

# Indian National Physics Olympiad – 2012

Roll Number: 

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INPhO – 2012

Date: 29<sup>th</sup> January 2012

Duration: **Three Hours**

Maximum Marks: **60**

Please fill in all the data below correctly. The contact details provided here would be used for all further correspondence.

Full Name (BLOCK letters) Ms. / Mr.: \_\_\_\_\_

Male / Female                      Date of Birth (dd/mm/yyyy): \_\_\_\_\_

Name of the school / junior college: \_\_\_\_\_

Class: XI/ XII    Board: ICSE / CBSE / State Board / Other

Address for correspondence (include city and PIN code): \_\_\_\_\_

\_\_\_\_\_ PIN Code: 

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Telephone (with area code): \_\_\_\_\_ Mobile: \_\_\_\_\_

Email address: \_\_\_\_\_

Besides the International Physics Olympiad, do you also want to be considered for the Asian Physics Olympiad? The APhO - 2012 will be held from April 30- May 07 and your presence will be required from April 20 to May 07. The IPhO selection camp will be held after May 07 and in principle you can participate in both olympiads.

Yes/No.

*I have read the procedural rules for INPhO and agree to abide by them.*

Signature

***(Do not write below this line)***

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## MARKS

| Que.  | 1 | 2 | 3 | 4 | 5 | Total |
|-------|---|---|---|---|---|-------|
| Marks |   |   |   |   |   |       |

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HOMI BHABHA CENTRE FOR SCIENCE EDUCATION

Tata Institute of Fundamental Research

V. N. Purav Marg, Mankhurd, Mumbai, 400 088

**Instructions:**

1. Write your Roll Number on every page of this booklet.
2. Fill out the attached performance card. **Do not detach it from this booklet.**
3. Booklet consists of 20 pages (excluding this sheet) and five (5) questions.
4. Questions consist of sub-questions. Write your **detailed answer** in the **blank space** provided below the sub-question and **final answer** to the sub-question in the **smaller box** which follows the blank space.
5. Extra sheets are also attached at the end in case you need more space. You may also use these extra sheets for rough work.
6. Computational tools such as calculators, mobiles, pagers, smart watches, slide rules, log tables etc. are **not** allowed.
7. **This entire booklet must be returned.**

**Table of Information**

|                                   |  |
|-----------------------------------|--|
| Speed of light in vacuum          | $c = 3.00 \times 10^8 \text{ m}\cdot\text{s}^{-1}$                             |
| Planck's constant                 | $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$                              |
| Universal constant of Gravitation | $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2\cdot\text{kg}^{-2}$         |
| Magnitude of the electron charge  | $e = 1.60 \times 10^{-19} \text{ C}$   |
| Mass of the electron              | $m_e = 9.11 \times 10^{-31} \text{ kg}$  |
| Stefan-Boltzmann constant         | $\sigma = 5.67 \times 10^{-8} \text{ W}\cdot\text{m}^{-2} \cdot \text{K}^{-4}$ |
| Permittivity constant             | $\epsilon_0 = 8.85 \times 10^{-12} \text{ F}\cdot\text{m}^{-1}$                |
| Value of $1/4\pi\epsilon_0$       | $= 9.00 \times 10^9 \text{ N}\cdot\text{m}^2\cdot\text{C}^{-2}$                |
| Permeability constant             | $\mu_0 = 4\pi \times 10^{-7} \text{ H}\cdot\text{m}^{-1}$                      |
| Acceleration due to gravity       | $g = 9.81 \text{ m}\cdot\text{s}^{-2}$   |
| Universal Gas Constant            | $R = 8.31 \text{ J}\cdot\text{K}^{-1}\cdot\text{mole}^{-1}$                    |
| Molar mass of air                 | $= 29.0 \text{ kg}\cdot\text{kmol}^{-1}$                                       |

1. Figure (1) shows a mechanical system free of any dissipation. The two spheres ( $A$  and  $B$ ) are each of equal mass  $m$ , and a uniform connecting rod  $AB$  of length  $2r$  has mass  $4m$ . The collar is massless. Right above the position of sphere  $A$  in Fig. (1) is a tunnel

