m. A glass plate of thickness $t_g = 3.60 \times 10^{-5}$ m and refractive index $\mu_g = 1.50$ is introduced in front of one of the slits. Note that the illuminations at S_1 and S_2 are from coherent sources with zero phase difference.
[2.5 + 2.5 + 1 + 2 + 2 = 10]



(a) Consider the point P on the screen at distance y from O ($S_1O = S_2O$; $OP = y$). Obtain the expression for the optical path difference Δx in terms of the refractive indices and the lengths mentioned in the problem.

$\Delta x =$

(b) Now let P denote the central maximum. Obtain the expression for y as a function of time.

$y =$

(c) Obtain the time (t_m) when central maximum is at point O, equidistant from S_1 and S_2 i. e. $S_1O = S_2O$.

$t_m =$

(d) What is the speed (v) of the central maxima when it is at O.

$v =$

(e) If monochromatic light of wavelength 6000 Å is used to illuminate the slits, determine the time interval (Δt) between two consecutive maxima at O before steady state is reached.

$\Delta t =$

3. A Carnot engine cycle is shown in the Fig. (2). The cycle runs between temperatures $T_H = \alpha T_0$ and $T_L = T_0$ $(\alpha > 1)$. Minimum and maximum volume at state 1 and state 3 are V_0 and nV_0 respectively. The cycle uses one mole of an ideal gas with $C_P/C_V = \gamma$. Here C_P and C_V are the specific heats at constant pressure and volume respectively. You must express all answers in terms of the given parameters $\{\alpha, n, T_0, V_0, \gamma\}$ and universal gas constant R.

[3 + 4 + 1 = 8]

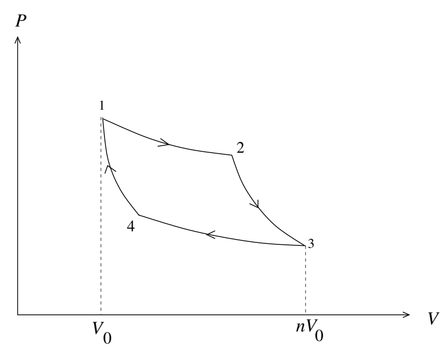


Figure 2:

(a) List $\{P, V, T\}$ of all the four states.

$P_1 =$	$P_2 =$	$P_3 =$	$P_4 =$
$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$
$T_1 =$	$T_2 =$	$T_3 =$	$T_4 =$

(b) Calculate the work done by the engine in each process: W_{12}, W_{23}, W_{34}, W_{41}.

$W_{12} =$
$W_{23} =$
$W_{34} =$
$W_{41} =$

(c) Calculate Q, the heat absorbed in the cycle.

$Q =$

4. Consider a modification of Coulomb's law by replacing it with the force between two charges q_1, q_2 separated by $\vec{r}$ given by

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r^2} + \frac{\beta}{r^3}\right]\hat{r}$$

where β is a constant. As far as possible express your answers in terms of the standard Bohr radius $a_o = 4\pi\epsilon_0\hbar^2/me^2$ where the symbols have their usual meanings.

[3 + 4 + 1 = 8]

(a) Obtain the Bohr radius (r_n) for this modified law.

$r_n =$

(b) Obtain the expression for the energy (E_n) for the n^{th} orbit of this modified law.

$E_n =$

(c) Take β to be small ($\beta = 0.1a_0$). Take the binding energy of the standard Bohr hydrogen atom to be 13.60 eV. Calculate the transition energy (ΔE) from $n = 2$ to $n = 1$ for this modified law. For your calculation you may ignore terms of order β^2 and higher.

$\Delta E =$

5. Consider the motion of electrons in a metal in the presence of electric ($\vec{E}$) and magnetic ($\vec{B}$) fields. Due to collisions there arises a "retarding" force on the electron which is modeled by $m\vec{v}/\tau$ where m is the electron mass, $\vec{v}$ its velocity and τ a typical collision time. Take the magnitude of the electron charge to be e (note e is positive). Ignore gravity.