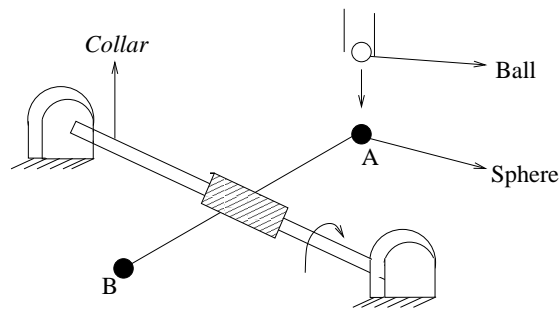


Figure 1:

from which balls each of mass  $m$  fall vertically at suitable intervals. The falling balls cause the rods and attached spheres to rotate. Sphere  $B$  when it reaches the position now occupied by sphere  $A$ , suffers a collision from another falling ball and so on. Just before striking, the falling ball has velocity  $v$ . All collisions are elastic and the spheres as well as the falling balls can be considered to be point masses. [Marks: 12]

- (a) Find the angular velocity  $\omega_{i+1}$  of the assembly in terms of  $\{\omega_i, v, \text{ and } r\}$  after the  $i^{\text{th}}$  ball has struck it. [4]

$$\omega_{i+1} =$$

- (b) The rotating assembly eventually assumes constant angular speed  $\omega^*$ . Obtain  $\omega^*$  in terms of  $v$  and  $r$  by solving the equation obtained in part (a). Argue how a constant  $\omega^*$  does not violate energy conservation.

[2]

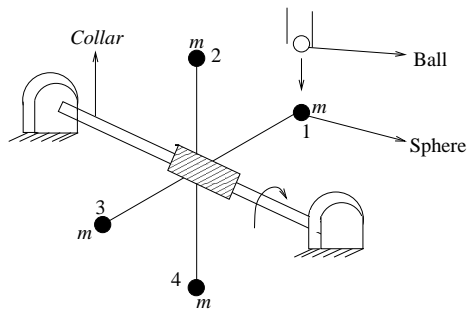
$$\omega^* =$$

Argument:

- (c) Solve the expression obtained in part (a) to obtain  $\omega_i$  in terms of  $\{i, v, \text{ and } r\}$ . [4]

$\omega_i =$

- (d) If instead of a pair of spheres, we have two pairs of spheres as shown in figure below. What would be the new constant angular speed  $\omega^*$  of the assembly (i.e. the answer corresponding to part (b)). [2]



$\omega^* =$

## 2. Rear view mirrors

Rear view mirrors in automobiles are generally convex. Suppose a car  $A$  moves with a constant speed of 40.0 kilometre per hour on a straight level road and is followed by another car  $B$  moving with the constant speed 60.0 kilometre per hour. At a given instant of time, we denote:

$x$  : distance of the car  $B$  from the mirror of car  $A$ ,

$y$  : distance of the car  $B$  from  $A$  as seen by the driver of  $A$  in the mirror,

$v_x$  : speed of approach of  $B$  relative to  $A$  and

$v_y$  : speed of approach of  $B$  as seen in the mirror of  $A$ .

[Marks: 6]

- (a) Obtain an expression for  $v_y$  in terms of  $x$ ,  $v_x$  and the radius of curvature  $R$  of the convex mirror used as the rear view mirror in the car  $A$ .

[3]

$v_y =$

- (b) Show a plot of
- $|v_y/v_x|$
- against
- $x$
- .

[2]

|   |
|---|
| $\left  \frac{v_y}{v_x} \right $ vs $x$ : |
|---|

- (c) If
- $R = 2.0$
- m, what is the speed of approach of
- $B$
- in kilometre per hour as seen by the driver of
- $A$
- in the mirror for
- $x = 2.0$
- m.

[1]

|         |
|---------|
| Speed = |
|---------|

### 3. Cloud formation condition

Consider a simplified model of cloud formation. Hot air in contact with the earth's surface contains water vapor. This air rises convectively till the water vapor content reaches its saturation pressure. When this happens, the water vapor starts condensing and droplets are formed. We shall estimate the height at which this happens. We assume that the atmosphere consists of the diatomic gases oxygen and nitrogen in the mass proportion 21:79 respectively. We further assume that the atmosphere is an ideal gas,  $g$  the acceleration due to gravity is constant and air processes are adiabatic. Under these assumptions one can show that the pressure is given by

$$p = p_0 \left( \frac{T_0 - \Gamma z}{T_0} \right)^\alpha \quad (1)$$

Here  $p_0$  and  $T_0$  is the pressure and temperature respectively at sea level ( $z = 0$ ),  $\Gamma$  is the lapse rate (magnitude of the change in temperature  $T$  with height  $z$  above the earth's surface, i.e.  $\Gamma > 0$ ). [Marks: 14]

- (a) Obtain an expression for the lapse rate  $\Gamma$  in terms of  $\gamma$ ,  $R$ ,  $g$  and  $m_a$ . Here  $\gamma$  is the ratio of specific heat at constant pressure to specific heat at constant volume;  $R$ , the gas constant; and  $m_a$ , the relevant molar mass. [4]

|            |
|------------|
| $\Gamma =$ |
|------------|



- (b) Estimate the change in temperature when we ascend a height of one kilometer? [1]

$\Gamma =$

- (c) Show that pressure will depend on height as given by Eq. (1). Find an explicit expression for exponent  $\alpha$  in terms of  $\gamma$ . [3]

$\alpha =$

- (d) According to this model what is the height to which the atmosphere extends? Take  $T_0 = 300$  K and  $p_0 = 1$  atm. [2]

- (e) The pressure at which vapor and liquid can co-exist is called the saturation vapor pressure  $p_s$ . The temperature dependence of  $p_s$  is given by the Clausius-Clapeyron equation

$$\frac{dp_s}{dT} = \frac{L}{T(v_2 - v_1)} \quad (2)$$

Where  $L$  is the latent heat of vaporisation and  $v_2$  and  $v_1$  are the specific volumes (volume per mass) of vapor and liquid respectively. Obtain an expression for  $p_s$  in terms of temperature  $T$ , gas constant  $R$ , molar mass of water vapour  $m_v$  and  $L$ . You may assume that water vapour also obeys ideal gas law. You can also use the fact that  $v_2 \gg v_1$  and ignore  $v_1$  in Eq. (2).

**[3]** $p_s =$ 

- (f) State the condition for condensation in terms of condensation height  $z_c$ .

**[1]**

4. A long solenoid of length  $l = 2.0$  m, radius  $r = 0.1$  m and total number of turns  $N = 1000$  is carrying a current  $i_0 = 20.0$  A. The axis of the solenoid coincides with the  $z$ -axis. [Marks: 14]

- (a) State the expression for the magnetic field of the solenoid and calculate its value? [1½]

Magnetic field =

Value of magnetic field =

- (b) Obtain the expression for the self-inductance ( $L$ ) of the solenoid. Calculate its value. [1½]

$L =$

Value of  $L =$

- (c) Calculate the energy stored ( $E$ ) when the solenoid carries this current? [1]

$E =$

- (d) Let the resistance of the solenoid be  $R$ . It is connected to a battery of emf  $e$ . Obtain the expression for the current ( $i$ ) in the solenoid. [2]

$i =$

- (e) Let the solenoid with resistance  $R$  described in part (d) be stretched at a constant speed  $v$  ( $l$  is increased but  $N$  and  $r$  are constant). State Kirchhoff's second law for this case. (**Note:** Do not solve for the current.) [2]

- 
- (f) Consider a time varying current  $i = i_0 \cos(\omega t)$  (where  $i_0 = 20.0\text{A}$ ) flowing in the solenoid. Obtain an expression for the electric field due to the current in the solenoid. (**Note:** Part (e) is not operative, i.e. the solenoid is not being stretched.) [3]

Electric field =

- (g) Consider  $t = \pi/2\omega$  and  $\omega = 200/\pi \text{ rad-s}^{-1}$  in the previous part. Plot the magnitude of the electric field as a function of the radial distance from the solenoid. Also, sketch the electric lines of force.

[3]

Plot:

Lines of force:

**5. Ionization of atoms**

A Rydberg hydrogenic atom is one in which the electron possesses a very large quantum number e.g.  $n = 100$ . Take the electron charge to be  $-e$  ( $e > 0$ ). The binding energy of the Rydberg electron may be taken as  $E_b = 10^{-3}$  eV. [Marks: 14]

- (a) Would a photon of angular frequency  $\omega_0 = 10^{10}$  rad·s<sup>-1</sup> ionize such an atom? [1]

Now consider the electron in the Rydberg hydrogenic atom to be unbounded and free for all practical purposes. Supposing such Rydberg atoms are injected uniformly into an oscillating electric field  $F_0 \cos \omega t \hat{k}$  provided by an electromagnetic wave. Let the speed of the electron at the time of injection ( $t = 0$ ), be  $v = 0$ .

- (b) Obtain an expression for the speed of the electron at a later time  $t$ . [2]

Speed of electron=

- (c) Obtain an expression for the average kinetic energy of the electron. [2]

Average kinetic energy =

- (d) Assuming that the criterion for photo-ionization is that the average kinetic energy exceed  $E_b$  estimate the value of the ionizing field ( $F_0$ ) for microwave radiation of angular frequency  $\omega_0 = 10^{10} \text{ rad}\cdot\text{s}^{-1}$ .