[1 + 2 + 2.5 + 3 + 2 + 1.5 = 12]

(a) State the equation of motion of the electron.

(b) Consider the case $\vec{E} = 0$ and $\tau \to \infty$. Obtain the expression for the angular cyclotron frequency ω_c and its numerical value for the case $B = 5.70$ T.

$\omega_c =$
Value of $\omega_c =$

(c) Consider the case of $\vec{B} = 0$ and $\vec{E} = E\hat{i}$. If n is the number of free (valence) electrons per unit volume, obtain the expression for the conductivity σ_0 of the sample. Obtain also the numerical value for the conductivity of Cu given that $n = 8.45 \times 10^{28}$ m^{-3} and $\tau = 2.48 \times 10^{-14}$ s. We assume steady state i.e. the acceleration dies down and terminal speed is attained.

$\sigma_0 =$

Value of $\sigma_0 =$

(d) Consider the case $\vec{E} = E_y\hat{j} + E_z\hat{k}$ ($E_x = 0$) and $\vec{B} = B\hat{k}$. Assume steady state and relate $\{j_x, j_y, j_z\}$ to $\{E_x E_y, E_z\}$. Here j's are the current densities (current per unit area).

$$\begin{aligned} j_x &= \sigma_{xy}E_y + \sigma_{xz}E_z \\ j_y &= \sigma_{yy}E_y + \sigma_{yz}E_z \\ j_z &= \sigma_{zy}E_y + \sigma_{zz}E_z \end{aligned}$$

where σ_{ij}'s are to be written in terms of σ_0, ω_c and τ.

$\sigma_{xy} =$	$\sigma_{xz} =$
$\sigma_{yy} =$	$\sigma_{yz} =$
$\sigma_{zy} =$	$\sigma_{zz} =$

(e) Sketch j_x (y-axis) versus B (x-axis) .

(f) Taking Cu as an example, for what value of the magnetic field will j_x be a maximum?

$B =$

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6. Two blocks, say B and C, each of mass m are connected by a light spring of force constant k and natural length L. The whole system is resting on a frictionless table such that $x_B = 0$ and $x_C = L$, where x_B and x_C are the coordinates of the blocks B and C respectively. Another block (named A) of mass M, which is travelling at speed V_0 collides head-on with the block B at an instant $t = 0$ (see Fig. (3)).

[3 + 2 + 2 + 3 = 10]

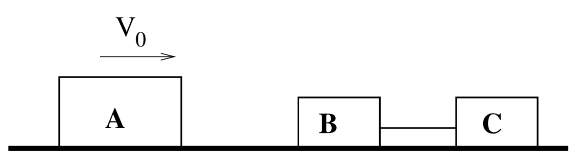

Figure 3:

(a) Obtain the velocities of blocks A, B and C just after the collision at $t = 0$? Express the velocities in terms of V_0 and $\gamma = m/M$. Assume that the collision is elastic.

$V_A =$
$V_B =$
$V_C =$

(b) Draw free body diagrams for the blocks B and C after the collison and write down the equation of motion.

Free body diagram:

Equation of motion:

(c) For $t > 0$ the positions of the blocks are given by

$$x_B = \alpha t + \beta \sin(\omega t) \quad (1)$$

$$x_C = L + \alpha t - \beta \sin(\omega t) \quad (2)$$

Find ω in terms of m and k. Express α and β in terms of V_0, γ and ω. (For this part, ignore the motion of block A.)

$\omega =$
$\alpha =$
$\beta =$

(d) Obtain the condition on γ such that the block A will collide with the block B again at some time $t > 0$?

Condition on γ:

7. **The Cubic Potential**: Consider a particle of mass m moving in one dimension under the influence of potential energy

$$u(x) = \frac{m\omega^2 x^2}{2} - \delta x - \frac{\alpha x^3}{3}$$

Here ω, δ and α are real and positive.

[6 + 3 = 9]

(a) Sketch typical plots of $u(x)$ and identify extrema if any.