[2]

$F_0 =$

Next consider the ionization of an atom by a steady electric field. Consider a hydrogen atom at rest (at  $z = 0$ ) in a uniform steady electric field  $F_0 \hat{k}$ . We take the potential energy due to the electric field to be zero at  $z = 0$ .

- (e) Write down the expression for the potential energy of the electron in this field.

[1]

Potential energy =

- (f) For this and subsequent parts take  $x = y = 0$ . Sketch the potential energy due to

this field along  $z$ - axis and between  $-1 < z < 1$ . Identify important points. To make this plot select a system of units where  $e = 1$ ,  $4\pi\epsilon_0 = 1$  and  $F_0 = 20$ . [3]

Plot:

(g) Let the energy of the electron confined in the atom be  $E$ . At what  $F_0$  would the atom ionize? [2]

$F_0 =$

(h) If we take  $E = 10^{-3}$  eV, estimate the value of  $F_0$ . Is this physically acceptable? [1]

$F_0 =$



# Indian National Physics Olympiad - 2012

## Solutions

Please note that alternate/equivalent solutions may exist. Brief solutions are given below.

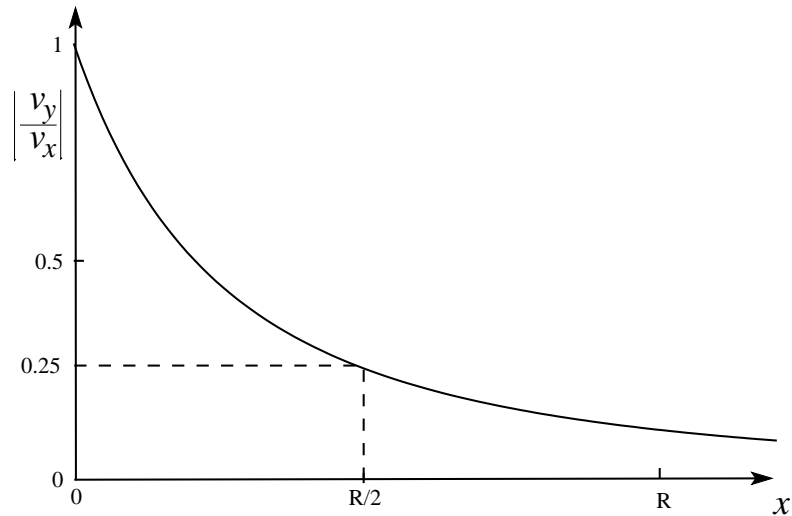
1. (a)  $\omega_{i+1} = \frac{7}{13}\omega_i + \frac{6}{13}\frac{v}{r}$   
 (b)  $\omega^* = \frac{v}{r}$

**Argument** : Initially  $\omega_i$  increases until it reaches a value  $v = \omega^*r$ , i.e. the speed of the falling ball. Thereafter the ball merely "touches" the sphere and does not impart it any momentum.

(c)  $\omega_i = \frac{v}{r} \left( 1 - \left( \frac{7}{13} \right)^i \right) \quad i=0,1,2,3,\dots$   
 or  $\omega_i = \frac{v}{r} \left( 1 - \left( \frac{7}{13} \right)^{i-1} \right) \quad i=1,2,3,\dots$

(d)  $\omega^* = \frac{v}{r}$

2. (a)  $v_y = \frac{R^2}{(2x + R)^2}v_x$   
 (b) See figure below:



(c) Speed = 2.22 km · hr<sup>-1</sup>

3. (a)  $\Gamma = \frac{m_a g (\gamma - 1)}{R \gamma}$   
 (b) For  $m_a = 29.0 \text{ kg} \cdot \text{kmol}^{-1}$  ;  $\Gamma =$  approx 10 K · km<sup>-1</sup>  
 (c)  $\alpha = \frac{\gamma}{\gamma - 1}$   
 (d) approximately 30.0 km  
 (e)  $p_s = p_{s0} \exp \left[ \frac{Lm_v}{R} \left( \frac{1}{T_{s0}} - \frac{1}{T} \right) \right]$   
 where  $T_{s0}$  and  $p_{s0}$  are the initial points for the integration. A convenient choice would be the triple point of water.

(f) At  $z_c$  atmospheric pressure should be equal to saturation pressure. Condition is

$$p_0 \left( \frac{T_0 - \Gamma z_c}{T_0} \right)^{\gamma/1-\gamma} = p_{s0} \exp \left[ \frac{Lm_v}{R} \left( \frac{1}{T_{s0}} - \frac{1}{T_0 - \Gamma z_c} \right) \right]$$

4. (a) Magnetic field = 
$$\begin{cases} \frac{\mu_0 N I}{l} \hat{k} & \rho < r \\ 0 & \rho > r \end{cases}$$

Value of magnetic field = 
$$\begin{cases} 1.26 \times 10^{-2} \text{ T} & \rho < r \\ 0 & \rho > r \end{cases}$$

where  $\rho$  is the radial distance.

(b) 
$$L = \frac{\mu_0 N^2 \pi r^2}{l}$$

Value of  $L = 1.97 \times 10^{-2} \text{ H}$

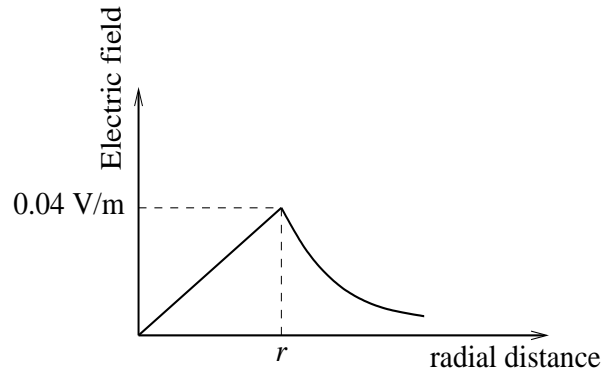
(c)  $E = 3.95 \text{ J}$

(d) 
$$i = \frac{e}{R} (1 - e^{-Rt/L}) + i_0 e^{-Rt/L} \quad \text{if } i_0 \neq 0$$

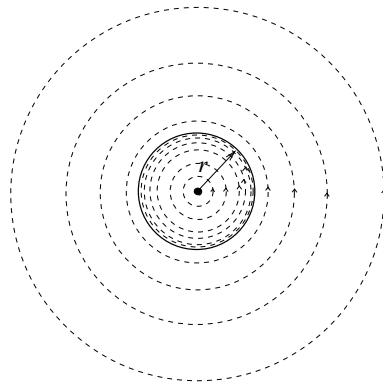
(e) 
$$e = iR + L \frac{di}{dt} - i \frac{Lv}{l + vt}$$
  
 where  $L = \mu_0 N^2 \pi r^2 / (l + vt)$

(f) Electric field = 
$$\begin{cases} \frac{\mu_0 N i_0 \omega \rho}{2l} \sin(\omega t) & \rho < r \\ \frac{\mu_0 N i_0 \omega r^2}{2\rho l} \sin(\omega t) & \rho > r \end{cases}$$

(g) The plot of  $E$  with radial distance:



Lines of forces: Note, the lines of force are dense upto  $\rho = r$  and increasingly sparse thereafter.



5. (a) Since  $\hbar\omega_0 < E_b$ , hence no ionisation by a single photon is possible.

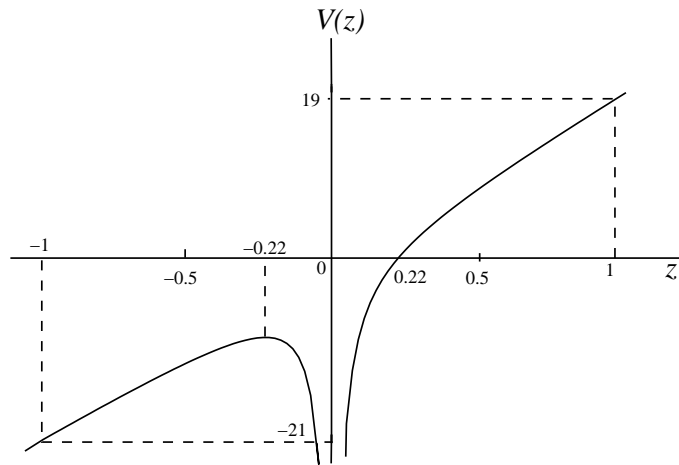
(b) Speed of electron =  $\left| -\frac{eF_0}{m\omega} \sin(\omega t) \right|$

(c) Average kinetic energy =  $\frac{e^2 F_0^2}{4m\omega^2}$

(d)  $F_0 = 1.5 \times 10^3 \text{ V} \cdot \text{m}^{-1}$

(e) Potential energy =  $-\frac{e^2}{4\pi\epsilon_0 r} + eF_0 z$

(f) See figure below:



(g)  $F_0 = \frac{E^2 \pi \epsilon_0}{e^3}$

(h) approx  $174 \text{ V} \cdot \text{m}^{-1}$  which is physically possible.